QCD Physics for Colliders

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Lecture 2

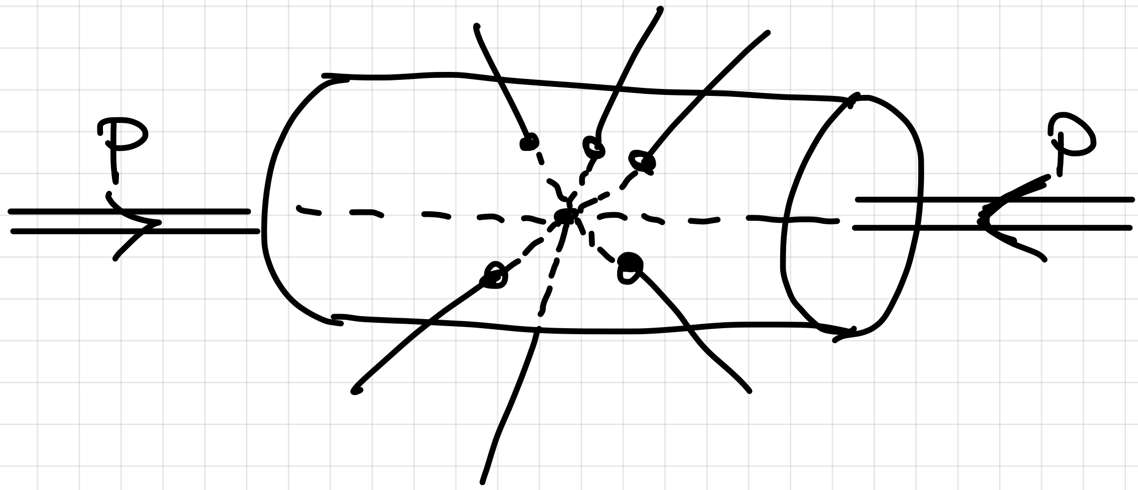
Collider kinematics

QCD Lagrangian

Kinematical variables at hadron colliders

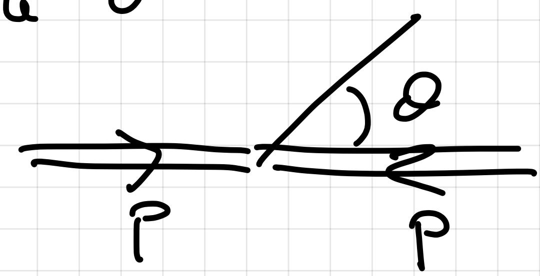
The variables that we use are suggested / constrained by the geometry of the machine / detectors

pp beams collide head-on at the center of cylindrical detectors (except LHCb, beam on fixed target in order to boost the collision frame and have longer decay lengths)

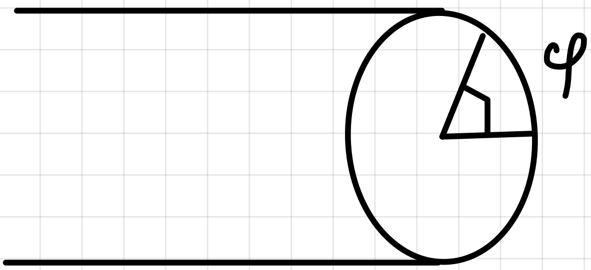


- we detect particles on the surface of a cylinder
- we cannot look down the beam pipe, so we privilege "transverse" quantities

We start by defining a polar angle θ



and an azimuthal angle φ



We often measure transverse energy

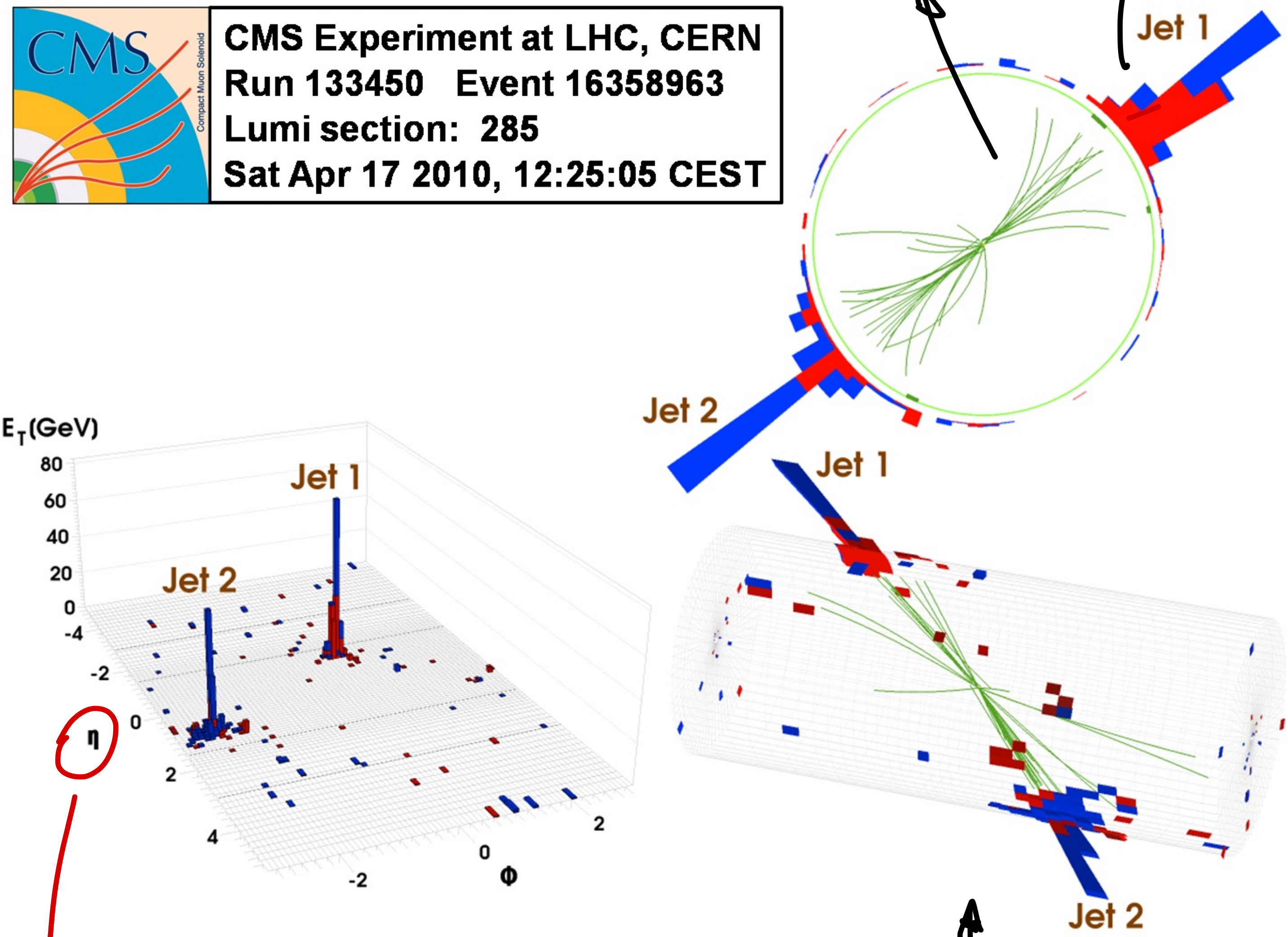
$$E_T \equiv E \sin\theta$$

or a transverse momentum $|\vec{p}_T|$

Example from CMS at LHC

energy deposits in calorimeter

tracks



cylinder open as a plane: "Lego plot"

cylinder

new variable, η, in place of the polar angle θ. More on this later

Also, we'll explain "jet" later

We write the momenta of the incoming protons as

$$P_A = \frac{\sqrt{s}}{2}(1, 0, 0, 1) \qquad P_B = \frac{\sqrt{s}}{2}(1, 0, 0, 1)$$

and of course $(P_A + P_B)^2 = 2P_A \cdot P_1 = S$ centre-of-mass energy squared.

We also anticipate that what collides are constituents of the protons with momenta

$$P_a = x_1 P_A \qquad P_b = x_2 P_B$$

so the cm energy of the collision is in fact

$$\sqrt{\hat{s}} = \sqrt{(x_1 P_A + x_2 P_B)^2} = \sqrt{x_1 x_2 2 P_A \cdot P_B} = \sqrt{x_1 x_2 S}$$

An outgoing momentum p^μ can be written as

$$p^\mu = (E, \underbrace{|\vec{p}|\sin\vartheta\cos\varphi, |\vec{p}|\sin\vartheta\sin\varphi}_{\vec{P}_T}, \underbrace{|\vec{p}|\cos\vartheta}_{P_{\parallel} = P_z})$$

with $E^2 = P_T^2 + P_{\parallel}^2 + m^2$

We introduce

Transverse mass $\quad m_T \equiv \sqrt{p_T^2 + m^2}$

and

Rapidity $\quad y \equiv \frac{1}{2} \log \frac{E+p_z}{E-p_z}$

and also $\quad y = \frac{1}{2}\log \frac{(E+p_z)(E+p_z)}{(E-p_z)(E+p_z)} = \frac{1}{2}\log \frac{(E+p_z)^2}{E^2-p_z^2}$

$$= \frac{1}{2}\log \frac{E+p_z}{m_T^2} = \log \frac{E+p_z}{m_T}$$

One can show that one can rewrite

$$p^\mu = (\underbrace{m_T \cosh y}_{E},\ p_T \cos\varphi,\ p_T \sin\varphi,\ \underbrace{m_T \sinh y}_{p_z})$$

[See explicit calculation in next page]

Proof

$$y = \frac{1}{2} \log \frac{E+p_z}{E-p_z}$$

$$\Rightarrow \quad e^{2y} = \frac{E+p_z}{E-p_z} = \frac{(E+p_z)^2}{E^2-p_z^2} = \frac{(E+p_z)^2}{p_T^2+m^2}$$

$$\Rightarrow \quad \boxed{e^{y} = \frac{E+p_z}{m_T}}$$

Also, $e^{2y} = \frac{E+p_z}{E-p_z} = \frac{E^2-p_z^2}{(E-p_z)^2} = \frac{m_T^2}{(E-p_z)^2}$

$$\Rightarrow \quad e^{y} = \frac{m_T}{E-p_z} \quad \Rightarrow \quad \boxed{e^{-y} = \frac{E-p_z}{m_T}}$$

$$\Rightarrow \quad \begin{array}{rl} m_T e^{y} = E+p_z & \\ m_T e^{-y} = E-p_z & \Big| + \\ \hline m_T(e^{y}+e^{-y}) = 2E & \end{array} \qquad \begin{array}{rl} m_T e^{y} = E+p_z & \\ m_T e^{-y} = E-p_z & \Big| - \\ \hline m_T(e^{y}-e^{-y}) = 2p_z & \end{array}$$

$$\Downarrow$$

$$E = m_T \frac{e^{y}+e^{-y}}{2} \qquad\qquad p_z = \frac{e^{y}-e^{-y}}{2}$$

$$= m_T \cosh y \qquad\qquad = m_T \sinh y \quad \text{OK}$$

Why rapidity?

When $x_1 \neq x_2$, the centre of mass of the collision is boosted with respect to the centre of mass of the pp system.

Rapidity has the property that it transforms additively under a boost:

$$y' = y + w$$

$\Rightarrow \quad \Delta y = y_1 - y_2 = \Delta y'$ invariant

Proof

Boost in positive z direction of $'$frame
Lorentz transformation

$$\begin{cases} \begin{pmatrix} E' \\ p_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} \\ p_T' = p_T \end{cases}$$

and one can rewrite the Lorentz transf. as

$$\gamma = \cosh w$$
$$\gamma\beta = \sinh w \qquad \left(\gamma = \frac{1}{\sqrt{1-\beta^2}} \right)$$

$$\Rightarrow E' = \gamma E - \beta\gamma p_z =$$
$$= (\cosh w) E - (\sinh w) p_z$$
$$= (\cosh w)\, m_T \cosh y - (\sinh w)\, m_T \sinh y$$
$$= m_T (\cosh w \cosh y - \sinh w \sinh y)$$
$$= m_T \cosh(y-w)$$

and $p_z' = m_T(\sinh y \cosh w - \cosh y \sinh w)$
$$= m_T \sinh(y-w)$$

$\Rightarrow$ the boosted momentum

$$p^{\mu\prime} = \left(m_T \cosh(y-w), \vec{p}_T, m_T \sinh(y-w) \right)$$

corresponds, as said, to a rapidity $y-w$.

Rapidity is useful because we can use it to rewrite the Lorentz-invariant phase space

$$d^4p\, \delta^+(p^2-m^2) = \frac{d^3p}{2E} = \frac{1}{2}\, dp_T^2\, dy\, d\varphi$$

One can show that, in the c.m. frame of a collision,

$$(y_{cm})_{max} = \log\left(\frac{\sqrt{s}}{m}\right)$$

For the LHC, $\sqrt{s} = 14$ TeV, and 1 GeV particle (typical hadron), $(y_{cm})_{max} \simeq 9$

In practice, less, because not all the energy of the machine goes into the collision

Pseudorapidity η

For a massless particle, $m=0$, we have

$$m_T = \sqrt{\cancel{m^2}+p_T^2} = p_T \quad \text{and} \quad E = |\vec{p}|$$

$$\Rightarrow \quad E = p_T \cosh y \quad \text{and} \quad p_z = p_T \sinh y$$

$$\Rightarrow \quad E + p_z = p_T(\cosh y + \sinh y) = p_T e^{y}$$

This means that, for $m=0$,

$$y = \log \frac{E+p_z}{p_T} = \frac{|\vec{p}| + |\vec{p}|\cos\theta}{|\vec{p}|\sin\theta}$$

$$= \log \frac{1+\cos\theta}{\sin\theta} = \log \frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}}$$

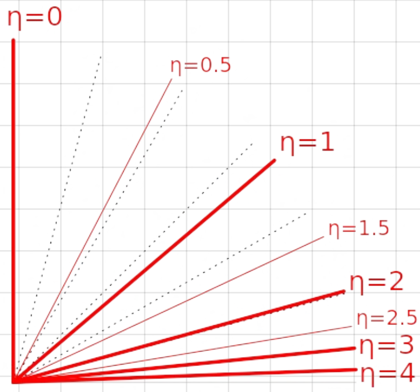
$$= \log \frac{1+\cos\theta}{\sqrt{(1+\cos\theta)}\sqrt{(1-\cos\theta)}}$$

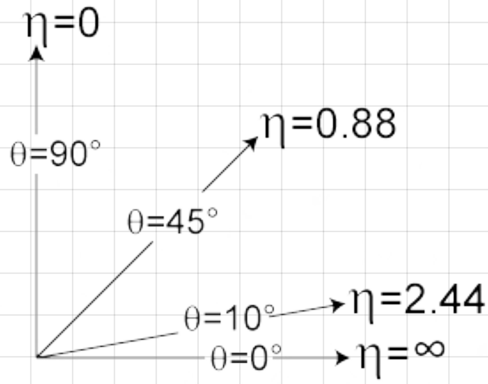
$$= \log \frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} = \log \frac{2\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}}$$

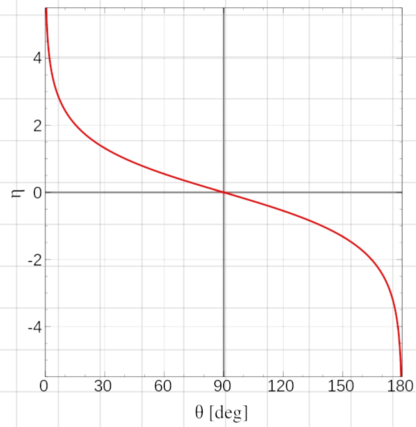
$$= -\log \operatorname{tg} \frac{\theta}{2} \equiv \eta$$

Pseudorapidity is a purely geometrical variable.

It has no special boost property, but it can be used to locate particles on the detector cylinder.







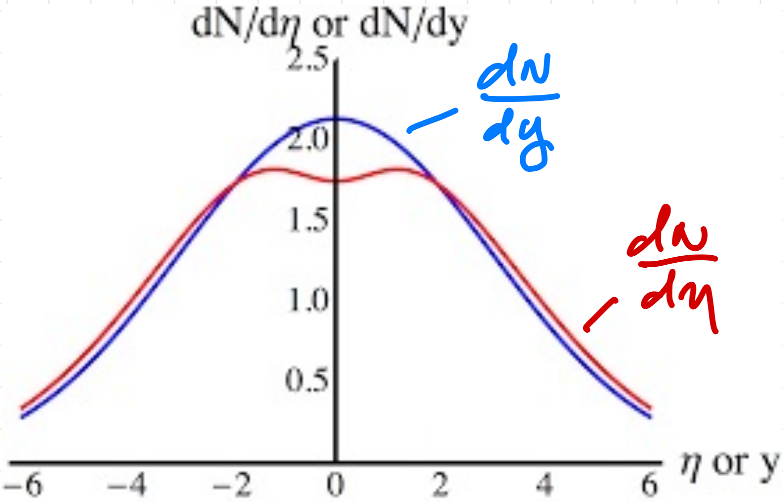
"Central events" ($P_z \simeq 0$) correspond to $\eta \simeq 0$ (i.e. $\theta \simeq \pi/2 = 90°$)

"Forward events" have large η (at the LHC the main components of a detector typically go until $\eta \simeq 2$-3, i.e. $\theta \simeq 5°$-$15°$)

When $E \gg m$, one can use pseudorap. (easily pinpointed in the detector) to approximate the rapidity of a particle, $\eta \simeq y$, while the measurement of y requires the knowledge of both E and P_z

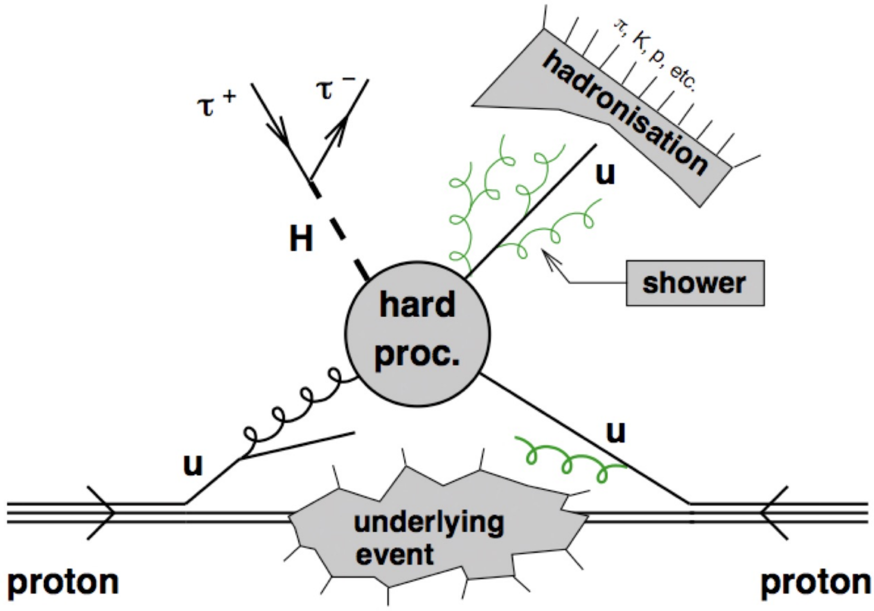
For massive particles, there is a Jacobian between rapidity and pseudorapidity distributions

$$\frac{dN}{d\eta} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \; \frac{dN}{dy}$$



QCD

With a better drawing (from Gavin Salam) we depict the QCD shells as



final state

hard interaction

initial state

We shall describe QCD tools and techniques to address these three stages (which are not fully independent/independently defined)

Note however that we shall concentrate on the <u>perturbative</u> parts of the description

→ won't talk about underlying event or hadronization

Also note that the fact that we can calculate anything at all in strong interactions should not be underestimated!

In the '50s / '60s:

"We are driven to the conclusion that the $\overbrace{\text{Hamiltonian method}}^{\text{QFT}}$ for strong interactions is dead and must be buried, although of course with deserved honor"

Lev Landau

"The correct theory [of strong interactions] will not be found in the next hundred years"

Freeman Dyson

The fact that today we have QCD and can calculated at least some selected observables with an $O(1\%)$ accuracy is nothing short of a huge achievent in HEP

What is QCD?

It is a non-abelian local gauge theory, with matter fermions in the fundamental representation of the $SU(3)_{colour}$ group (the quarks) and eight gauge fields (the gluons) in the adjoint representation

The lagrangian is invariant under the local gauge transformations

$$\psi_b \to \psi_a = e^{i\theta^c(x) t^c_{ab}} \psi_b$$

$$A^c_\mu t^c \to A^c_\mu t^c + \frac{1}{g}\left(\partial_\mu \theta^c(x)\right) t^c + i\left[t^c, t^D\right]\theta^c(x) A^D_\mu$$

The t^c_{ab} matrices are the 8 generators of the fundamental representation of the SU(3) group

Will try to use upper case for adj. rep. and lower case for fund. rep.

$$(t^c)_{ab}$$

index of the adjoint representation, 1 --- 8

indices of the fundamental representation, 1 --- 3

The group is defined by the Lie algebra of its generators:

$$[t^A, t^B] = i\underbrace{f^{ABC}}_{\text{structure constants}} t^C$$

Problem 9.2 Gell-Mann matrices.
The Gell-Mann matrices,

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \qquad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix},$$

are the generalization to SU(3) of the Pauli matrices. The quantities $\frac{1}{2}\lambda_i$ are the generators of the $\mathfrak{su}(3)$ Lie algebra (*i.e.* they form a basis for this algebra).

The (conventional) normalisation of the generators is such that

$$\mathrm{Tr}(t^A t^B) = \frac{1}{2}\delta^{AB} \equiv T_F \delta^{AB}$$

$$\Rightarrow \quad t^A = \frac{\lambda^A}{2}$$

This also fixes the f^{ABC} (or viceversa)

The QCD Lagrangian is similar to QED, but with crucial differences

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma_\mu A^\mu \psi$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathcal{L}_{QCD} = \sum_{\text{flavours}} \bar{\psi}_i(i\not{\partial} - m_i)\psi_i - \frac{1}{4}F^C_{\mu\nu}F^{C,\mu\nu} - \sum_{fl.} g\,\bar{\psi}_i\gamma_\mu A^{C,\mu} t^C \psi_i$$

with $F^C_{\mu\nu} = \partial_\mu A^C_\nu - \partial_\nu A^C_\mu + g f^{CAB} A^A_\mu A^B_\nu$

and f^{ABC} are the structure constants of the Lie group defined by the commutation rules of its generators

[Note that the generator matrix t^C_{ab} acts onto the colour indices of a fermion:

$$t^C \psi_i = (t^C)_{ab}\,\psi_{bi} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

colour index (b)

flavour index (i)
]

What are the differences with QED?

Expanding the QCD Lagrangian we get

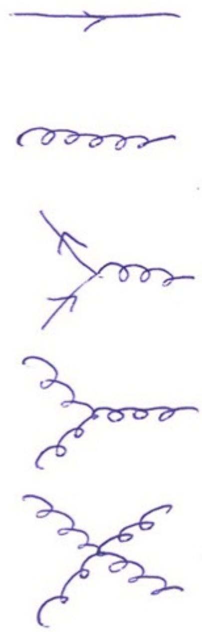
$$\Rightarrow \mathcal{L} = \sum \bar\psi_i (i\partial\!\!\!/ - m_i)\psi_i$$

$$- \frac{1}{4} \sum (\partial_\mu A_\nu^C - \partial_\nu A_\mu^C)^2$$

$$+ g \bar\psi_i \gamma^\mu A_\mu^C t^C \psi_i$$

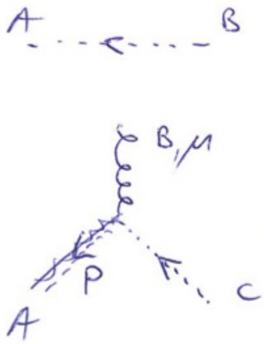
$$- g f^{ABC} (\partial_\mu A_\nu^A) A^{\mu B} A^{\nu C}$$

$$- \frac{g^2}{4} f^{EAB} A_\mu^A A_\nu^B f^{ECD} A^{\mu C} A^{\nu D}$$



(+ ghosts ...) Faddeev-Popov

$$\mathcal{L}_{ghost} = \bar c^A (-\partial^\mu D_\mu^{AC}) c^C$$



$$\frac{i\delta^{AB}}{p^2}$$

$$-g f^{ABC} p^\mu$$