

QUANTUM GENERALIZED HYDRODYNAMICS

PAOLA RUGGIERO

With: P. Calabrese, B. Doyon, J. Dubail

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June 11th, 2020



UNIVERSITÉ
DE GENÈVE

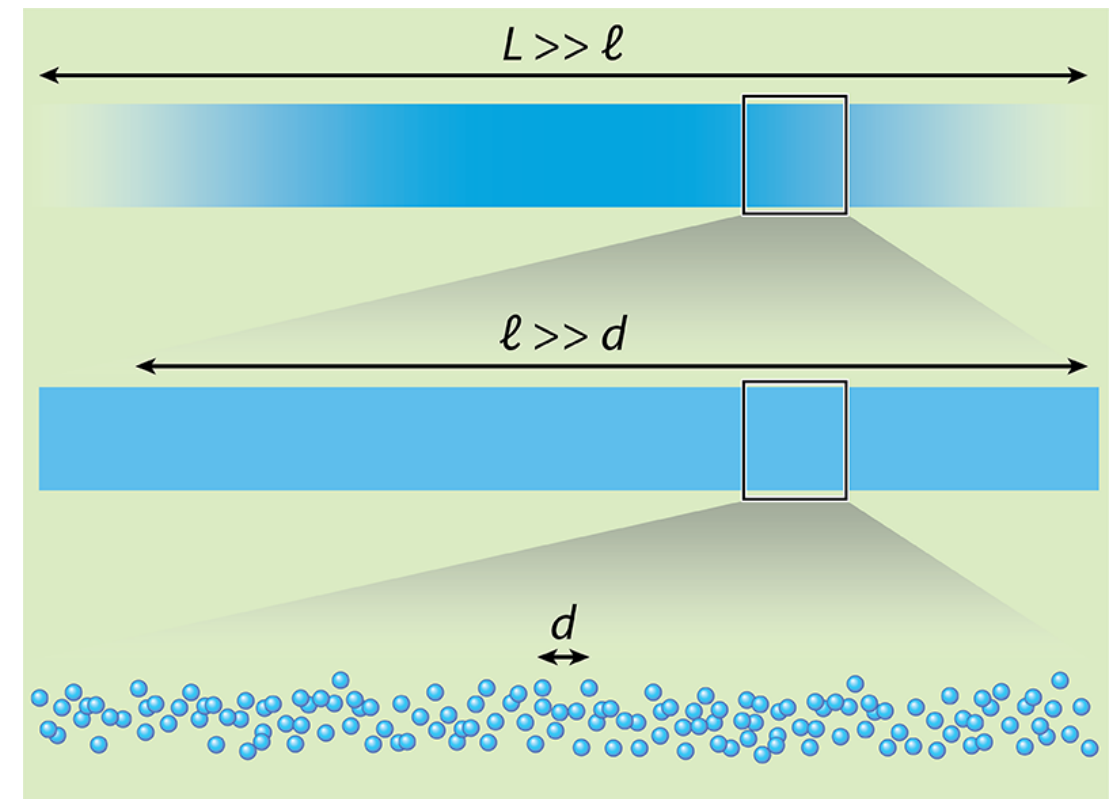
HYDRODYNAMICS: THE EULER SCALE

SEPARATION OF LENGTH SCALES

- Macroscopic scale : $L = \rho / \partial_x \rho$
- Microscopic scale : d
- **MESOSCOPIC scale:** $d \ll \ell \ll L$

SEPARATION OF TIME SCALES

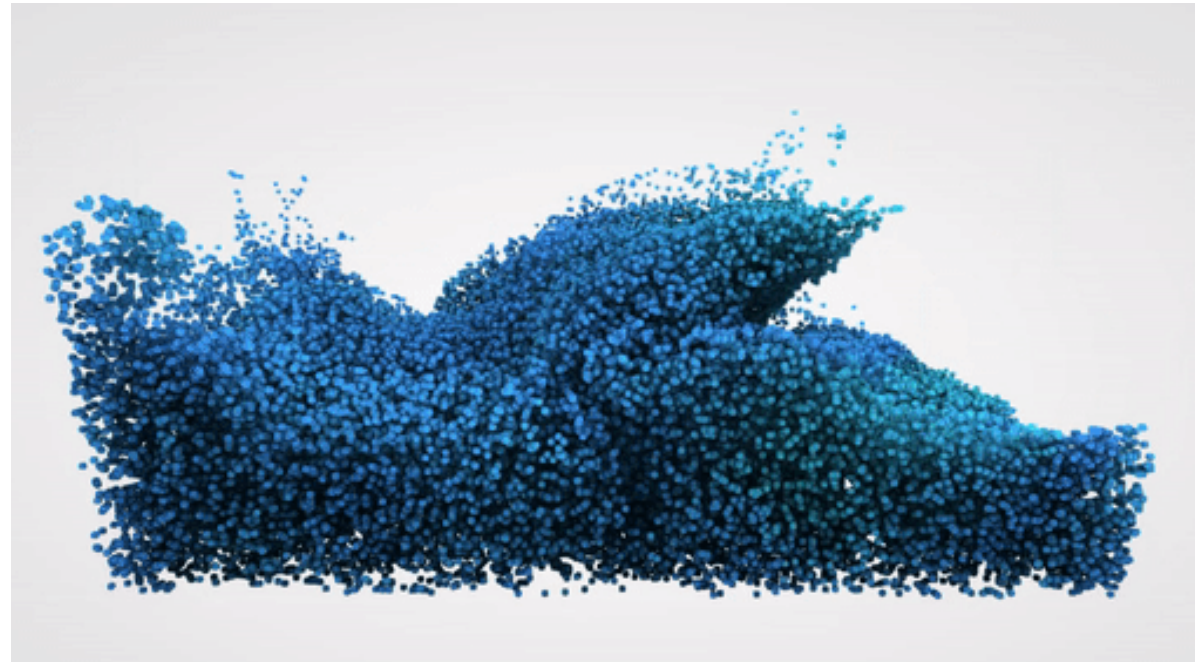
- Macroscopic time : $T = \rho / \partial_t \rho$
- Microscopic time : τ_{relax}
- **MESOSCOPIC time:** $\tau_{relax} \ll t \ll T$



[PICTURE FROM: DUBAIL, PHYSICS 9, 153]

H. SPOHN, *Large Scale Dynamics of Interacting Particles* (1991)

(CONVENTIONAL) HYDRODYNAMICS



$$\langle O(x) \rangle_t = \langle O(x) \rangle_{\rho_{Gibbs}}, \quad \rho_{Gibbs} = \rho_{Gibbs}(x, t)$$

CONSERVATION LAWS:

- Mass
- Momentum
- Energy



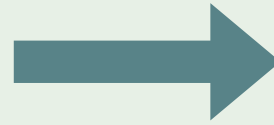
CONTINUITY EQUATIONS:

$$\begin{cases} \partial_t \rho + \partial_x j & = 0 \\ \partial_t \rho_P + \partial_x j_P & = 0 \\ \partial_t \rho_E + \partial_x j_E & = 0. \end{cases}$$

GENERALIZED HYDRODYNAMICS (GHD)

CONSERVATION LAWS:

$$Q_i = \int q_i(x) dx \quad i = 1, 2, \dots$$



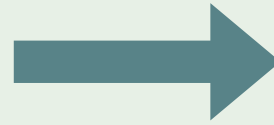
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EIGENSTATES:

$$|\theta_1, \dots, \theta_N\rangle$$

"Rapidities"

TDL



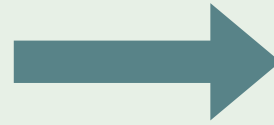
$$n(\theta)$$

"Occupation function"

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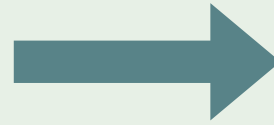
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$$\partial_t n(\theta) + v^{eff}(\theta) \partial_x n(\theta) = 0 \quad + \quad v^{eff}(\theta) = \frac{(E')^{dr}(\theta)}{(p')^{dr}(\theta)}$$

$$f^{dr}(\theta) = f(\theta) + \int \frac{d\theta'}{2\pi} \frac{d\phi(\theta - \theta')}{d\theta} n(\theta') f^{dr}(\theta')$$

CASTRO-ALVAREDO, DOYON, YOSHIMURA, PRX 6, 041065 (2016)
BERTINI, COLLURA, DE NARDIS, FAGOTTI, PRL 117, 207201 (2016)

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"FREE" GHD

$$\theta \rightarrow p, \quad v^{eff} \rightarrow p,$$

$$n(x, \theta; t) \rightarrow n(x, p; t)$$

$$\langle O(x) \rangle_t = \langle O(x) \rangle_{stat, \rho_{GGE}}$$

GHD \equiv Evolution Wigner function

BETTELHEIM, ABANOV, WIEGMANN, *PRL* **97**, 246402 (2006)

BETTELHEIM, ABANOV, WIEGMANN, *J. PHYS. A* **41** 392003 (2008)

BETTELHEIM, WIEGMANN, *PHYS. REV. B* **84**, 085102 (2011)

BETTELHEIM, GLAZMAN, *PRL* **109**, 260602 (2012)

PROTOPOPOV, GUTMAN, SCHMITTECKERT,

MIRLIN, *PRB* **87**, 045112 (2013)

$$\partial_t n(\theta) + v^{eff}(\theta) \partial_x n(\theta) = 0 \quad +$$

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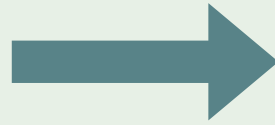
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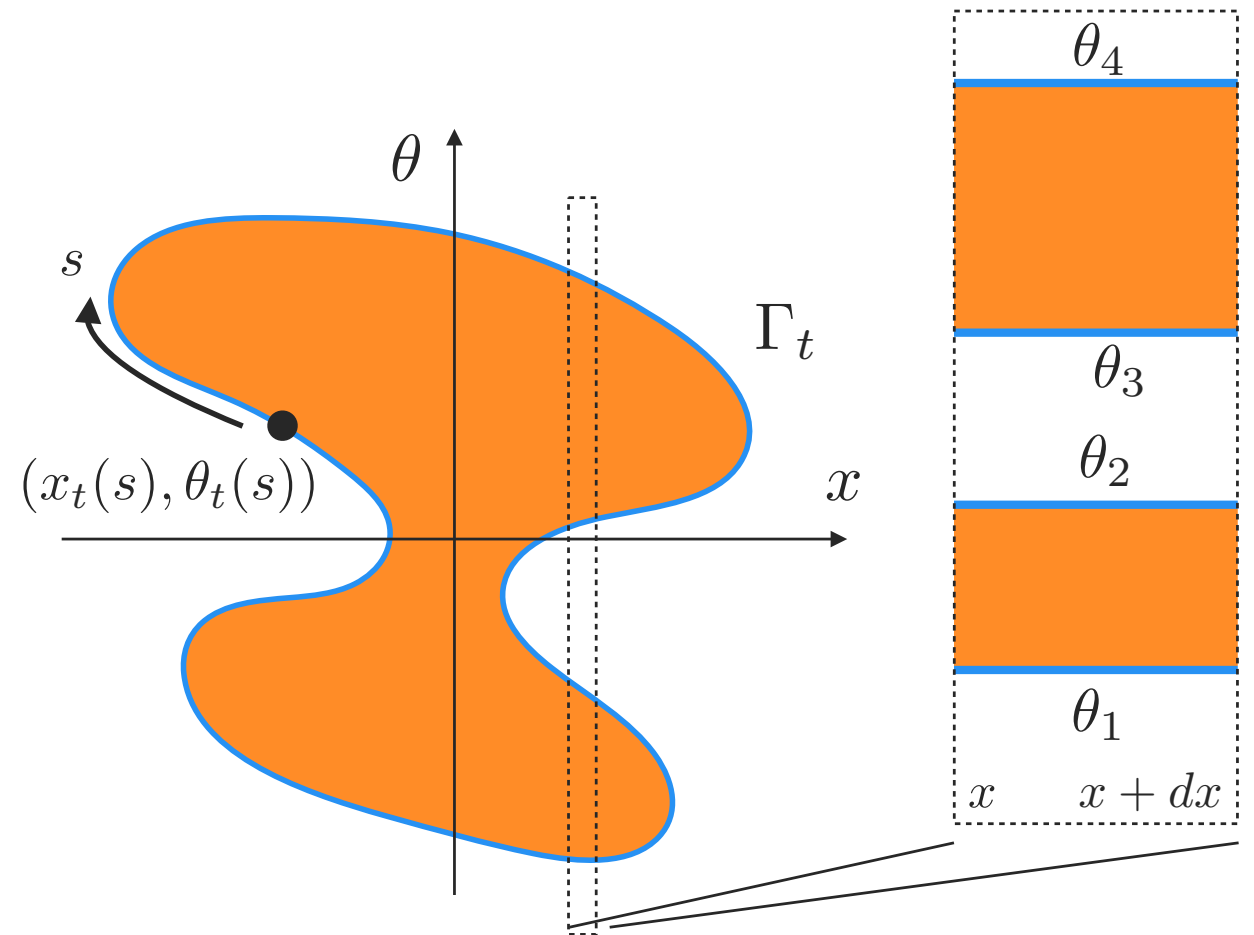
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ZERO-ENTROPY GHD

DOYON, DUBAIL, KONIK, YOSHIMURA, PRL119, 195301 (2017)

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$$n(x, \theta) = \begin{cases} 1 & \text{if } (x, \theta) \text{ in } \Gamma_t \\ 0 & \text{otherwise.} \end{cases}$$



DOYON, DUBAIL, KONIK, YOSHIMURA, PRL119, 195301 (2017)

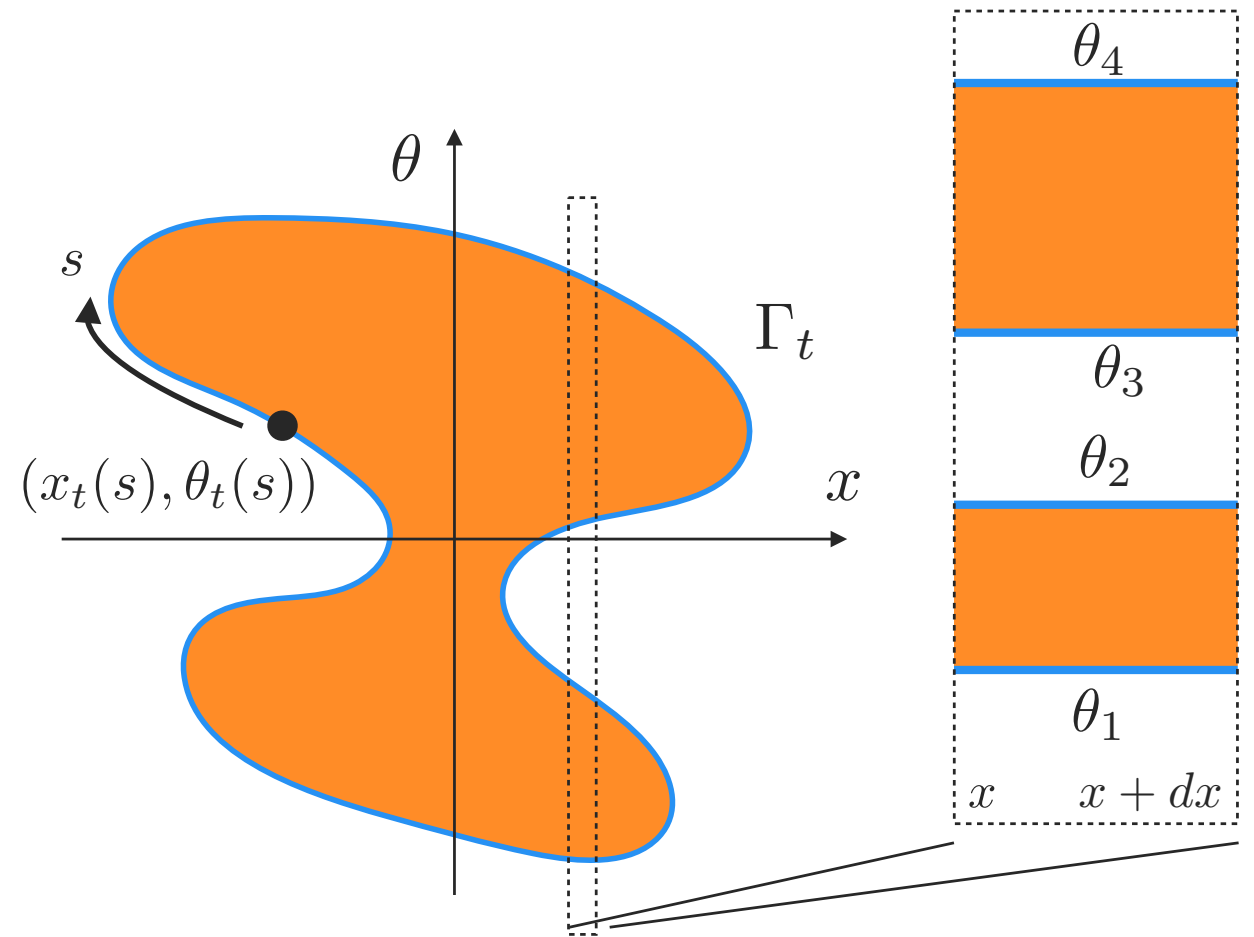


ZERO-ENTROPY GHD

EXAMPLES:

Ground states within inhomogeneous potentials

$$n(x, \theta) = \begin{cases} 1 & \text{if } (x, \theta) \text{ in } \Gamma_t \\ 0 & \text{otherwise.} \end{cases}$$



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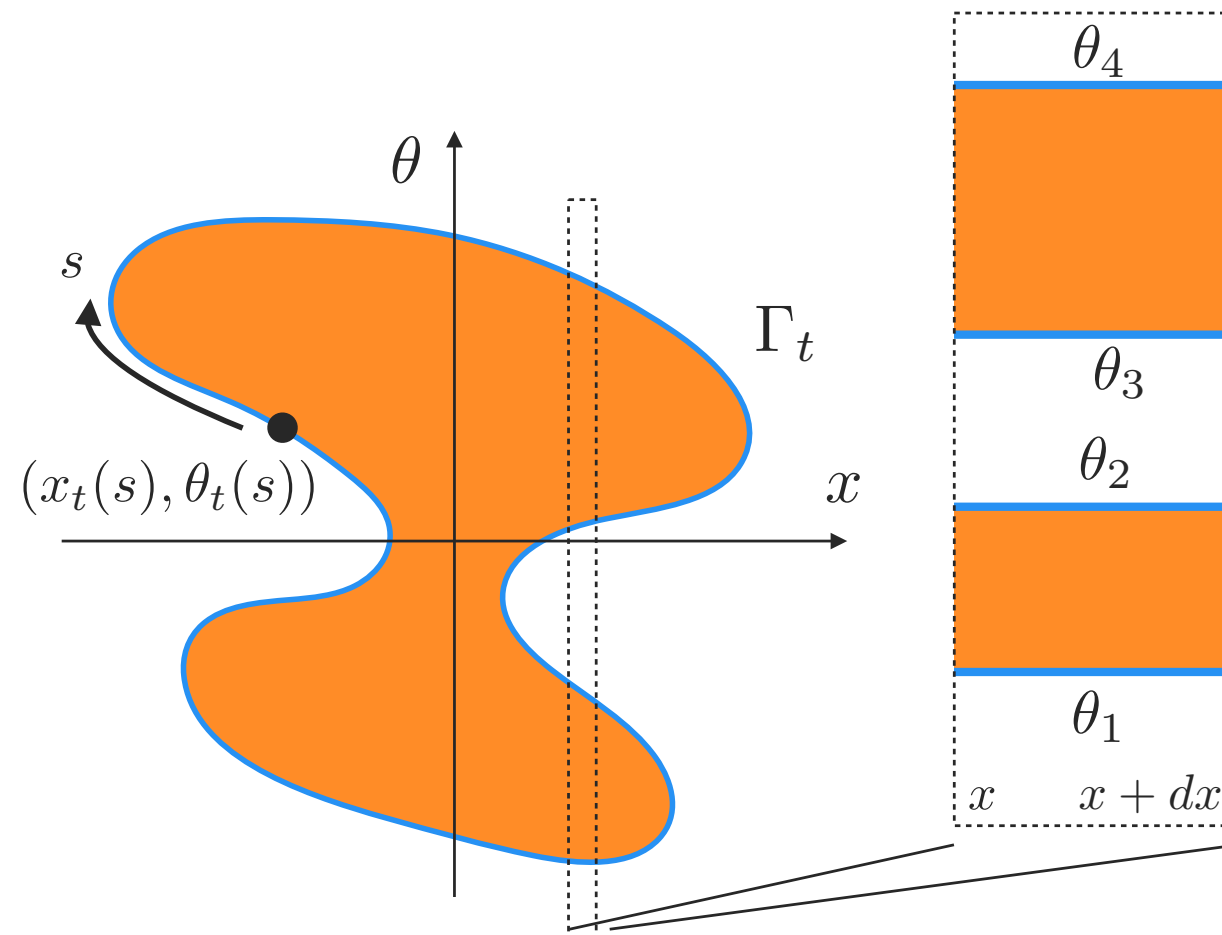
"Split Fermi seas"

FOKKEMA, ELIENS, CAUX, PRA 89, 033637 (2014)

ELIENS, CAUX, J.PHYS.A 49, 495203 (2016)

VLIJM, ELIENS, CAUX, SCIPost 1, 008 (2016)

$$\theta \in [\theta_1, \theta_2] \cup \dots \cup [\theta_{2q-1}, \theta_{2q}]$$



DOYON, DUBAIL, KONIK, YOSHIMURA, PRL119, 195301 (2017)

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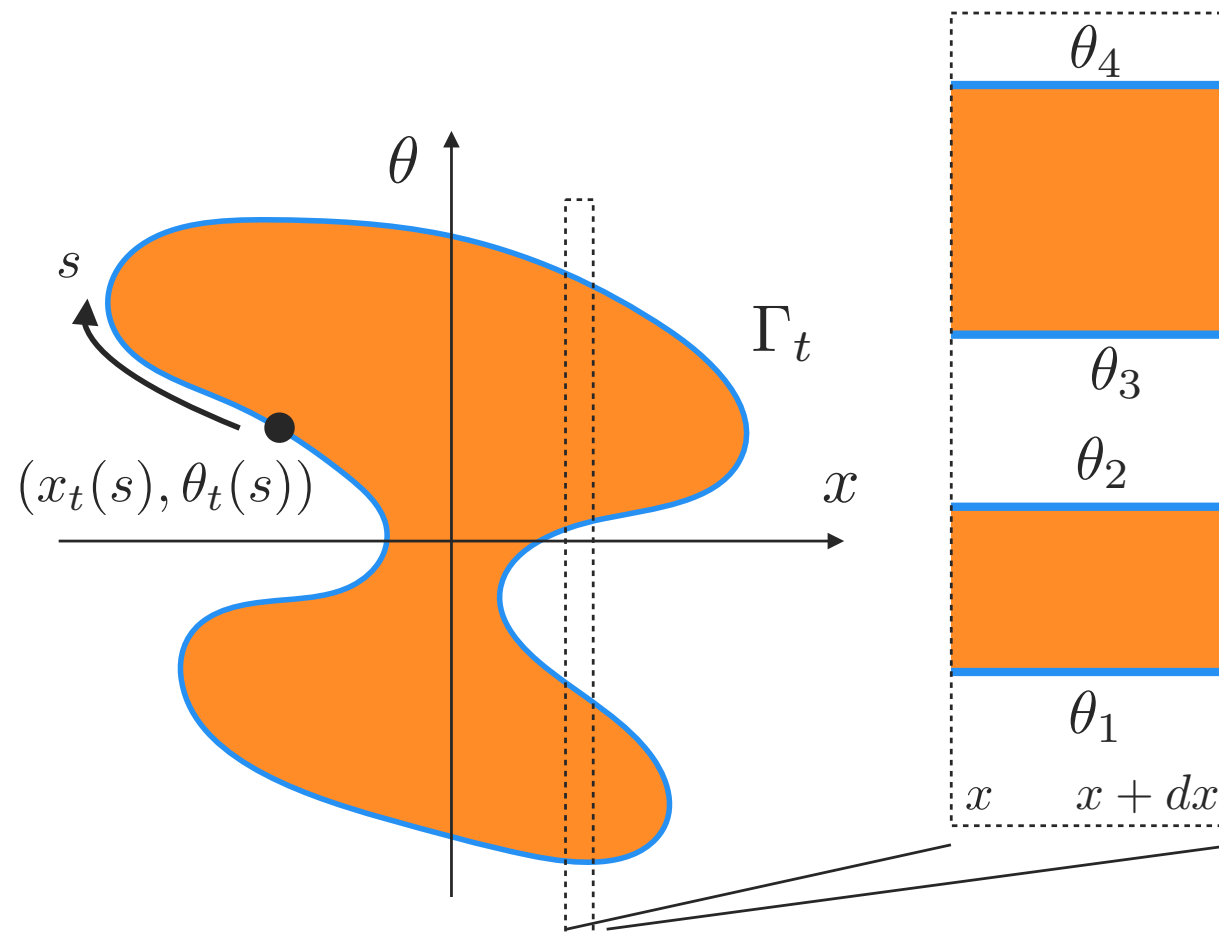
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FINITE-DIMENSIONAL GHD

$$\partial_t \theta_j + v_{\{\theta\}}^{eff}(\theta_j) \partial_x \theta_j = 0$$

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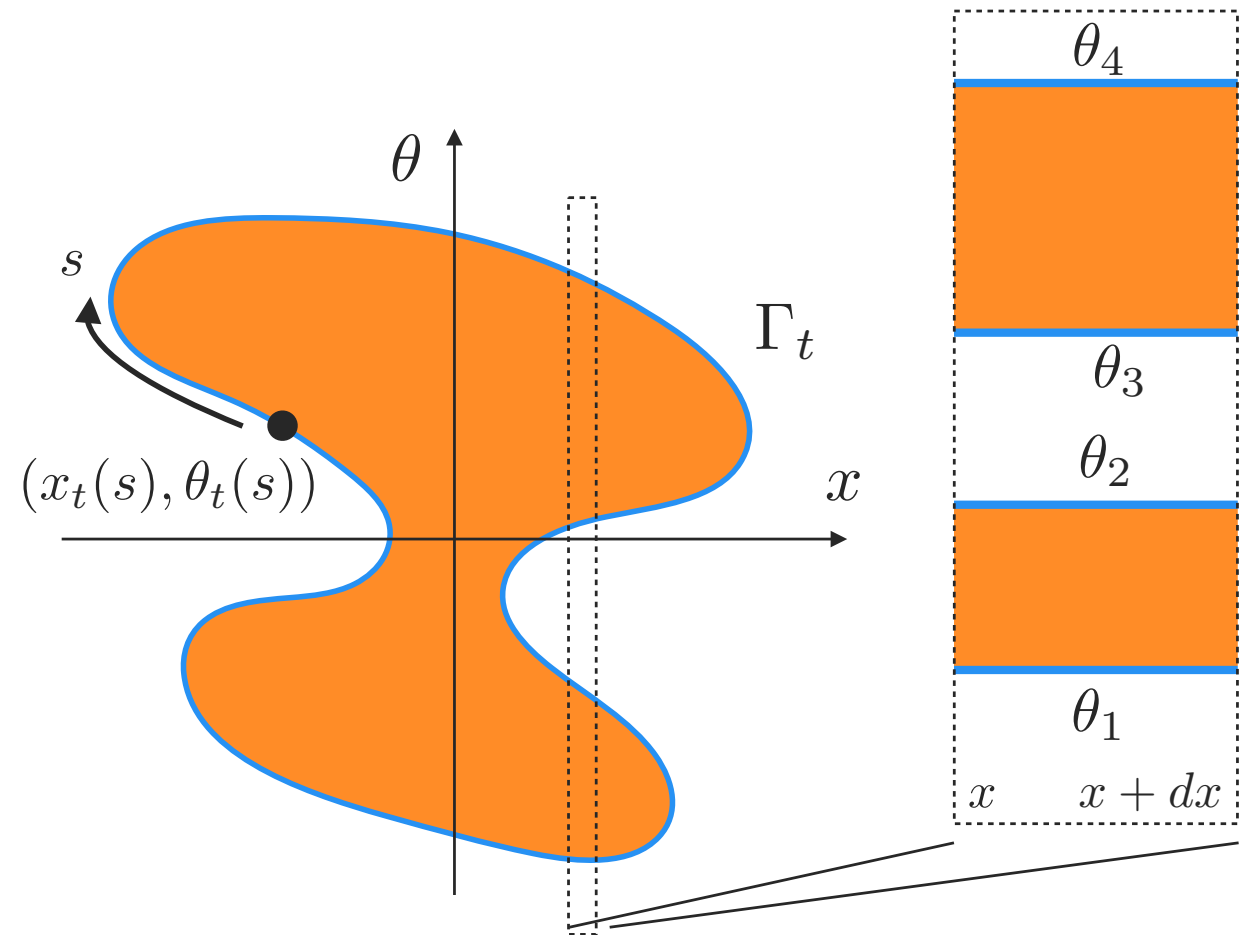
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Conventional HD!

DOYON, DUBAIL, KONIK, YOSHIMURA, PRL 119, 195301 (2017)

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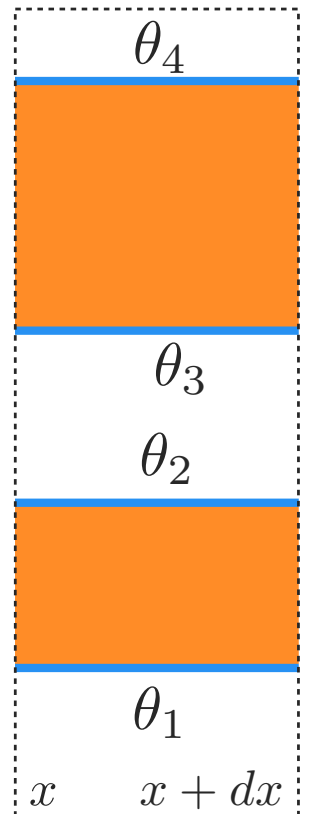
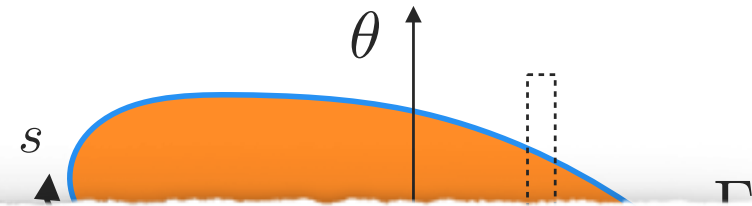
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BULCHANDANI, VASSEUR, KARRASCH, MOORE, PRL 119, 220604 (2017)
 DOYON, YOSHIMURA, SciPost 2, 014 (2017)
 DOYON AND H. SPOHN, J. STAT. 2017, 073210 (2017)
 FAGOTTI, PRB 96, 220302 (2017)
 GOPALAKRISHNAN, HUSE, KHEMANI, VASSEUR, PRB 98, 220303 (2018)
 DE NARDIS, BERNARD, DOYON, PRL 121, 160603 (2018)
 BERTINI, FAGOTTI, PIROLI, CALABRESE, J. PHYS. A 51, 39LT01 (2018)
 ILIEVSKI, DE NARDIS, MEDENJAK, PROSEN, PRL 121, 230602 (2018)
 BASTIANELLO, ALBA, CAUX, PRL 123, 130602 (2019)
 PANFIL, PAWELCZYK, SciPost CORE 1, 002 (2019)
 SCHEMMER, BOUCHOULE, DOYON, DUBAIL, PRL 122, 090601 (2019)
 MOLLER, SCHMIEDMAYER, SciPost 8, 041 (2020)
 FAGOTTI, SciPost PHYS. 8, 048 (2020)

...

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DOYON, DUBAIL, KONIK, YOSHIMURA, PRL 119, 195301 (2017)

"QUANTUM" HYDRODYNAMICS ?

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CLASSICAL EQUATIONS



QUANTIZE THEM

"QUANTUM" HYDRODYNAMICS ?



CLASSICAL EQUATIONS



QUANTIZE THEM

- **Superfluidity** LANDAU, *PHYS. REV.* 60, 356 (1941)
- **Bose-Einstein condensates** BOGOLYUBOV, *J. PHYS.* 11 (1947), 23
- **Superconductivity** BARDEEN, COOPER, SCHRIEFFER, *PHYS REV.* 104 (4)
- **Hall liquids** WIEGMANN, ABANOV, *PRL* 113, 034501 (2014)
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Different problem:

Corrections
to GHD equations:

FAGOTTI,
SCIPOST PHYS. 8, 048 (2020)

DEAN, LE DOUSSAL, MAJUMDAR, SCHEHR,
EPL, 126 20006 (2019)

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≠

HERE:

Look at propagation of linear
sound waves on top of GHD
and quantise them:

! We talk about "**quantum fluids**"
in the same sense in which
a Luttinger Liquid is a quantum fluid !

QUANTUM GENERALIZED HYDRODYNAMICS

$$H = \sum_{i=1}^N \left[-\frac{\hbar^2}{2} \partial_{x_i}^2 + V(x_i) \right] + g \sum_{i < j} \delta(x_i - x_j) \quad n(x, \theta) = \begin{cases} 1 & \text{if } (x, \theta) \text{ in } \Gamma_t \\ 0 & \text{otherwise.} \end{cases}$$

QUANTUM GENERALIZED HYDRODYNAMICS

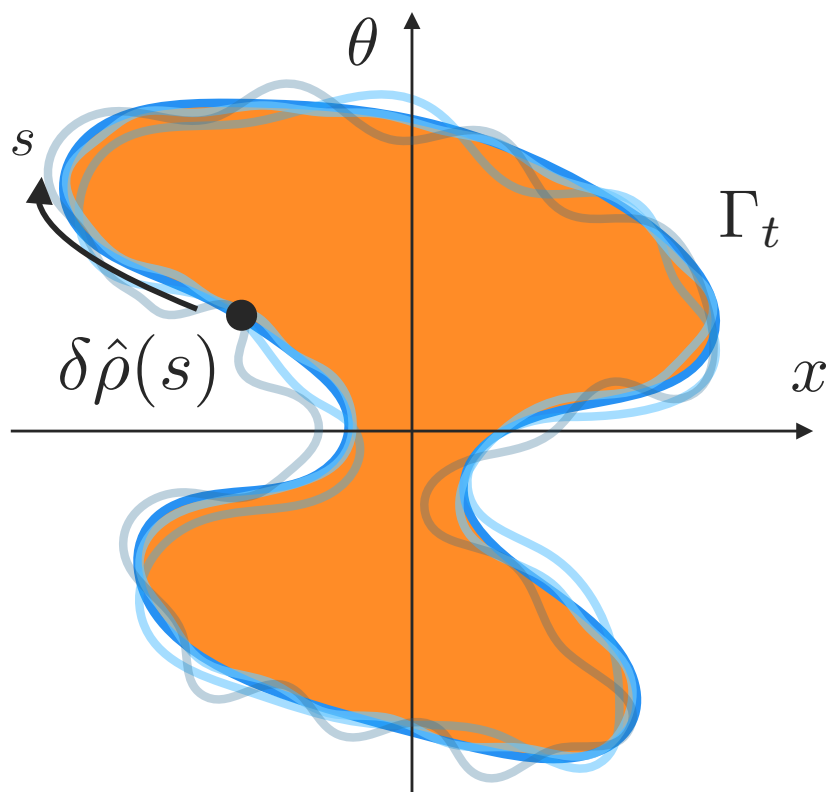
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- PLAN:**
1. Look at *fluctuations* around a (zero-entropy) GHD background
 2. Quantize them in a semiclassical fashion.

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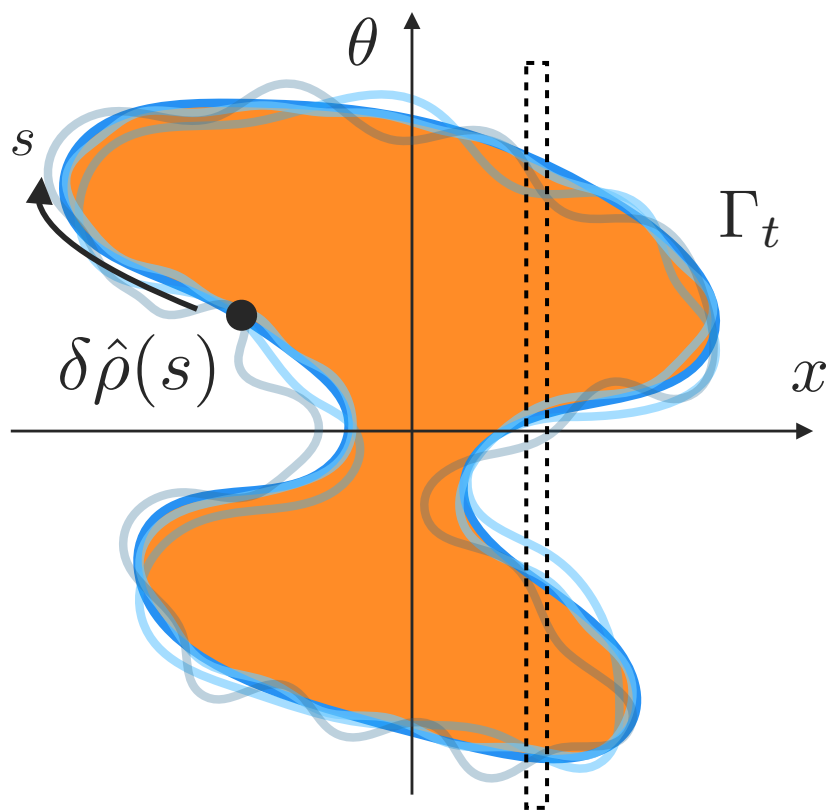
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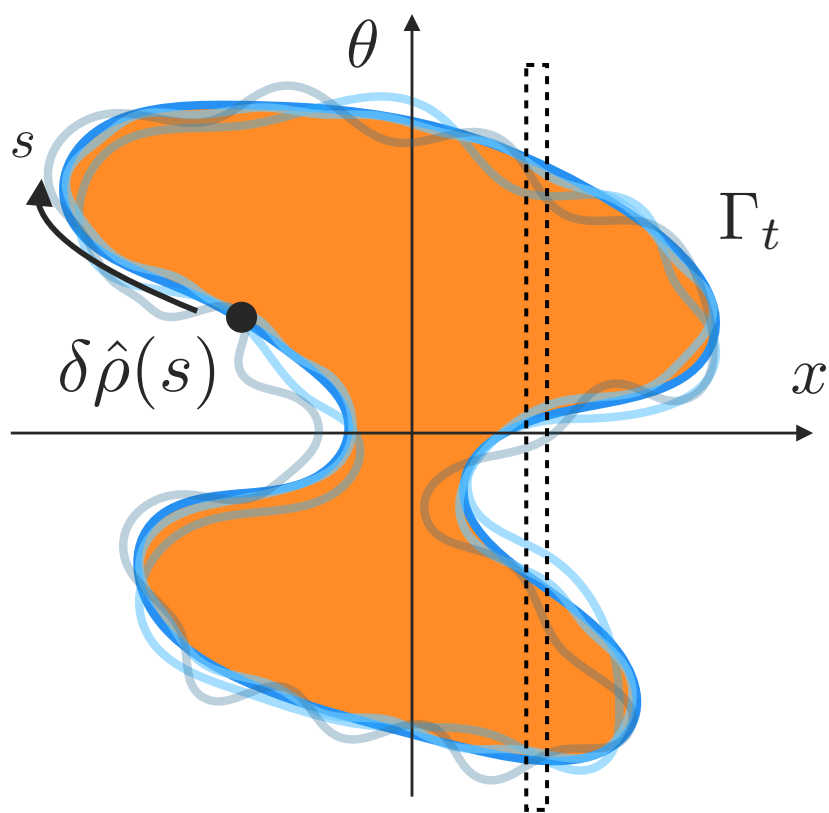


1. $\theta_a \rightarrow \theta_a + \delta\theta_a$

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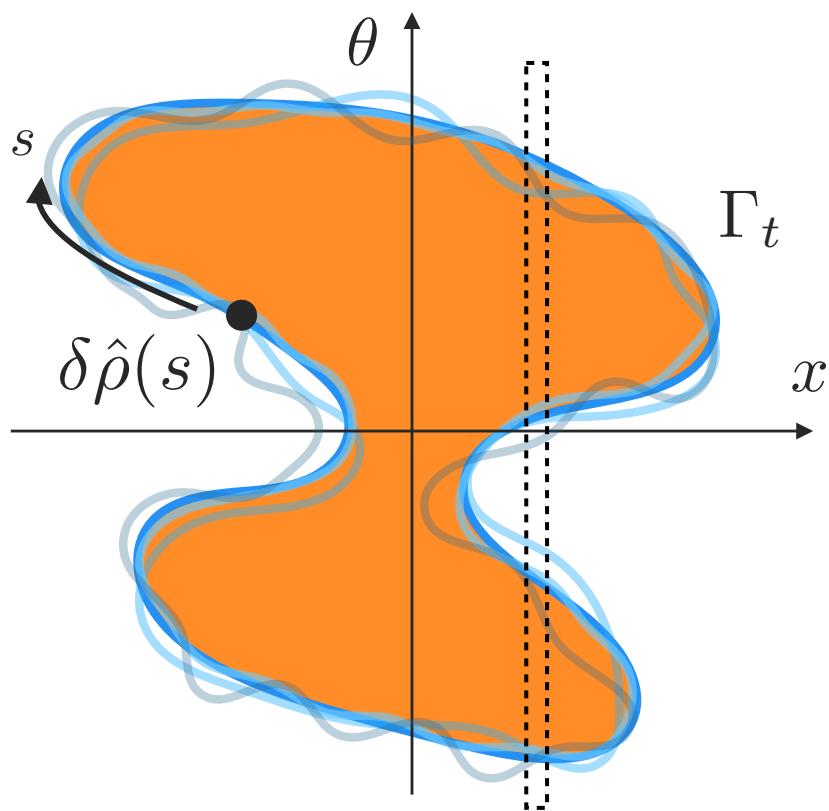


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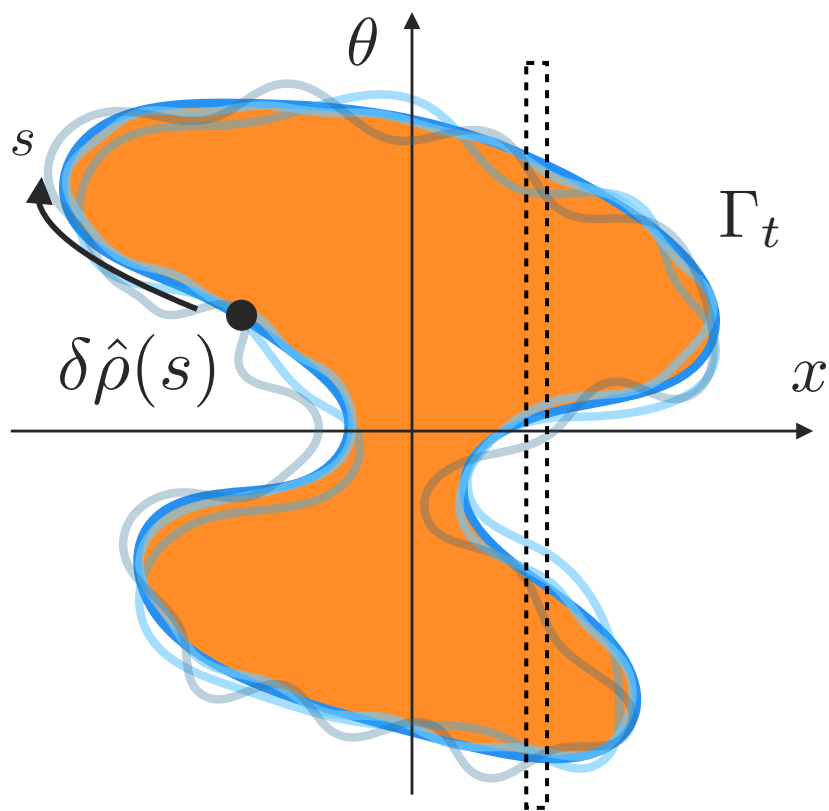
$$\partial_t p_a + \partial_x \epsilon_a = 0$$

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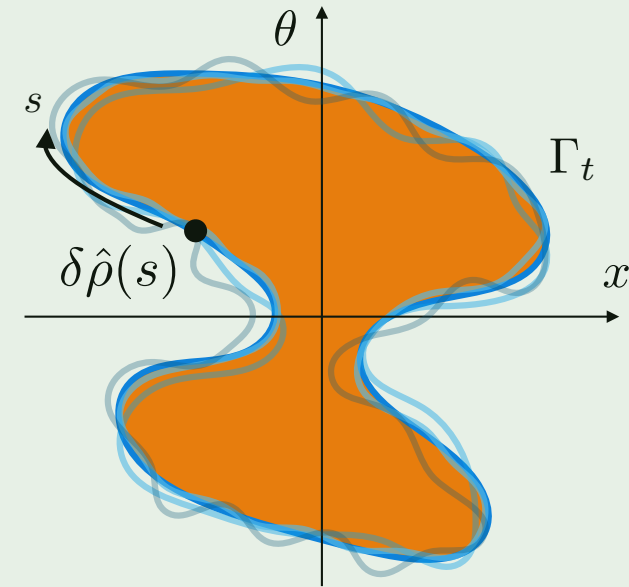
$$2. \delta p_a \rightarrow \delta \hat{p}_a : e^{iS} \approx e^{iS_{\text{classical}} + i \sum_{ab} S_{ab}^{(2)} \delta p_a \delta p_b}$$

QUANTUM GENERALIZED HYDRODYNAMICS

"QGHD HAMILTONIAN"

$$\hat{H}[\Gamma_t] = \frac{1}{4\pi\hbar} \int dx \sum_{a,b} \delta\hat{p}_a(x) A^{ab} \delta\hat{p}_b(x)$$

$$[\delta\hat{p}_a(x), \delta\hat{p}_b(y)] = -i\sigma_a 2\pi\hbar^2 \delta_{ab} \delta'(x-y)$$

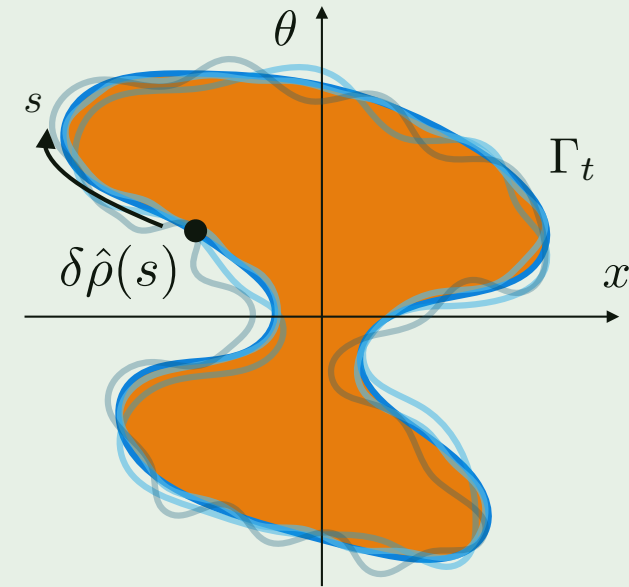


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Time-dependent, spatially inhomogeneous, multicomponent
LUTTINGER LIQUID

QUANTUM (conventional) HYDRODYNAMICS

$$H = \sum_{i=1}^N \left[-\frac{\hbar^2}{2} \partial_{x_i}^2 + V(x_i) \right] + g \sum_{i<j} \delta(x_i - x_j), \quad g \rightarrow \infty$$

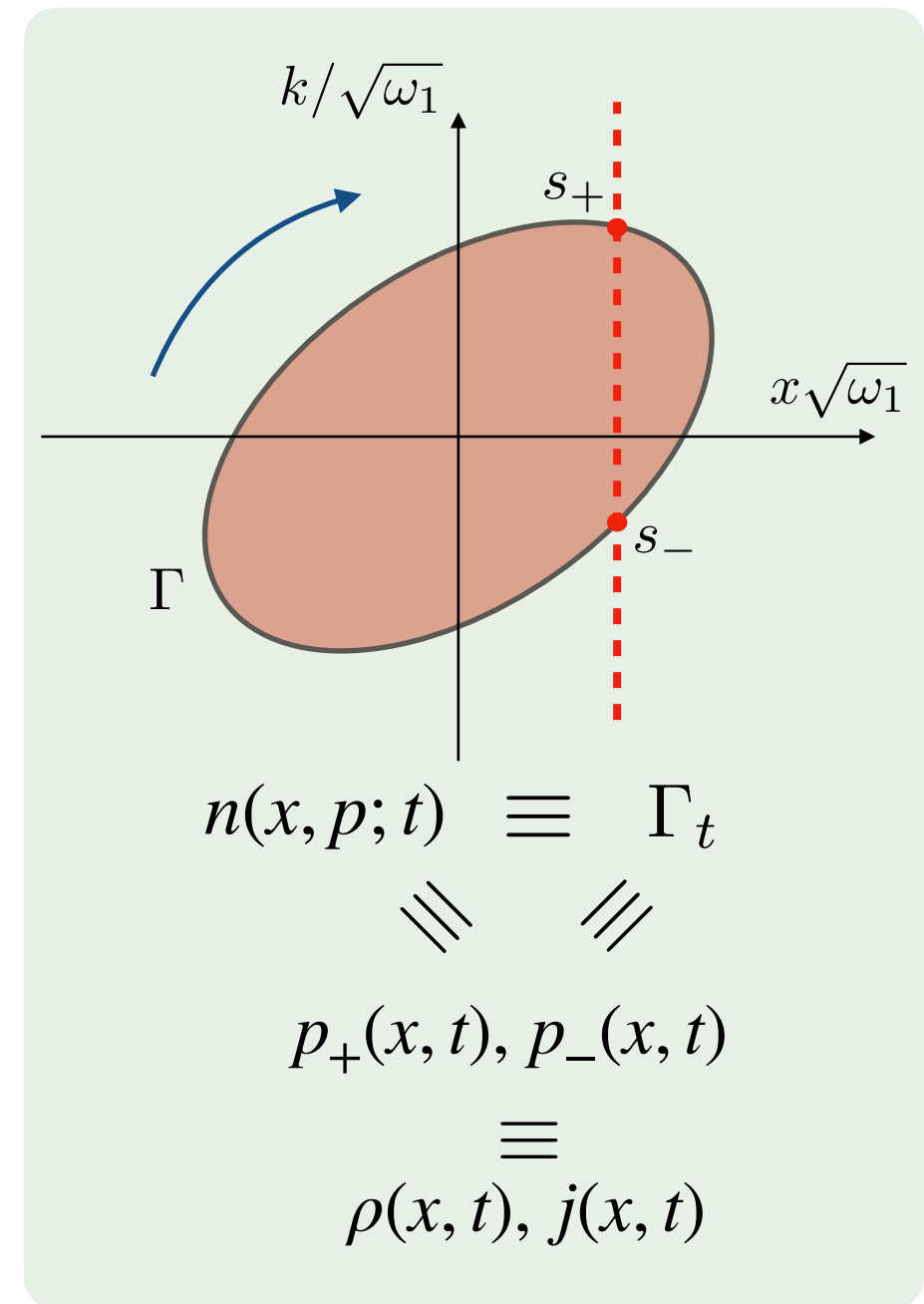
$$V(x) = \frac{\omega^2 x^2}{2}, \quad \omega_0 \rightarrow \omega_1$$

PLAN: 1. Look at fluctuations around hydrodynamics
2. Quantize them in a semiclassical fashion.

$$1. (\rho, j) \rightarrow (\rho + \delta\rho, j + \delta j)$$

$$2. (\delta\rho, \delta j) \rightarrow (\delta\hat{\rho}, \delta\hat{j}) : e^{iS} \approx e^{iS^{(0)} + iS^{(2)} + \dots}$$

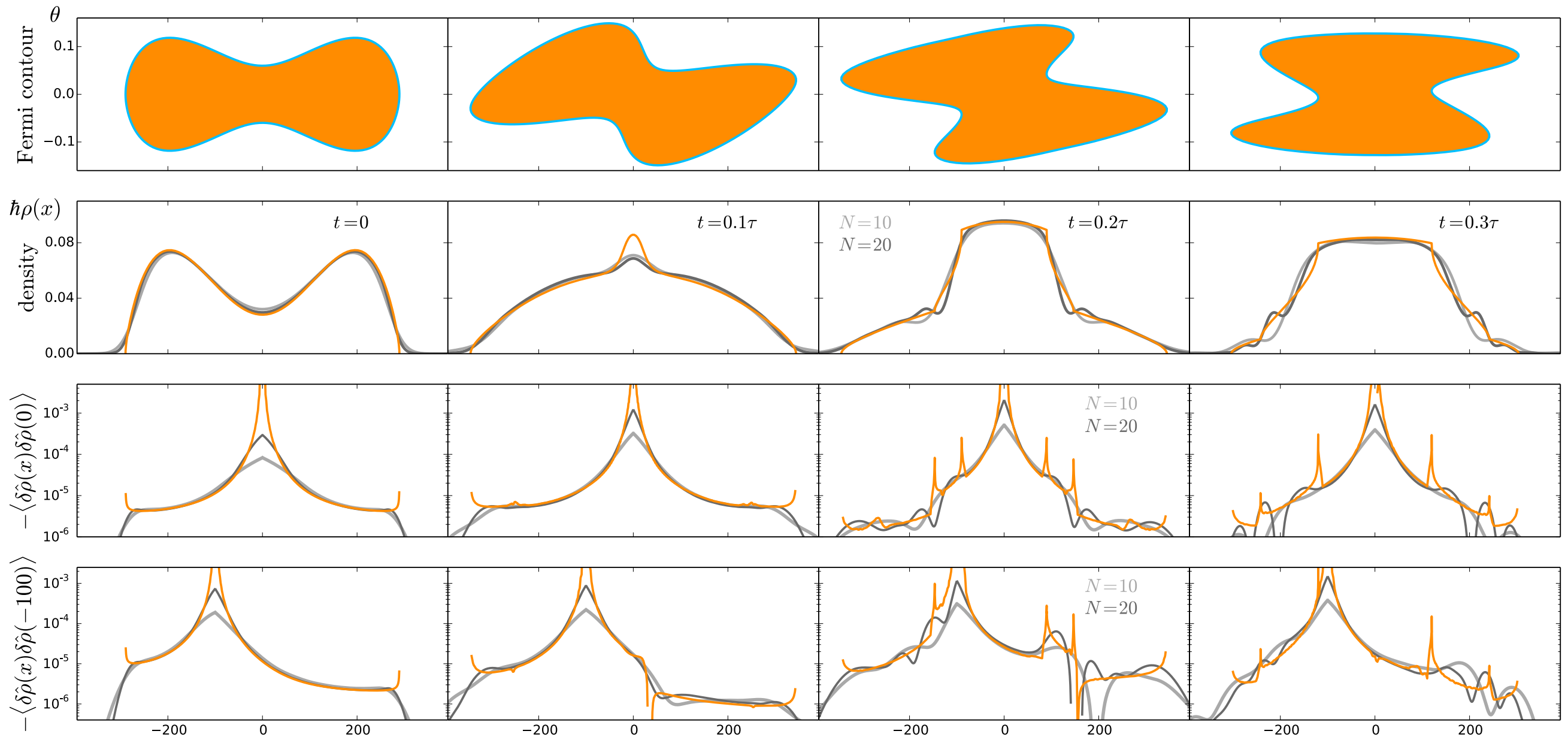
$$S^{(2)} = \frac{1}{8\pi} \int dx dt \left[\frac{m}{\pi\rho} \delta j^2 - 2 \frac{mj}{\pi\rho^2} \delta\rho \delta j + \left(\frac{mj^2}{\pi\rho^3} - \frac{1}{\pi} \partial_\rho^2 \varepsilon \right) \delta\rho^2 \right]$$



**Time-dependent,
inhomogeneous
LUTTINGER LIQUID**

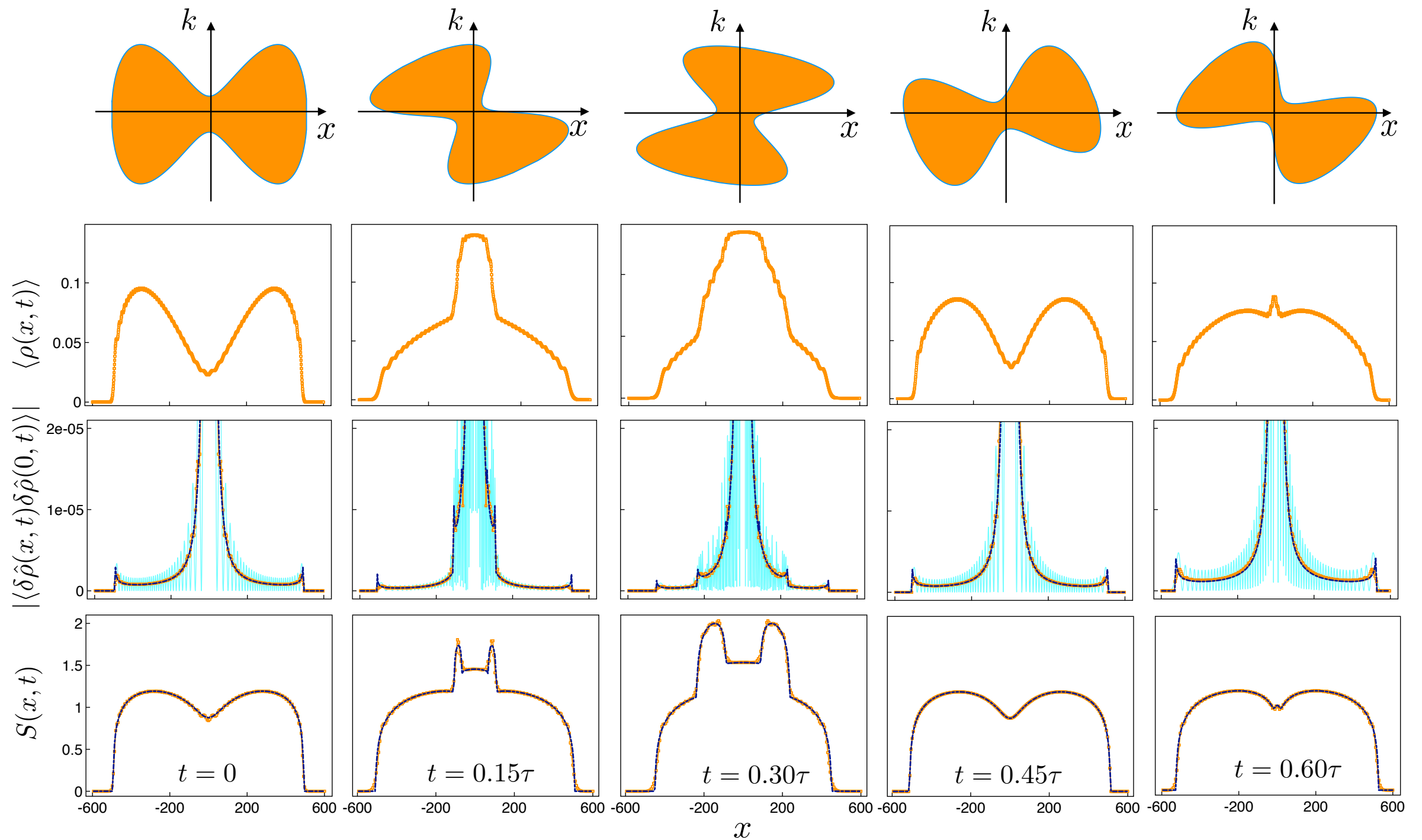
QGHD: Numerical check in the Lieb-Lininger model

Quench: $V_0(x) = a_4x^4 - a_2x^2 \rightarrow V(x) = \omega^2x^2/2$



PR, CALABRESE, DOYON, DUBAIL, PRL 124, 140603 (2020)

FREE QGHD: checks in the Tonks-Girardeau limit



PR, CALABRESE, DOYON, DUBAIL, IN PREPARATION

SUMMARY

1. GHD provides a more efficient way to describe interacting quantum particles in 1D
2. Still, it misses important quantum effects
3. QGHD gives a way to construct quantum fluctuations around GHD
4. This is not the end of the story...

THANK YOU.