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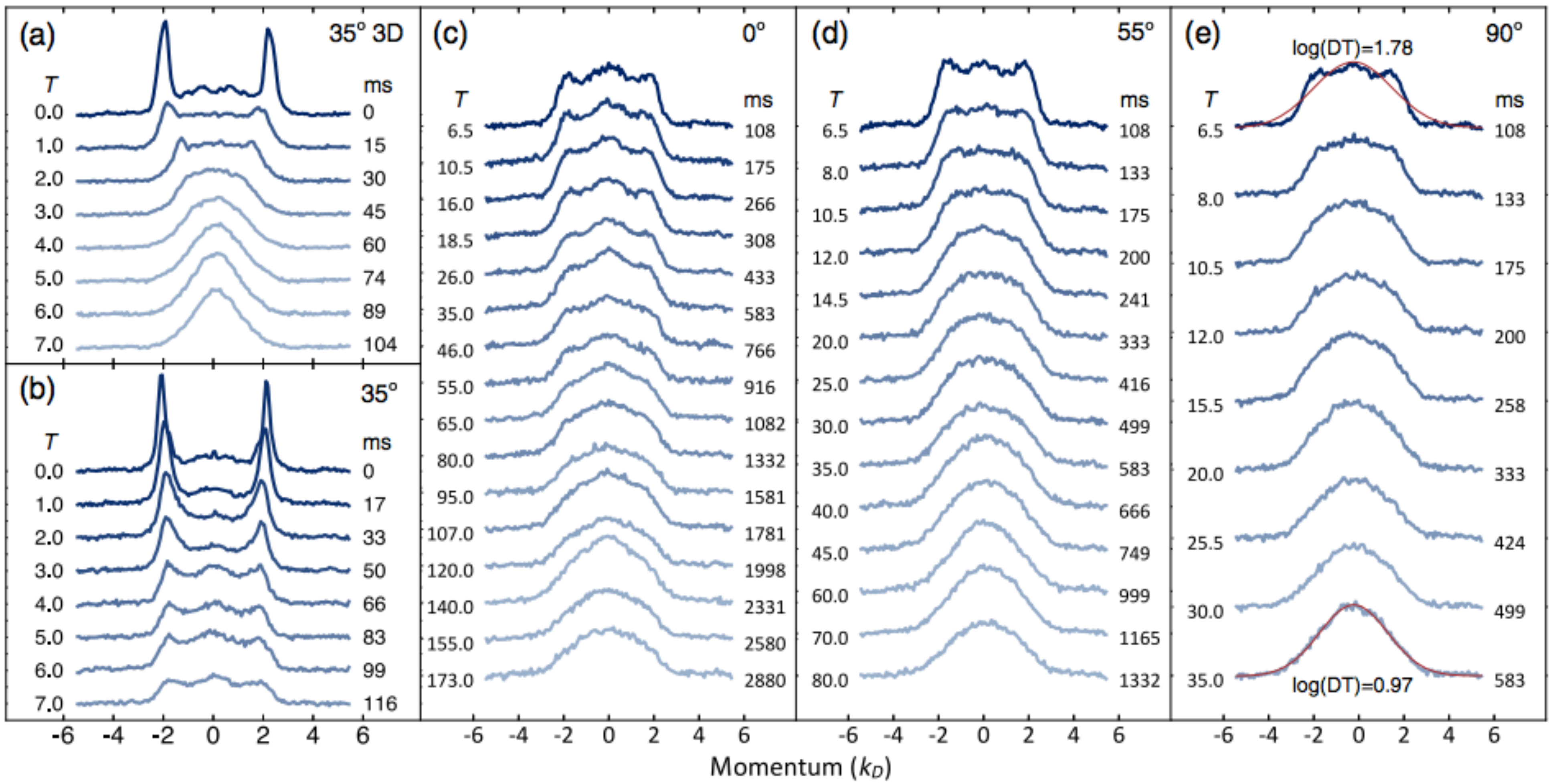
integrability breaking

AJ Friedman, SG, R Vasseur, *Phys. Rev. B* **101**, 180302(R)
J Lopez-Piqueres, B Ware, SG, R Vasseur, arXiv:2005.13546

why is this problem interesting?

- Experimental systems are not perfectly integrable; need to understand rates
  [Tang, Kao, Li, Seo, Mallayya, Rigol, SG, Lev, PRX (2018); cf. Moller, Dubail talks]



- Theoretically unclear what happens in many cases of interest
  [cf. De Nardis talk, also “prethermalization” generally (Abanin, De Roeck, Huveneers)]

why is this problem hard?

- Why not just write down an rhs for GHD/Bethe-Boltzmann equation?

$$\partial_t n_\lambda(x,t) + v^{\text{eff}}[\{n\}]\partial_x n_\lambda(x,t) = I[\{n\}]$$

- Two questions
  - **Hard problem**
    What is the collision integral? (Involves matrix elements between many-body states)
    Collision integral generally lies outside GHD: e.g., umklapp scattering
  - **Relatively easy problem**
    How do we extract transport coefficients for the remaining conserved quantities?

(relatively) easy problem

- An integrable system has infinitely many conserved quantities

$$\partial_t Q_i \underset{\text{gradient expansion}}{=} I(\{Q_j\})$$

(relatively) easy problem

- An integrable system has infinitely many conserved quantities

$$\partial_t Q_i \quad = \quad I(\{Q_j\})$$

gradient expansion

- Linearize: $$\partial_t \delta Q_i \quad = \quad \left.\frac{\partial I}{\partial Q_j}\right|_{\rho_{\mathrm{eq.}}} \delta Q_j \quad \equiv \quad -\Gamma_{ij}\delta Q_j$$

specified by residual charges

- Residual conserved charges are zero modes of Gamma

d.c. conductivity

$$\partial_t \delta Q_i = -\Gamma_{ij} \delta Q_j$$

- Continuity equation for residual charges

$$\partial_t q_\alpha + \partial_x j_\alpha = 0$$

Greek indices for residual charges

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- Drude response in integrable systems comes from overlap of current with conserved charges ("hydrodynamic projections")

$$J_\alpha = A_{\alpha i} Q_i + \ldots$$

stuff that decays at late times

- Kubo formula:

$$\sigma^{\text{d.c.}}_{\alpha\beta} \propto \int dt \langle J_\alpha(t) J_\beta(0) \rangle = A_{\alpha i} A_{\beta j} \int dt \langle Q_i(t) Q_j(0) \rangle + \ldots$$

diverges in integrable limit

d.c. conductivity

$$\sigma_{\alpha\beta}^{\mathrm{d.c.}} \propto \int dt \langle J_\alpha(t) J_\beta(0) \rangle = A_{\alpha i} A_{\beta j} \int dt \langle Q_i(t) Q_j(0) \rangle + \ldots$$

- Integrable: $T\sigma_{\alpha\beta}(\omega) = A_{\alpha i} C_{mn} (A^T)_{j\beta} \delta(\omega) + \ldots$

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- For sufficiently weak integrability breaking:

$$\langle \delta Q_i(t) \delta Q_j(0)\rangle = [e^{-\Gamma t}]_{ik} \langle \delta Q_k \delta Q_j\rangle$$

d.c. conductivity

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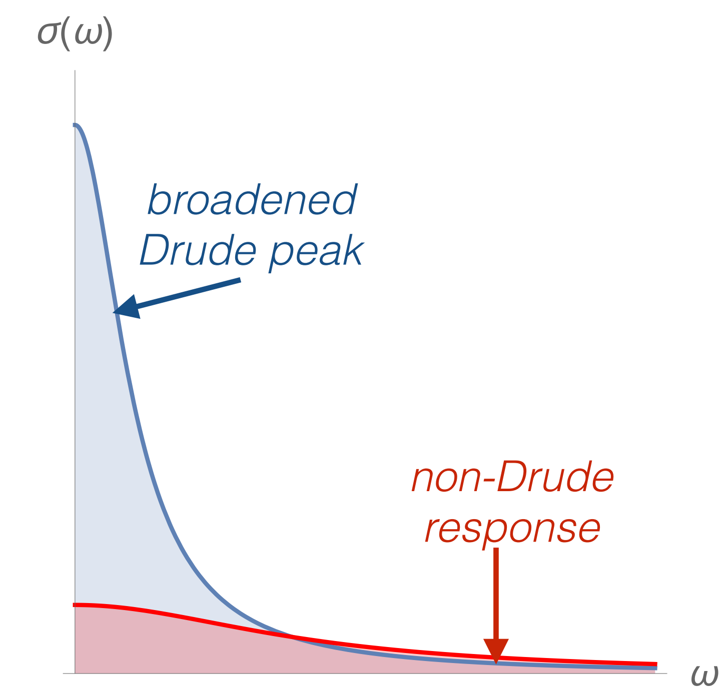
- For sufficiently weak integrability breaking:

$$\langle \delta Q_i(t) \delta Q_j(0) \rangle = [e^{-\Gamma t}]_{ik} \langle \delta Q_k \delta Q_j \rangle$$

- Final expression ("broadened Drude peak"):

$$T\sigma^{\text{d.c.}}_{\alpha\beta} = \left(\mathbf{A} \mathbf{\Gamma}^{-1} \mathbf{C} \mathbf{A}^T \right)_{\alpha\beta}$$

inverse restricted to fast subspace



- By Einstein relation, related to diffusion matrix $D_{\alpha\beta} = \left(\mathbf{A} \mathbf{\Gamma}^{-1} \mathbf{A} \right)_{\alpha\beta}$

hydrodynamics and noise

- Continuity equations for the residual charges: $\partial_t q_\alpha + \partial_x (A_{\alpha n} q_n) = 0$
- Decay of other charges:

$$\partial_t q_n + \partial_x (A_{n\alpha} q_\alpha) = -\Gamma_{nm} q_m + \xi \Rightarrow q_m = (\Gamma^{-1})_{mn} \left[\partial_x (A_{n\alpha} q_\alpha) + \xi\right]$$

- Stationarity of C_{mn} fixes the noise strength (a la fluctuating hydro)
- Combines to give a noisy diffusion equation with $D_{\alpha\beta} = \left(\mathbf{A}\mathbf{\Gamma}^{-1}\mathbf{A}\right)_{\alpha\beta}$
- In principle this can be extended to a nonlinear diffusion equation where the matrices are nontrivial functions of space

hard problem: what is Gamma?

$$\hat{H} = \hat{H}_0 + h_i(x)\hat{q}_i(x) + J_{ij}(x)\hat{q}_i(x)\hat{q}_j(x) + \ldots$$

- In general: depends on matrix elements to rearrange many particles with (in principle) large momentum transfer

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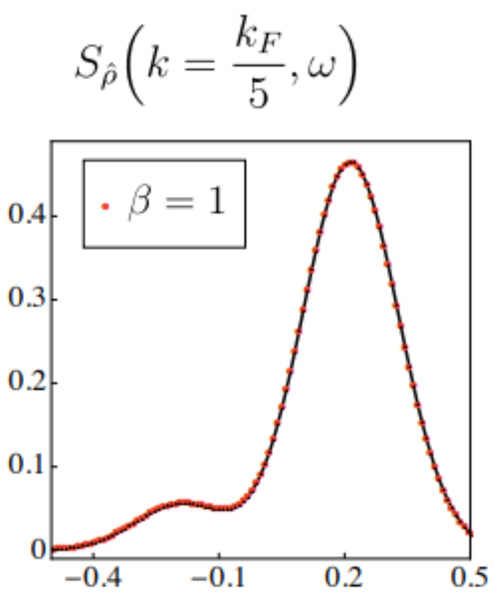
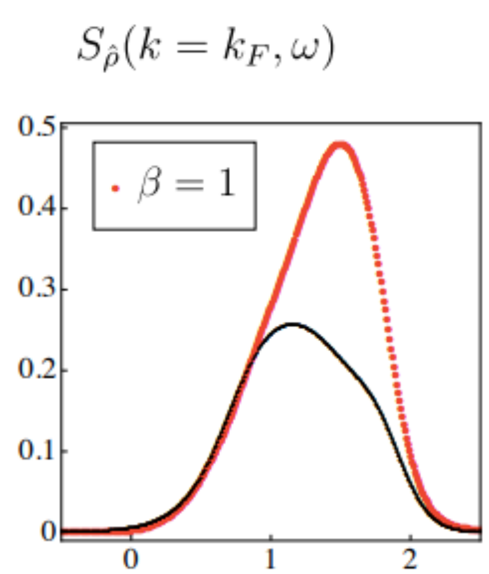
- Hydrodynamic limit:





  - Smoothly varying potentials and/or long-range interactions

  - Dominated by small-momentum scattering, which rearranges few quasiparticles [De Nardis and Panfil; De Nardis, Bernard, Doyon]

  - Matrix elements depend on hydrodynamic data, e.g.: $\langle \psi | \hat{q}_\alpha | \psi ; \theta \to \theta' \rangle = q^{\mathrm{dr}}(\theta)$

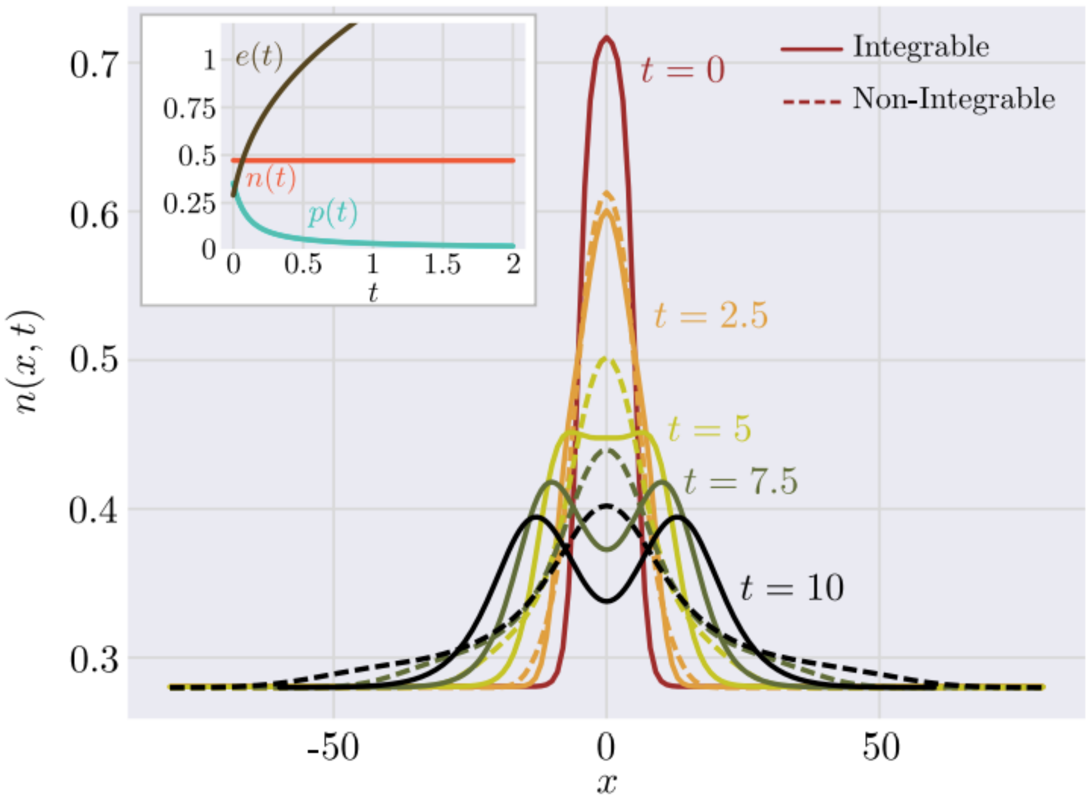
$$\langle\psi|\hat{q}_\alpha|\psi;\theta\to\theta'\rangle = q^{\rm dr}(\theta)$$

slowly fluctuating noise

- Suffices to work with 1ph excitations

$$\partial_t n_\lambda(x,t) + v^{\rm eff}[\{n\}]\partial_x n_\lambda(x,t) = I[\{n\}]$$

$$\mathcal{I}_\lambda = \rho^{\rm tot}_\lambda \int d\varphi |\tilde{V}(k_{\lambda+\varphi}-k_\lambda)|^2 |\tilde{\eta}(\varepsilon_{\lambda+\varphi}-\varepsilon_\lambda)|^2 \rho^{\rm tot}_{\lambda+\varphi} h_0^{\rm dr}(\lambda)$$
$$\times h_0^{\rm dr}(\lambda+\varphi)[n_{\lambda+\varphi}(1-n_\lambda) - n_\lambda(1-n_{\lambda+\varphi})], \qquad (1\text{:}$$



- Noise causes "rapidity diffusion"

- Noise strength is proportional to dressed charge of quasiparticles

- Can also do slow static potentials, interactions, cases with no Euler-scale hydro (gapped XXZ)

Friedman, SG, Vasseur, PRB (2020)
cf. Bastianello, De Nardis, De Luca arXiv:2003.01702; Durnin, Bhaseen, Doyon, arXiv:2004.11030
Bouchoule, Doyon, Dubail, arXiv:2006.03583

generalized relaxation time approximation

- GHD assumes that the system is locally in a generalized Gibbs ensemble
  - GGE in 1-1 correspondence with quasiparticle distribution functions
- Impose local equilibration at some rate:

$$\hat{\rho}_{\mathrm{GGE}}(t+\Delta t) = (1-\gamma\Delta t)\hat{\rho}_{\mathrm{GGE}}(t) + \gamma\Delta t\hat{\rho}_{\mathrm{Gibbs}}\left[\hat{\rho}_{\mathrm{GGE}}(t)\right]$$

Lopez-Piqueres, Ware, SG, Vasseur

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- Faithful treatment of nonlinearities
- Algorithm: evaluate residual charges in the instantaneous GGE, find Gibbs state that matches these residual charges
- In principle can also let relaxation rate be a function of residual charges

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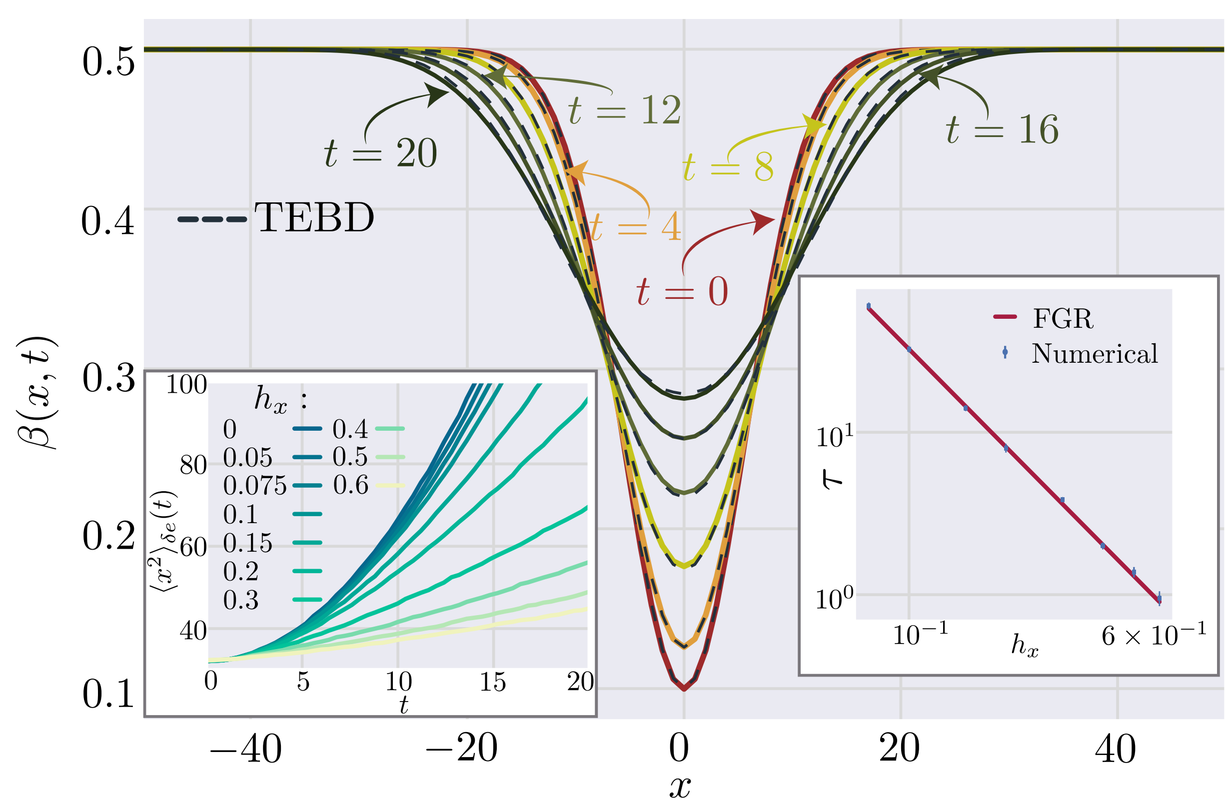
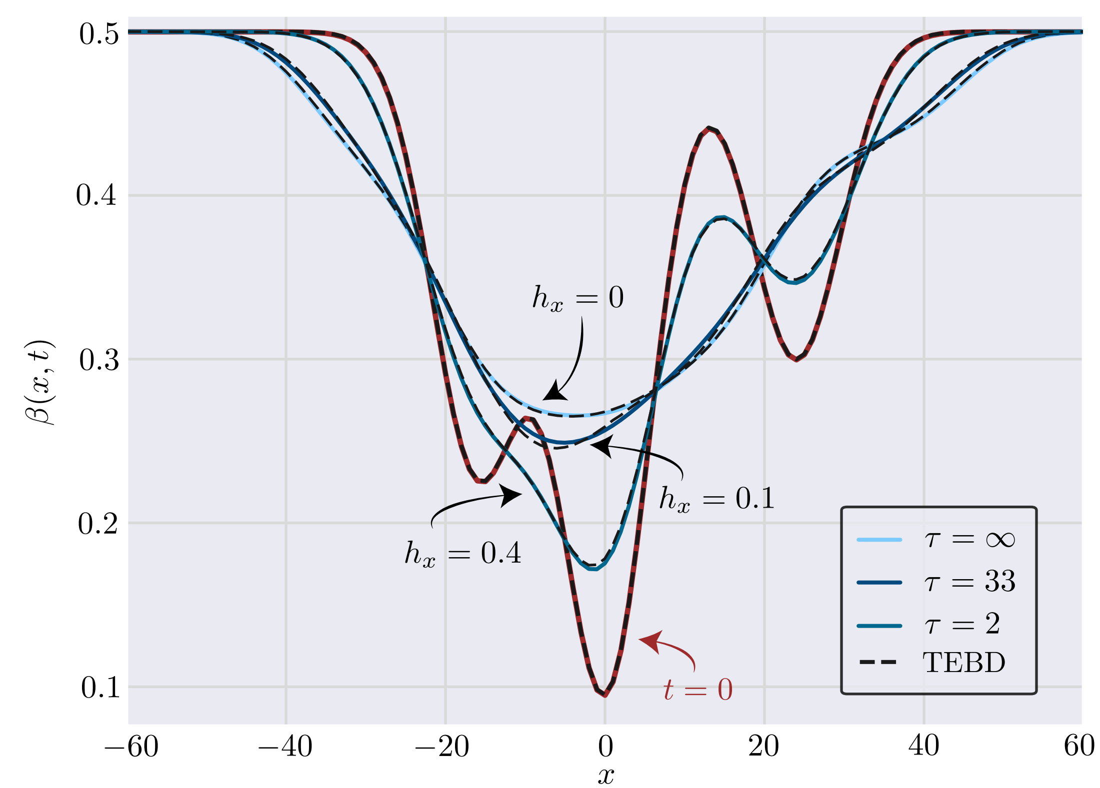
does it work?

- Energy transport in easy-axis XXZ $\hat{H} = \sum_i (\hat{S}^x_i \hat{S}^x_{i+1} + \hat{S}^y_i \hat{S}^y_{i+1} + \Delta \hat{S}^z_i \hat{S}^z_{i+1}) + h_x (-1)^i \hat{S}^x_i,$
- Extract relaxation time vs *h_x* by evolving a gaussian initial state
- Same relaxation time works for essentially *all* initial states





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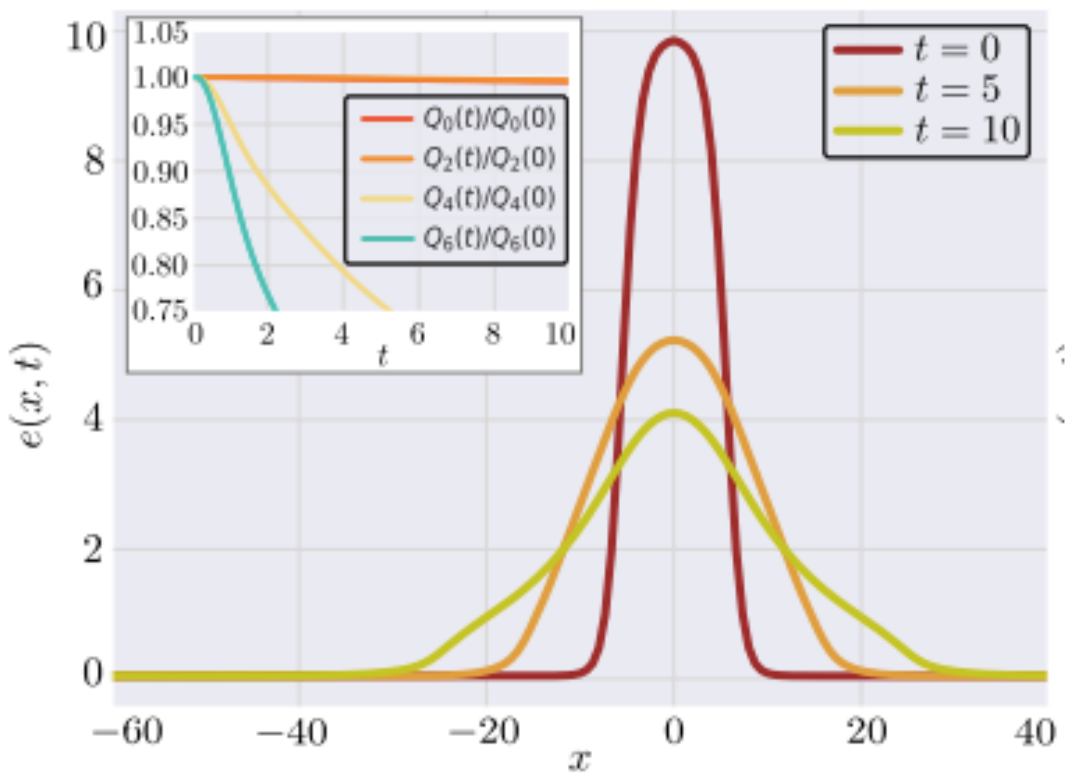
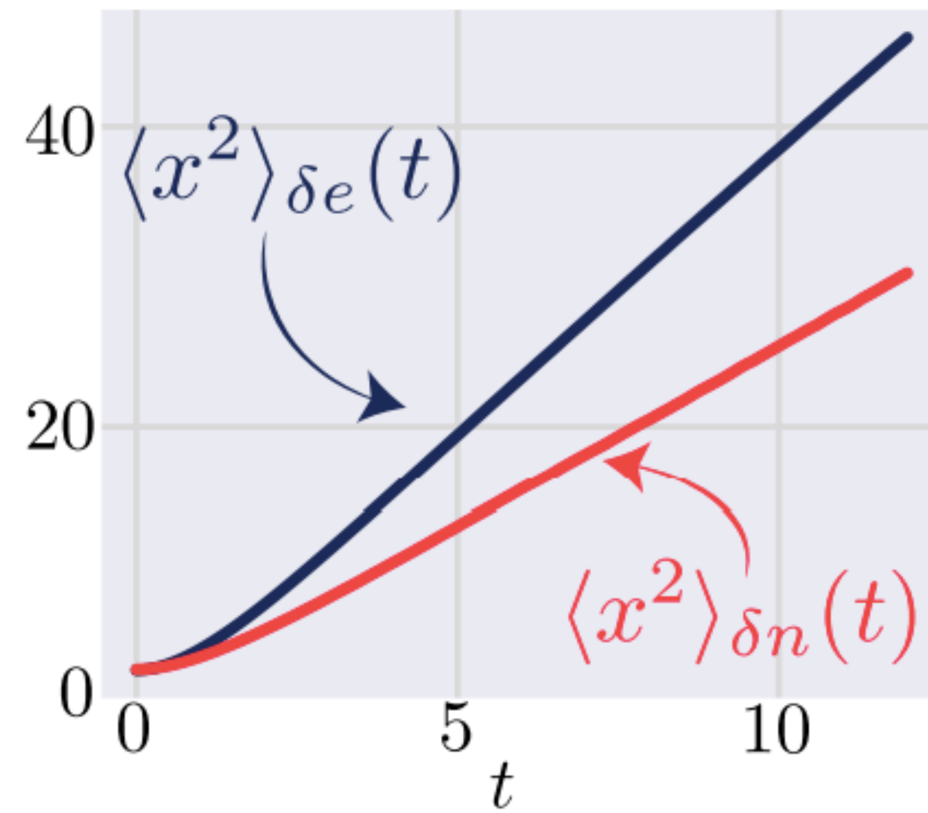
1d bose gases





- Captures non-Gaussianity of profiles
- Captures distinct transport coefficients for distinct charges

outlook

- Integrability breaking is a hard problem because it depends on matrix elements that are fundamentally outside GHD
  - Except when the perturbations are slowly varying in space and time
- We care about *qualitative* phenomena
  - Crossovers from ballistic to diffusive transport, “dressing” effects
  - Generalized relaxation time approximation captures all the physics essential to hydro
  - Need to extend GRTA to include noise (KPZ?)
  - Key question: integrability-breaking effects in cases with anomalous integrable dynamics