

## Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers



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## Representing Numbers

How values can be coded ?

In a digital circuit each signal can take 2 values (0, 1) (Boolean world)

A vector of  $n$  bits can represent up to  $2^n$  values



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## Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

- The character 'F'
- The character 'Φ'
- The number 46
- The number 70
- The number 123

Any symbol in a set where the  $Card = 256$



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## Representing Numbers

How values can be coded ?

by itself a code has no signification



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**Representing Numbers**

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

- Natural numbers
- Relative numbers
- Rational numbers
- Real numbers




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**Representing Numbers**

How can I represent a Natural number ?

I need at least  $n$  bits for a Natural ranging from 0 to  $2^n - 1$

Standards



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**Representing Numbers**

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2

$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$



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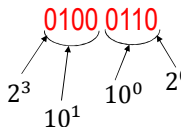
**Representing Numbers**

How can I represent a Natural number ?


Binary Coded Decimal :

The bits represent the successive powers of 2  
The quartets represent the successive powers of 10

$0100\ 0110 = (2^1 + 2^2) 10^0 + (2^2)10^1 = 46$



packed



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## Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal - Unpacked :

The bits represent the successive powers of 2  
The bytes represent the successive powers of 10  
In each byte the 4 Msb are 0

$$0100\ 0110 = \text{Illegal} \quad \underbrace{0000\ 0110}_{\substack{10^0 \\ 2^0}} = 6$$



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## Representing Numbers

How can I represent a Relative number ?

Sign + Value

The bits represent the successive powers of 2  
The Msb represents the sign (1 means negative)



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## Representing Numbers

How can I represent a Relative number ?

Sign+Value :

$$+ \quad \begin{array}{c} 0100\ 0110 \\ \swarrow \quad \searrow \\ 2^6 \quad \quad 2^0 \end{array} = 2^1 + 2^2 + 2^6 = 70$$

$$- \quad \begin{array}{c} 1100\ 0110 \\ \swarrow \quad \searrow \\ 2^6 \quad \quad 2^0 \end{array} = -1 \times (2^1 + 2^2 + 2^6) = -70$$



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## Representing Numbers

How can I represent a Relative number ?

2's complemented

The bits represent the successive powers of 2  
The Msb represents  $-2^{n-1}$



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## Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$\begin{array}{c} \nearrow \quad \nearrow \quad \nwarrow \\ -2^7 \quad 2^6 \quad 2^0 \\ 0100\ 0110 = 2^1 + 2^2 + 2^6 = 70 \end{array}$$

$$1100\ 0110 = 2^1 + 2^2 + 2^6 - 2^7 = -58$$



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## Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

$$\begin{array}{r} \phantom{2^7} \phantom{2^6} \phantom{2^5} \phantom{2^4} \phantom{2^3} \phantom{2^2} \phantom{2^1} \phantom{2^0} \\ \phantom{2^7} \phantom{2^6} \phantom{2^5} \phantom{2^4} \phantom{2^3} \phantom{2^2} \phantom{2^1} \phantom{2^0} \\ 2^7 = 1 + 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 \\ 2^7 = 1 + 2^0 + \phantom{2^1} \phantom{2^2} \phantom{2^3} \phantom{2^4} \phantom{2^5} \phantom{2^6} + 70 \\ -70 = 1 + 2^0 + \phantom{2^1} \phantom{2^2} \phantom{2^3} \phantom{2^4} \phantom{2^5} - 2^7 \\ -70 = \phantom{2^1} \phantom{2^2} \phantom{2^3} \phantom{2^4} \phantom{2^5} - 2^7 \end{array}$$

$$1011\ 1010$$



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## Representing Numbers

How can I represent a Real number ?

Range  
Precision



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## Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

$$\begin{array}{c} \nearrow \quad \nearrow \quad \nearrow \quad \nearrow \\ -2^3 \quad 2^0 \quad 2^{-1} \quad 2^{-4} \\ 0100.0110 = 2^2 + 2^{-2} + 2^{-3} = 4.375 \end{array}$$



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## Representing Numbers

How can I represent a Real number ?

Wide range  
High precision

Floating Point



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## Representing Numbers

How can I represent a Real number ?

Normalized scientific representation

$$R = (-1)^S \times M \times 2^E$$

S : Sign (1 if negative)

M : Mantissa (  $\in [1, 2 [$  )

E : Exponent (relative number)



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## Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

	Single Precision 32 bits	Double Precision 64 bits
S : Sign (1 if negative)	1 bit	1 bit
M : Mantissa ( $\in [1, 2 [$ )	23 bits	52 bits
E : Exponent	8 bits	11 bits



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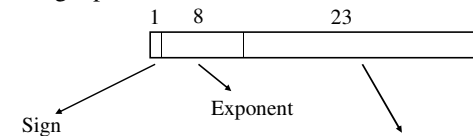
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## Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision :



Relative number between  
-127 and 128

The code 0000 0000  
means -127

Natural Binary Code  
by Excess of 127

Fraction  
Fixed point positive real number  
1. ....

The 1 is not represented !!



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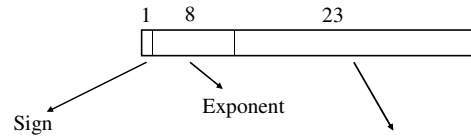
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## Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 1111 1111  
(128) means  $\pm\infty$  or  
an error

Fraction  
.000...000 means  $\pm\infty$   
other values mean error (NaN)



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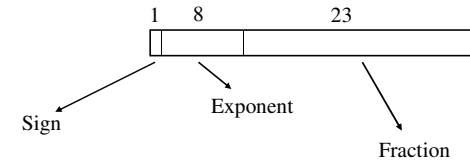
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## Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Range and precision



$$R \in ]-2^{128}, 2^{128} [$$

$$\text{Precision} = 2^{-24+E}$$



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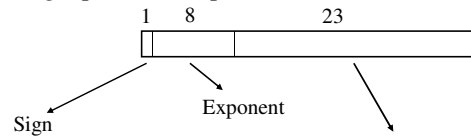
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## Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 0000 0000 (-127)  
indicates denormalized  
Mantissa

0.00 ... 000 means 0



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