Universal aspects of low-temperature transport from GHD

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Universal aspects of low-temperature transport from GHD

- B. Bertini, L. Piroli, and P. Calabrese,
  Universal Broadening of the Light Cone in Low-Temperature Transport,
  Phys. Rev. Lett. **120**, 176801 (2018)

- B. Bertini and L. Piroli,
  Low-temperature transport in out-of-equilibrium XXZ chains,
  J. Stat. Mech. 033104 (2018)

- M. Mestyán, B. Bertini, L. Piroli, and P. Calabrese,
  Spin-charge separation effects in the low-temperature transport of one-dimensional Fermi gases,
  Phys. Rev. B **99**, 014305 (2019)

Motivations

- GHD introduced in
  - B. Bertini, M. Collura, J. De Nardis, M. Fagotti,
    Transport in Out-of-Equilibrium XXZ Chains: Exact Profiles of Charges and Currents,
    Phys. Rev. Lett. **117**, 207201 (2016)
  - O. A. Castro-Alvaredo, B. Doyon, T. Yoshimura,
    Emergent Hydrodynamics in Integrable Quantum Systems Out of Equilibrium,
    Phys. Rev. X **6**, 041065 (2016)
- Original motivations:
  - study of quantum quenches in inhomogeneous settings

Quantum quenches

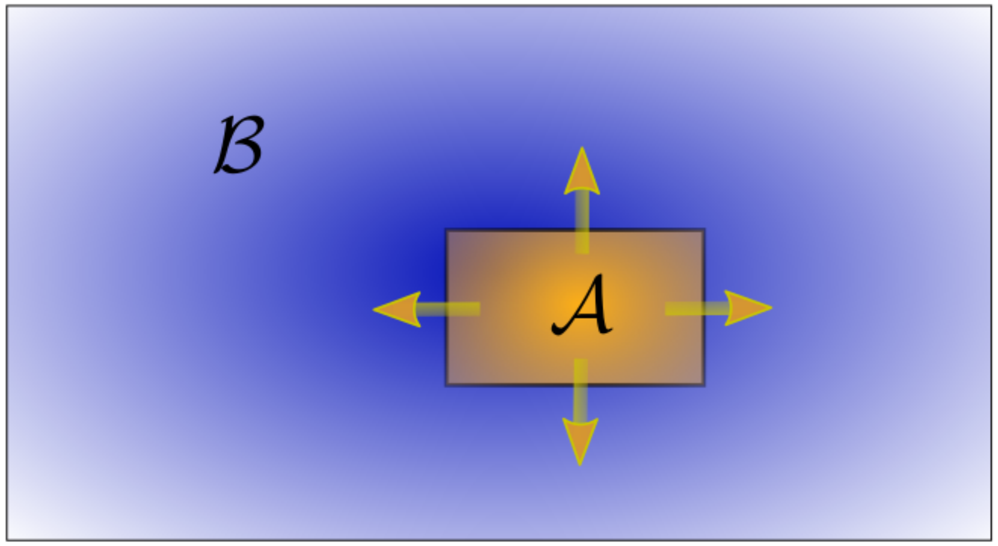
- Well-defined initial state $|\Psi_0\rangle$, e.g. ground-state of

$$H(g) = \sum_j h_j(g)$$

- At time $t = 0$, evolve with different Hamiltonian, e.g. $g \to g'$
- Compute local correlations $\langle \mathcal{O}_{j,\ldots,j+q}(t) \rangle$
- Ideal homogeneous systems
- Lots of no's:
    - no disorder
    - no trapping potential
    - no external driving

Quantum quenches

- Obvious motivations from experimental and foundational point of view
- Physical intuition: equilibration occurs locally



- Thermalization expected for generic systems
- Different behavior in the presence of conservation laws

M. Rigol, V. Dunjko, M. Olshanii, Nature **452**, 854 (2008)
M. Rigol, V. Dunjko, V. Yurovsky, M. Olshanii, Phys. Rev. Lett. **98**, 050405 (2007)

- Integrable systems: extensive number of local conservation laws

$$H = \sum_j h_j \qquad Q^{(k)} = \sum_j q_j^{(k)} \quad \Rightarrow \quad \left[H, Q^{(k)}\right] = 0$$

finite support (of $q_j^{(k)}$)

- Exactly solvable: spectrum can be computed via Bethe Ansatz

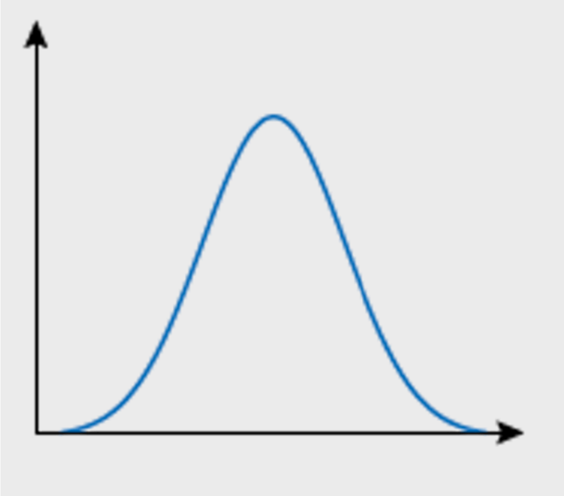
- Description in terms of stable quasi-particles

  - Finite systems: $$E = \sum_{j=1}^{N} \varepsilon(\lambda_j)$$ "quasi-momenta" (λ_j)

  - TD limit: each eigenstate associated with distribution function of quasi-momenta $\rightarrow \rho(\lambda)$

- Thermal states $\rightarrow \rho_T(k) \propto \dfrac{1}{1 + e^{\varepsilon(k)/K_B T}}$

"Dressed" dispersion relation



- Quench problem conceptually solved for integrable systems:
  - at large times, system described by a GGE

$$\hat{\rho}_{\mathrm{GGE}} \propto e^{\sum_k \beta_k Q_k}$$

associated with (non-thermal) distribution functions $\rho(\lambda)$

- Recent developments allow us to extract correlations from $\rho(\lambda)$

- Several approaches developed to compute $\rho(\lambda)$ in practice

F. H. L. Essler and M. Fagotti, J. Stat. Mech. 064002 (2016)
J.-S. Caux, J. Stat. Mech. 064006 (2016)
E. Ilievski, M. Medenjak, T. Prosen, L. Zadnik, J. Stat. Mech. 064008 (2016)

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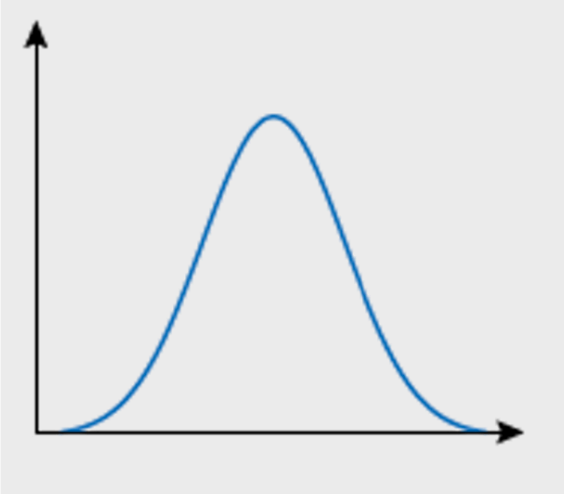
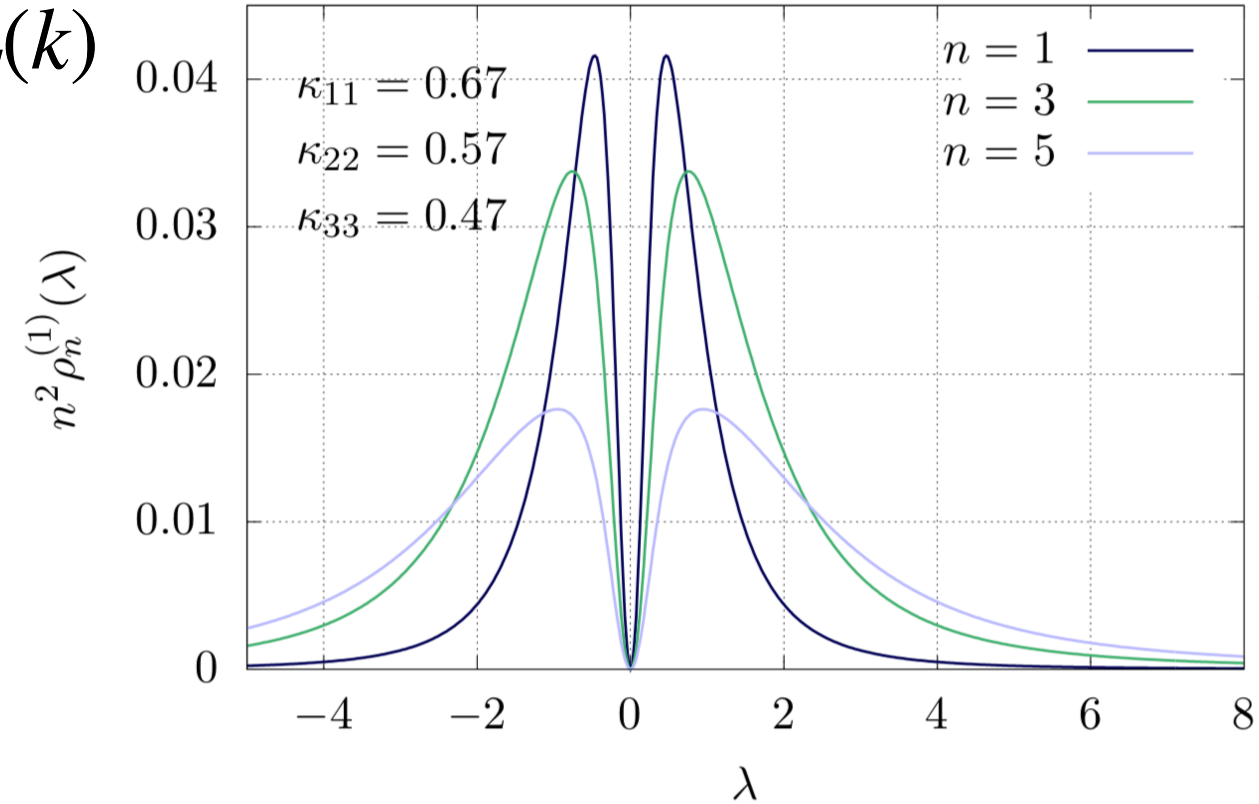


- **GGE states** $\rightarrow \rho_{GGE}(k) \neq \rho_T(k)$



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- What happens in inhomogeneous systems?

F. H. L. Essler and M. Fagotti, J. Stat. Mech. 064002 (2016)
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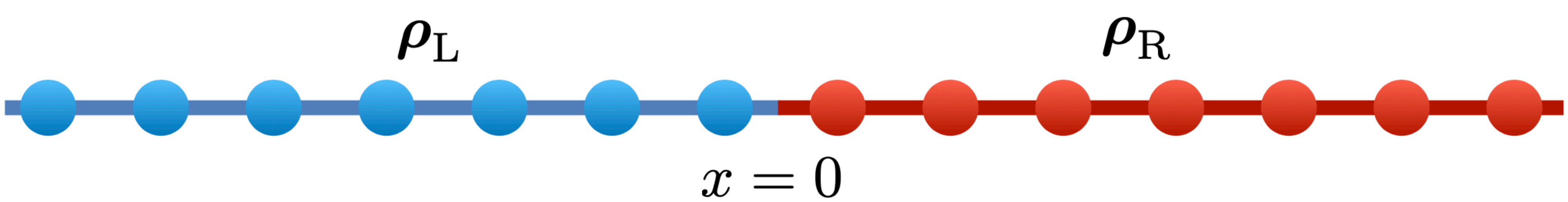
Inhomogeneous quenches

- Simplest setting: bipartition protocols



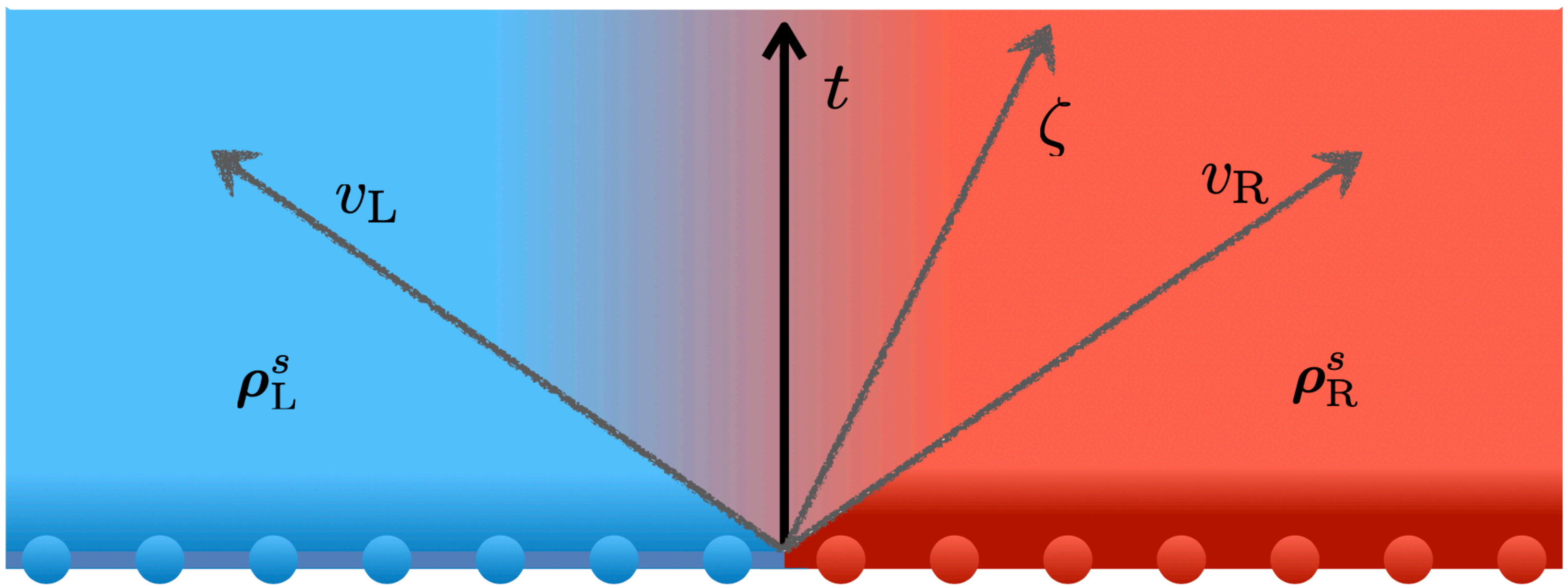
- Ex: $\boldsymbol{\rho}_{\mathrm{L}} = \frac{1}{Z_{\mathrm{L}}} e^{-\beta_{\mathrm{L}} \sum_{x<0} \boldsymbol{h}_x + \mu_{\mathrm{L}} \sum_{x<0} \boldsymbol{s}_x^z}$ $\quad$ $\boldsymbol{\rho}_{\mathrm{R}} = \frac{1}{Z_{\mathrm{R}}} e^{-\beta_{\mathrm{R}} \sum_{x>0} \boldsymbol{h}_x + \mu_{\mathrm{R}} \sum_{x>0} \boldsymbol{s}_x^z}$

- Let evolve with integrable Hamiltonian at time $t = 0$

- What happens at large times?

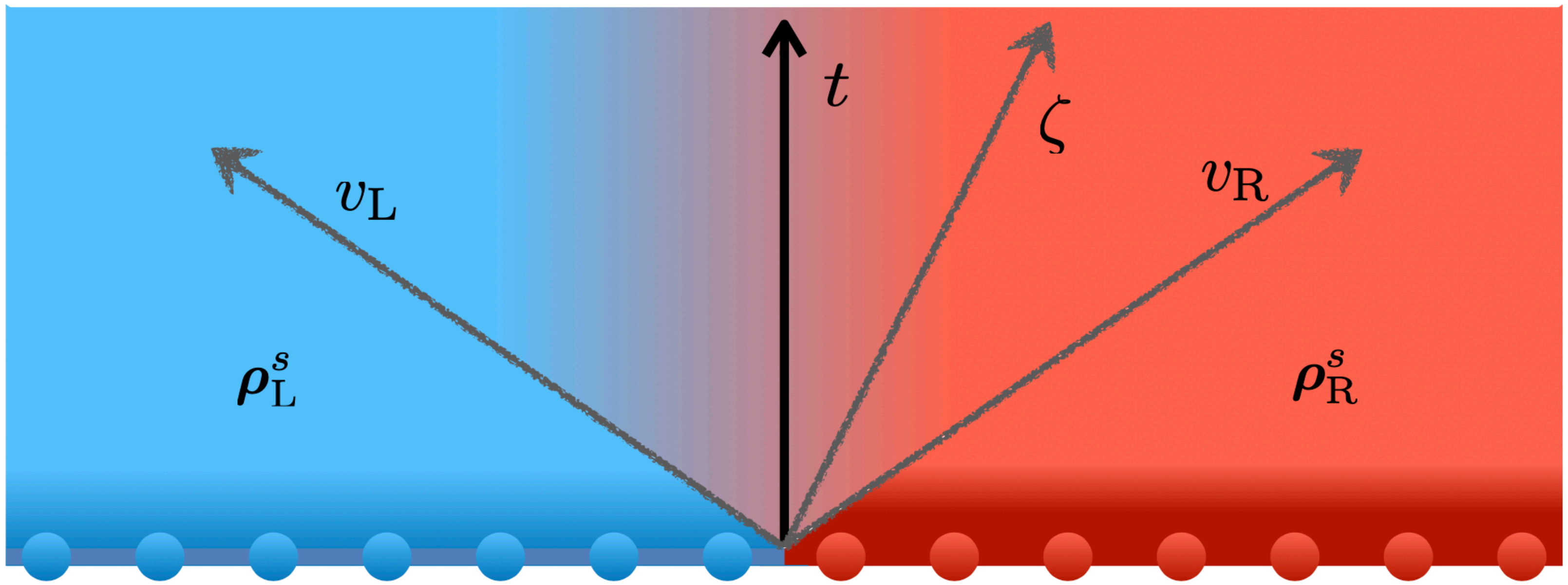
- The problem has a long history: previously studied in free theories and conformal systems

The GHD solution



- Postulate emergence of space-time dependent stationary states $\rho_{x,t}(\lambda)$
- Intuition:
  - information carried by quasi-particles moving from the two sides
  - frames at different velocities receive different sets of quasi-particles
- $\Rightarrow$ in the limit $x, t \to \infty$ stationary states only depend on $\zeta = x/t$

The GHD solution



- States $\rho_{x,t}(\lambda)$ fixed by constraints from conservation laws:
  - continuity equations:

$$\partial_x \langle Q \rangle_\zeta + \partial_t \left\langle J_Q \right\rangle_\zeta = 0$$

  - expectation values:

$$\langle Q \rangle = \int \mathrm{d}k \rho_\zeta(k) q(k) \qquad \left\langle J_Q \right\rangle = \int \mathrm{d}k \rho_\zeta(k) q(k) v(k)$$

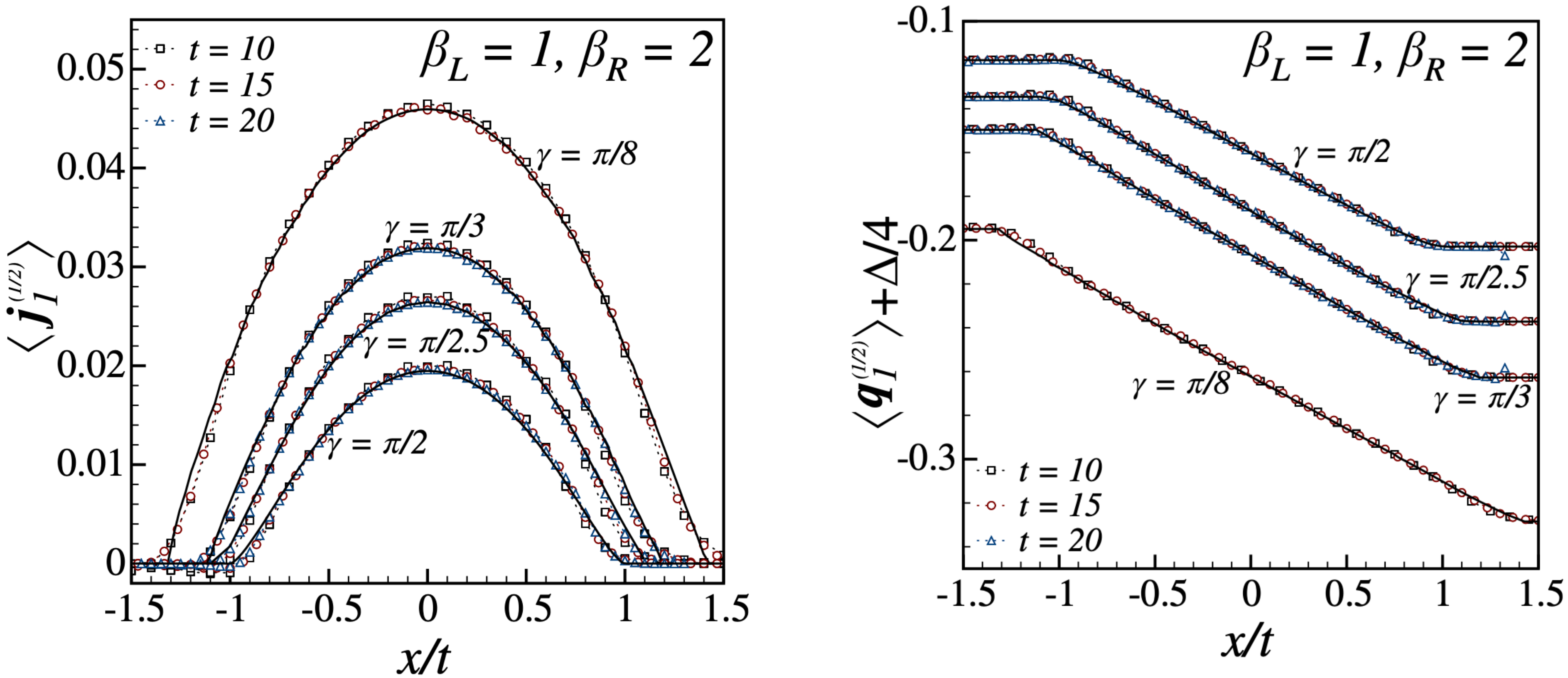
B. Bertini, M. Collura, J. De Nardis, M. Fagotti, Phys. Rev. Lett. **117** (2016)
O. A. Castro-Alvaredo, B. Doyon, T. Yoshimura, Phys. Rev. X **6** (2016)

- Final result:

$$\partial_t \rho_\zeta(\lambda) + \partial_x \Big(v_\zeta(\lambda) \rho_\zeta(\lambda) \Big) = 0$$

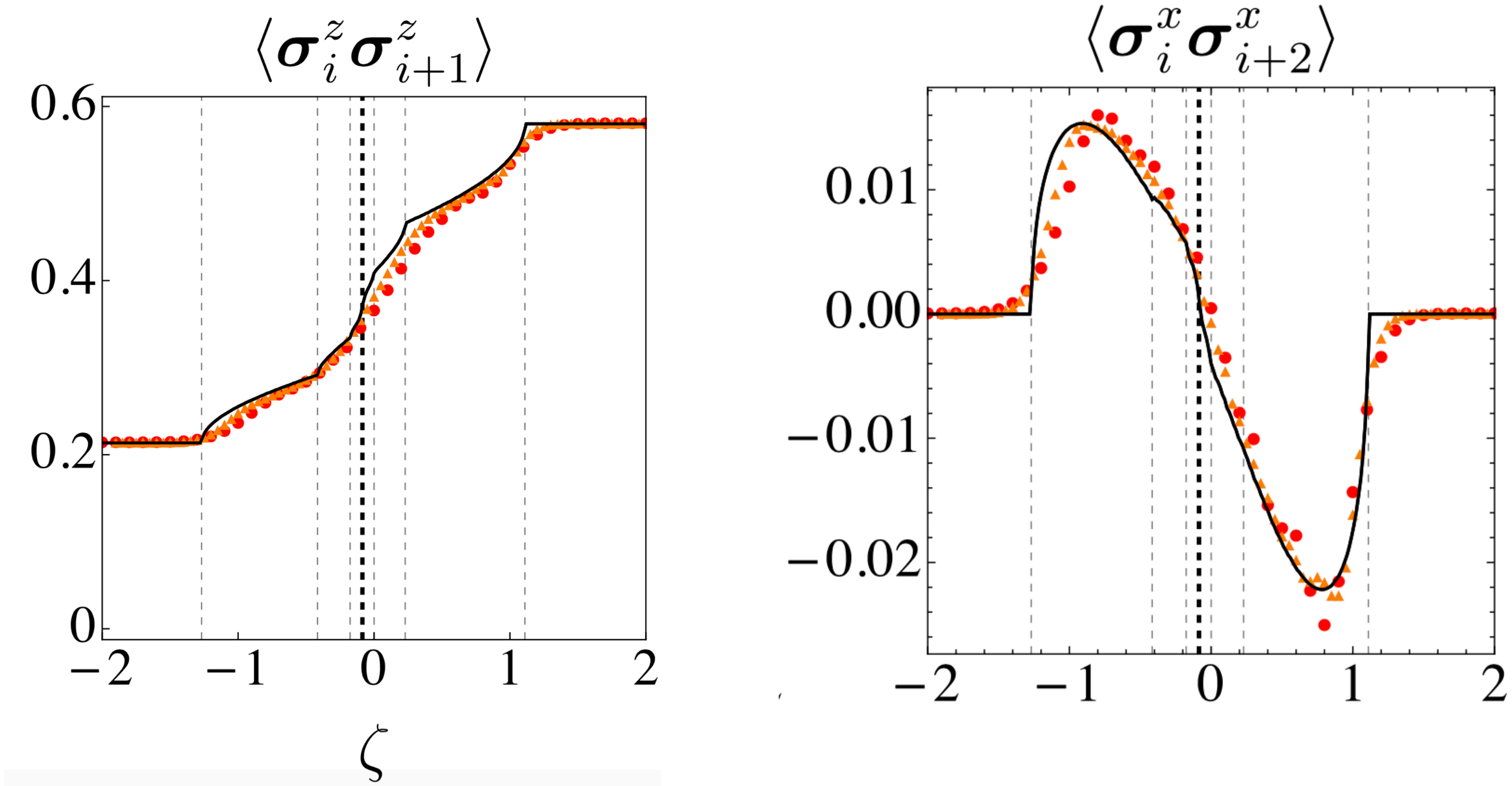
$$\lim_{\zeta \to \pm\infty} \rho_\zeta(\lambda) = \rho_{L,R}(\lambda)$$

- Equations in general solved numerically
- Predictions are claimed to be exact for $t \to \infty$



B. Bertini, M. Collura, J. De Nardis, M. Fagotti, Phys. Rev. Lett. **117** (2016)

- In principle, all local observables can be computed



- Early literature: "phenomenology" of transport profiles:
    - characteristic features
    - universal aspects
    - solvable limits

L. Piroli, J. De Nardis, M. Collura, B. Bertini, M. Fagotti, Phys. Rev. B **96** (2017)

- This talk:
    - focus on low-temperature regime of bipartition protocols

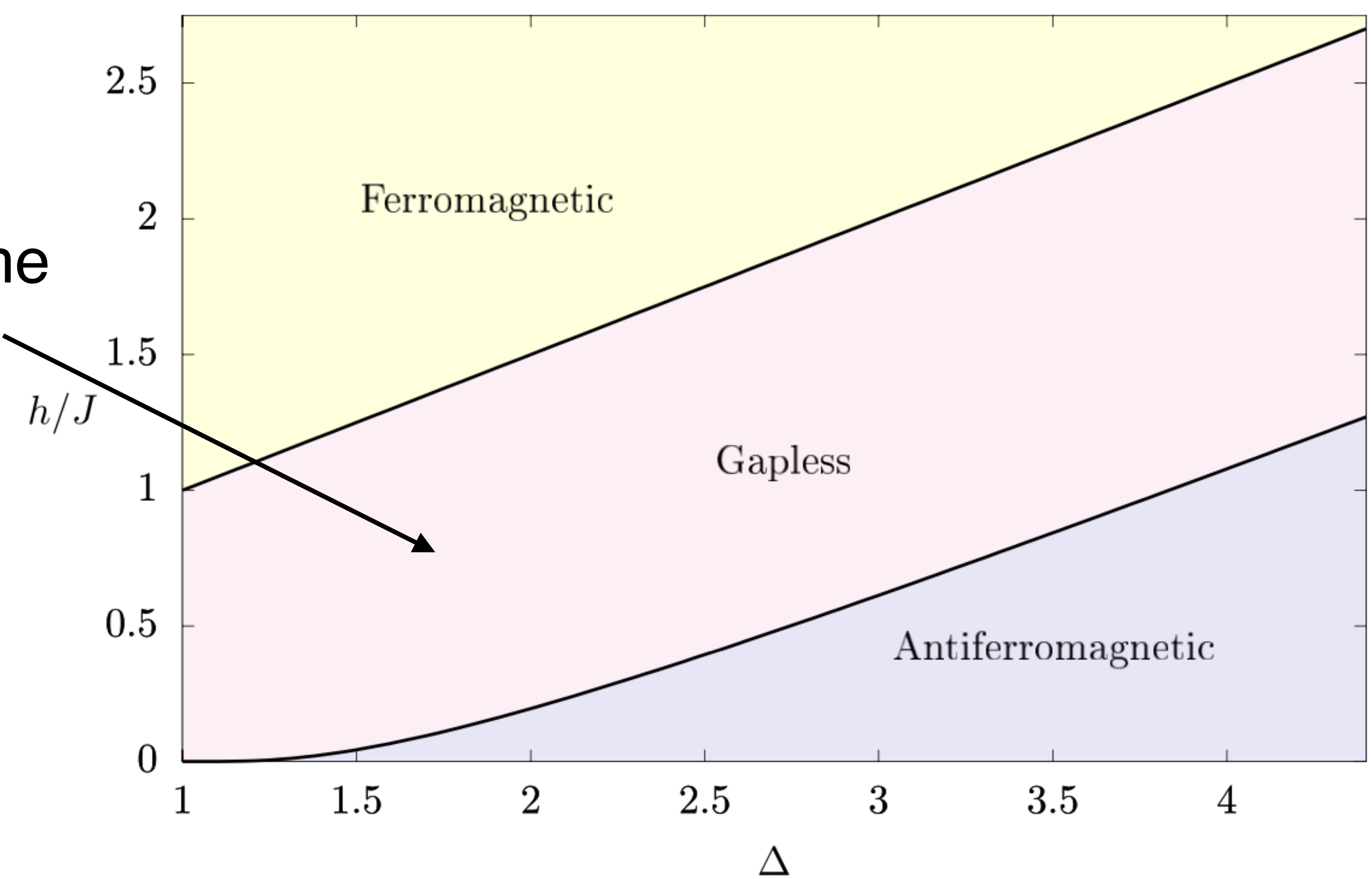
- Main points:
    - test GHD against previous CFT predictions
    - discovery of new universal in the Luttinger-Liquid class
    - spin-charge separation effects in multi-species integrable models

B. Bertini and L. Piroli, J. Stat. Mech. 033104 (2018)
B. Bertini, L. Piroli, and P. Calabrese, Phys. Rev. Lett. **120**, 176801 (2018)
M. Mestyán, B. Bertini, L. Piroli, and P. Calabrese, Phys. Rev. B **99** 014305 (2019)

- Consider XXZ Heisenberg chain

$$\boldsymbol{H} = \frac{J}{4}\sum_{j=-L/2}^{L/2-1}\left[\boldsymbol{\sigma}_j^x\boldsymbol{\sigma}_{j+1}^x + \boldsymbol{\sigma}_j^y\boldsymbol{\sigma}_{j+1}^y + \Delta\left(\boldsymbol{\sigma}_j^z\boldsymbol{\sigma}_{j+1}^z - 1\right)\right] - h\sum_{j=-L/2}^{L/2-1}\left(\boldsymbol{\sigma}_j^z - 1\right)$$

Focus on gapless regime



- Focus on XXZ Heisenberg chain

$$\boldsymbol{H} = \frac{J}{4}\sum_{j=-L/2}^{L/2-1}\left[\boldsymbol{\sigma}_j^x\boldsymbol{\sigma}_{j+1}^x + \boldsymbol{\sigma}_j^y\boldsymbol{\sigma}_{j+1}^y + \Delta\left(\boldsymbol{\sigma}_j^z\boldsymbol{\sigma}_{j+1}^z - 1\right)\right] - h\sum_{j=-L/2}^{L/2-1}\left(\boldsymbol{\sigma}_j^z - 1\right)$$

- TBA equations

$$T_{L/R}\log\eta_j(\lambda) = e_j(\lambda) + T_{L/R}\sum_k\left[T_{jk} * \log\left(1+\eta_k^{-1}\right)\right](\lambda)$$

- Bethe equations

$$\rho_{j,\zeta}(\lambda) + \rho_{j,\zeta}^h(\lambda) = a_j(\lambda) - \left[\sum T_{jk} * \rho_{k,\zeta}\right](\lambda)$$

$$\vartheta_j(\lambda) = \rho_j\left(\rho_j + \rho_j^h\right)^{-1}$$

- GHD solution

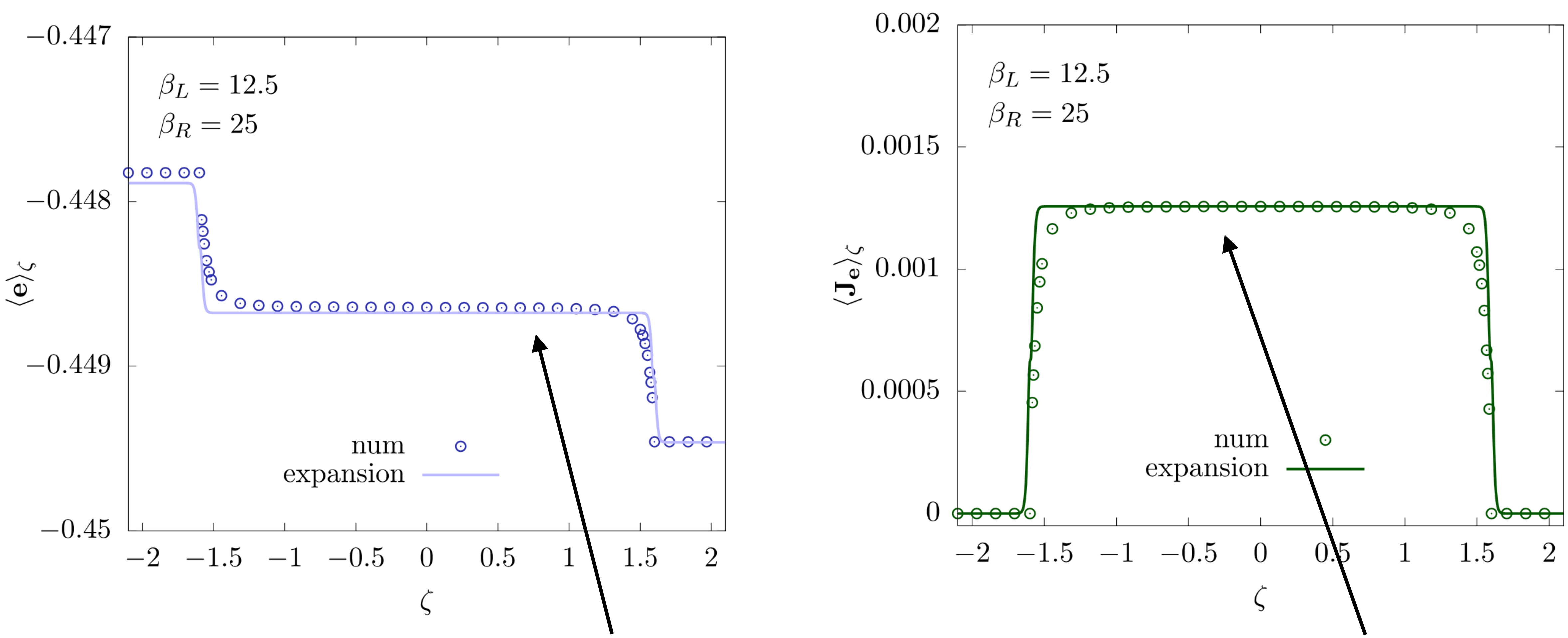
$$\vartheta_{j,\zeta}(\lambda) = \vartheta_j^R(\lambda)H\left(\zeta - v_{j,\zeta}(\lambda)\right) + \vartheta_j^L(\lambda)H\left(v_{j,\zeta}(\lambda) - \zeta\right)$$

- Velocities

$$v_j(\lambda)\rho_j^t(\lambda) = \frac{a_j'(\lambda)}{2\pi} - \left[\sum_k T_{jk} * v_k\rho_k\right](\lambda)$$

Gapless regime

- Three-step profiles for energy density and currents:



$$\langle e \rangle_{\text{ness}} = \frac{\pi}{12}\left(T_l^2 + T_r^2\right) \qquad \langle j \rangle_{\text{ness}} = \frac{\pi}{12}\left(T_l^2 - T_r^2\right)$$

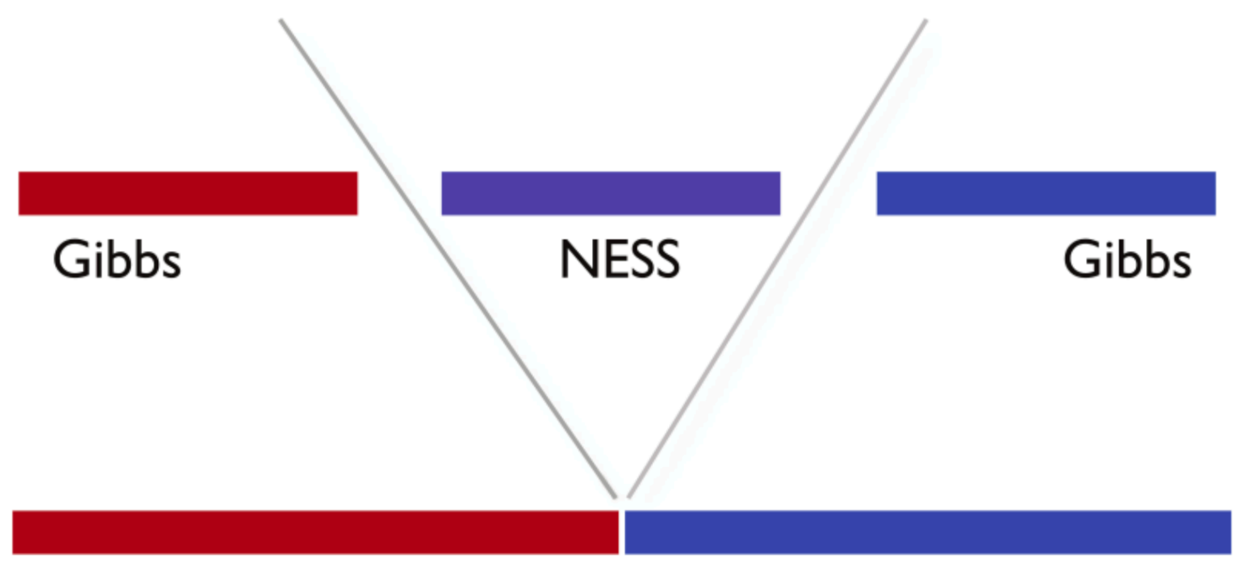
B. Bertini and L. Piroli, J. Stat. Mech. 033104 (2018)

Gapless regime

- We recover analytical results by CFT

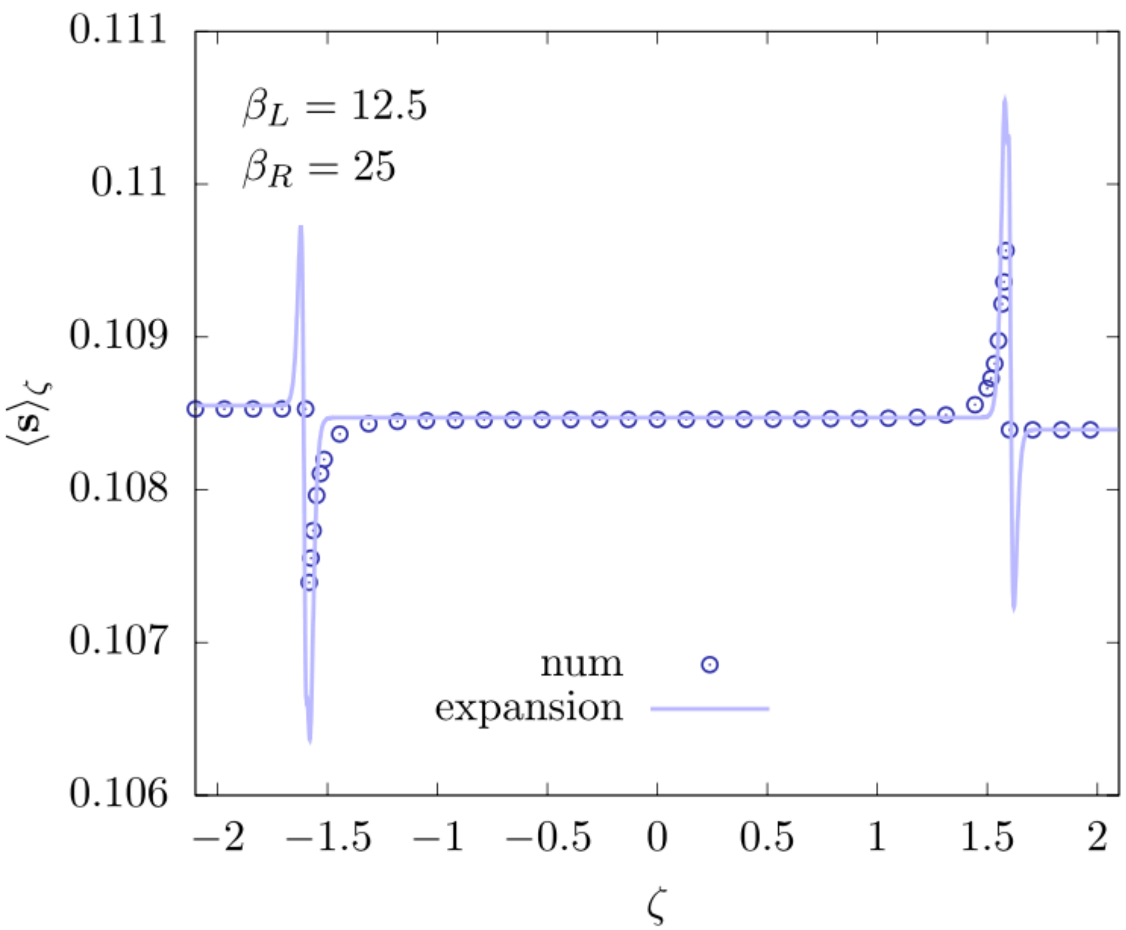
$$\langle e \rangle_{\rm ness} = \frac{c\pi}{12}\left(T_l^2 + T_r^2\right) \qquad \langle j \rangle_{\rm ness} = \frac{c\pi}{12}\left(T_l^2 - T_r^2\right)$$

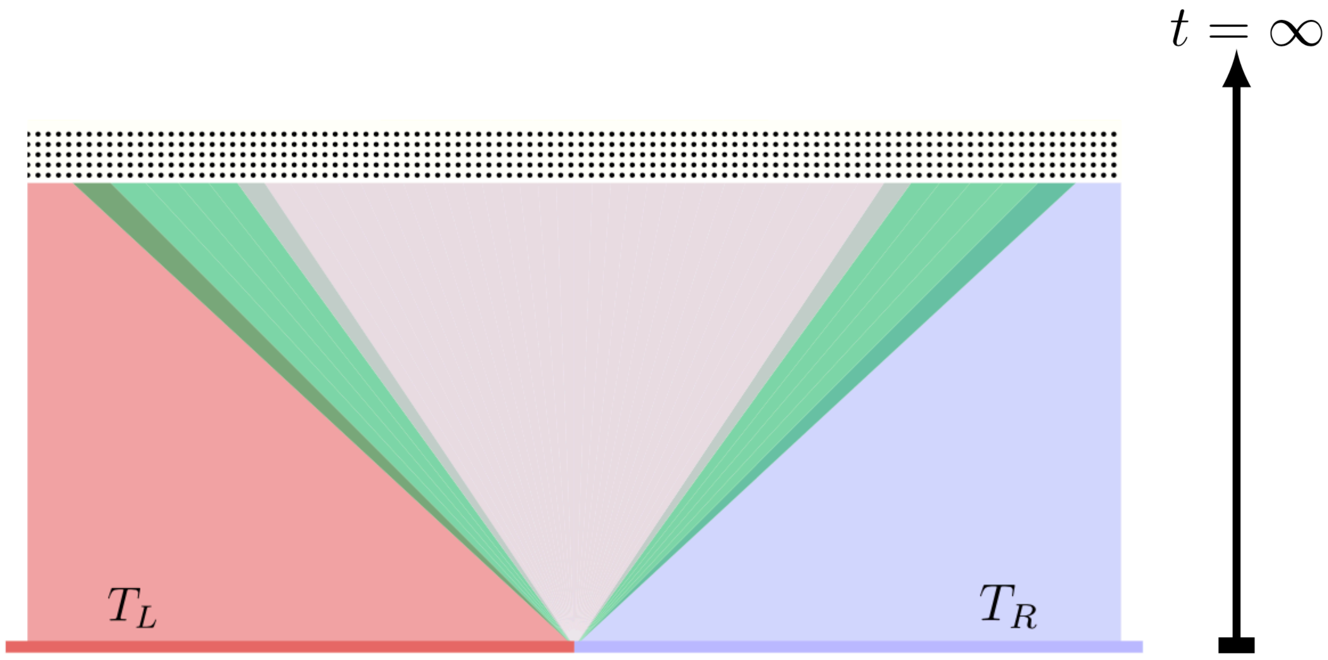
central charge



D. Bernard and B. Doyon, J. Phys. A: Math. Theor. **45**, 362001 (2012)
D. Bernard and B. Doyon, J. Stat. Mech. (2016) 064005

- Universal broadening of the light-cone for generic observables





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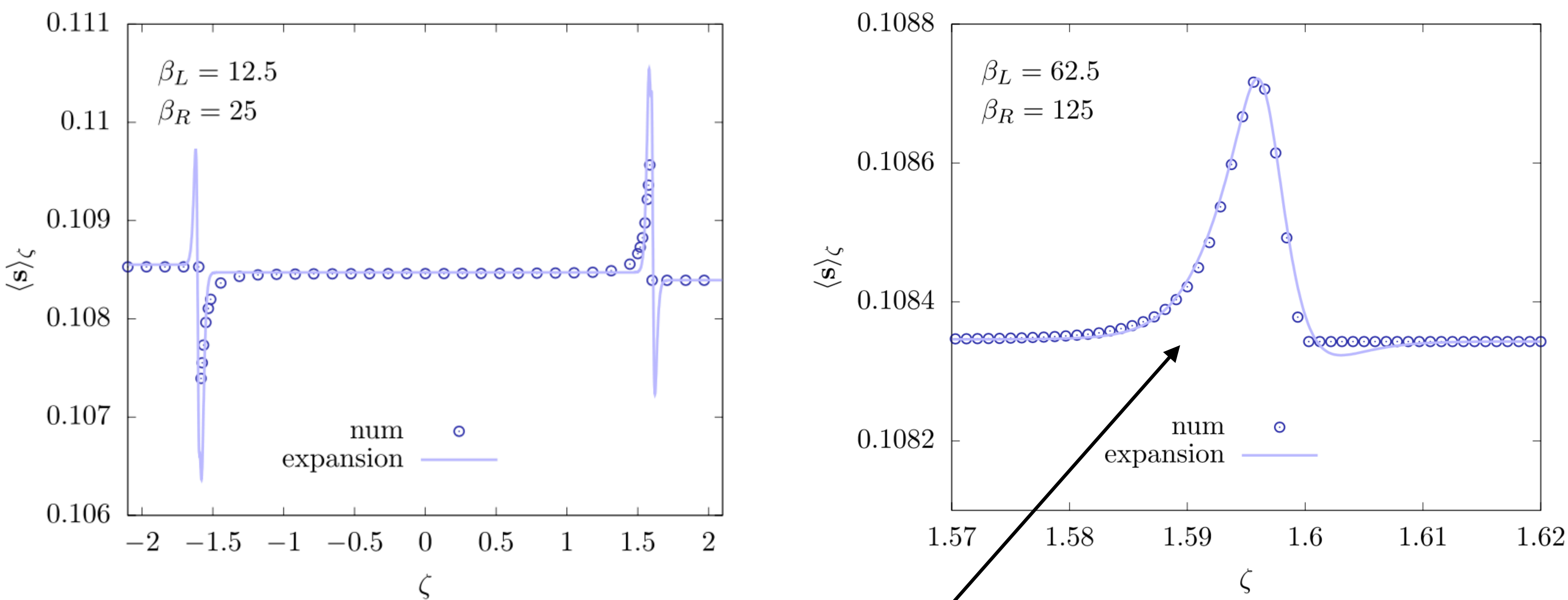


$$\mathscr{D}(z) \equiv T_L \log\left(1 + e^{z/T_L}\right) - T_R \log\left(1 + e^{z/T_R}\right)$$

All observables proportional to the same function

B. Bertini and L. Piroli, J. Stat. Mech. 033104 (2018)
B. Bertini, L. Piroli, and P. Calabrese, Phys. Rev. Lett. **120** 176801 (2018)

Non-linear Luttinger liquid description

- The broadening of the light cone can be obtained from universal non-linear Luttinger liquid description

- Starting from linear Luttinger liquid, dominant irrelevant term introduces a curvature in the dispersion of left and right movers

$$\varepsilon(k) = vk + \frac{1}{2m_*}k^2 \qquad p(k) = k$$

- Simple calculation yields

$$\langle \mathbf{q} \rangle_\zeta = \langle \mathbf{q} \rangle_{\mathrm{GS}} + \frac{\alpha_1}{2\pi v^2}\mathscr{D}\left[m_* v(\zeta - v)\right] + \frac{\alpha_2}{2\pi v^2}\mathscr{D}\left[m_* v(\zeta + v)\right]$$

non-perturbative effect in m_*

A. Imambekov, L. I. Glazman, Science **323**, 228 (2009)

- Low-temperature analysis gives qualitatively different picture in gapped phases

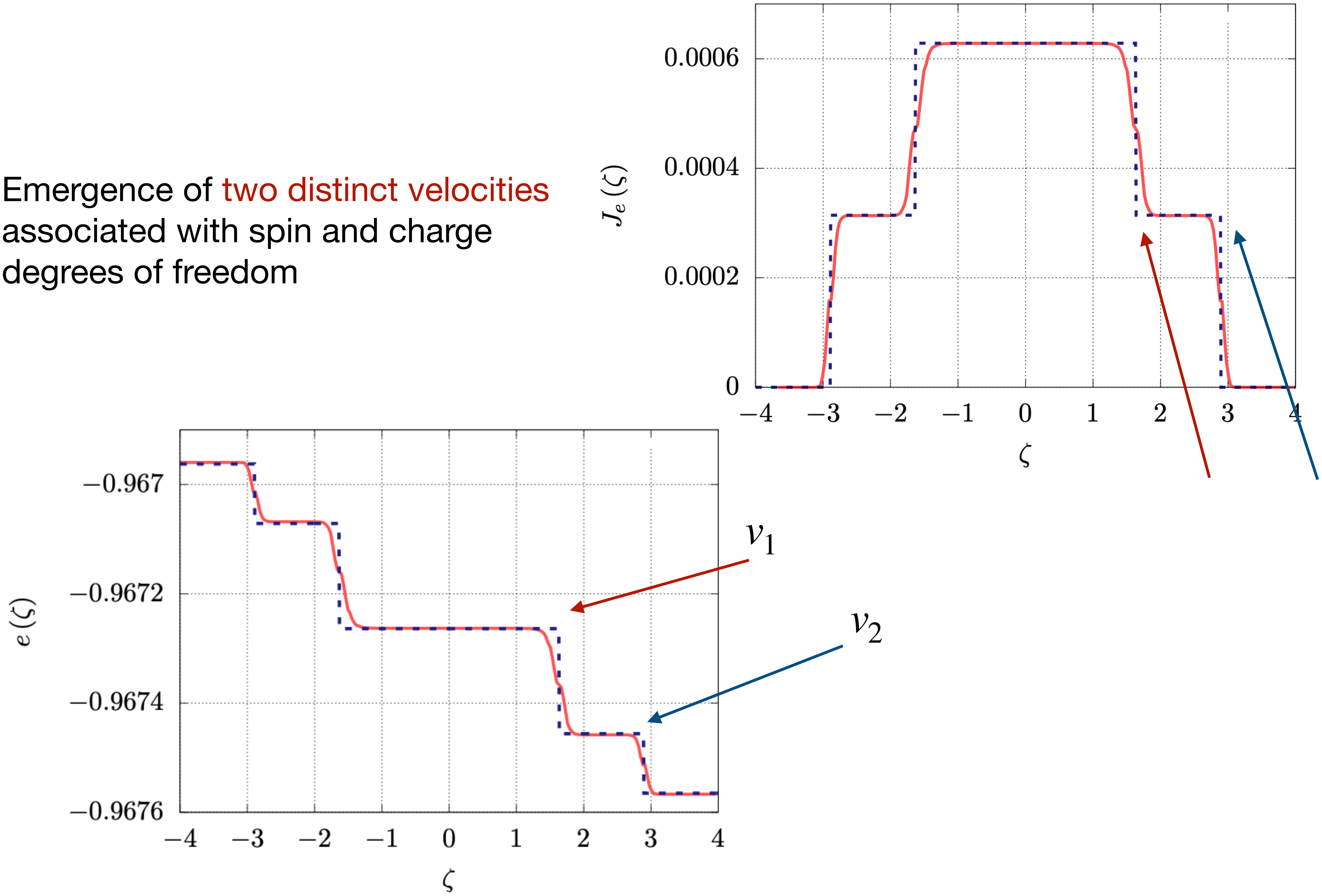
- Particularly interesting features emerging in multi-component "nested" integrable systems

- Prototypical example: Yang-Gaudin model of spinful fermions

$$\hat{H} = -\int_{-L/2}^{L/2} dx \left[\sum_{\alpha=\pm} \psi_\alpha^\dagger(x) \left(\partial_x^2 + A + \alpha h\right) \psi_\alpha(x) \right] + c \int_{-L/2}^{L/2} dx \left[\sum_{\alpha,\beta=\pm} \psi_\alpha^\dagger(x) \psi_\beta^\dagger(x) \psi_\beta(x) \psi_\alpha(x) \right]$$

M. Mestyán, B. Bertini, L. Piroli, P. Calabrese, Phys. Rev. B **99**, 014305 (2019)

Emergence of two distinct velocities associated with spin and charge degrees of freedom



Outlook

- Many works have now extended GHD to more general settings (trapping potentials, inhomogeneities, dephasing..)

- GHD predictions for trap quenches observed in 1D Bose gases!
[M. Schemmer, I. Bouchoule, B. Doyon, J. Dubail, PRL **122** (2019)]

- Do low-temperature features survive in the presence of experimentally feasible settings?

- Ex: spin-charge separation effects in trapped multi-component Fermi gases*? [G. Pagano, et al. Nature Phys **10** (2014)]

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*Ongoing work with Stefano Scopa, Pasquale Calabrese

Thank you for your attention!