



MAX School on Advanced Materials and Molecular Modelling with QUANTUM ESPRESSO

Chasing saddle points: the NEB method



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The Abdus Solorn International Centre for Theoretical Physics



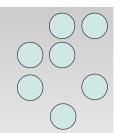


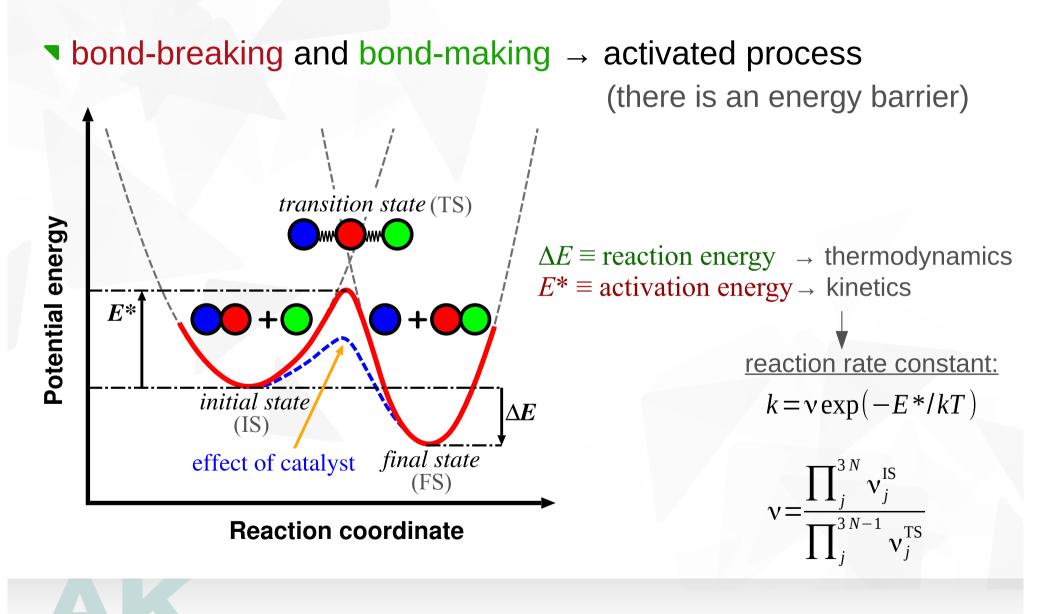


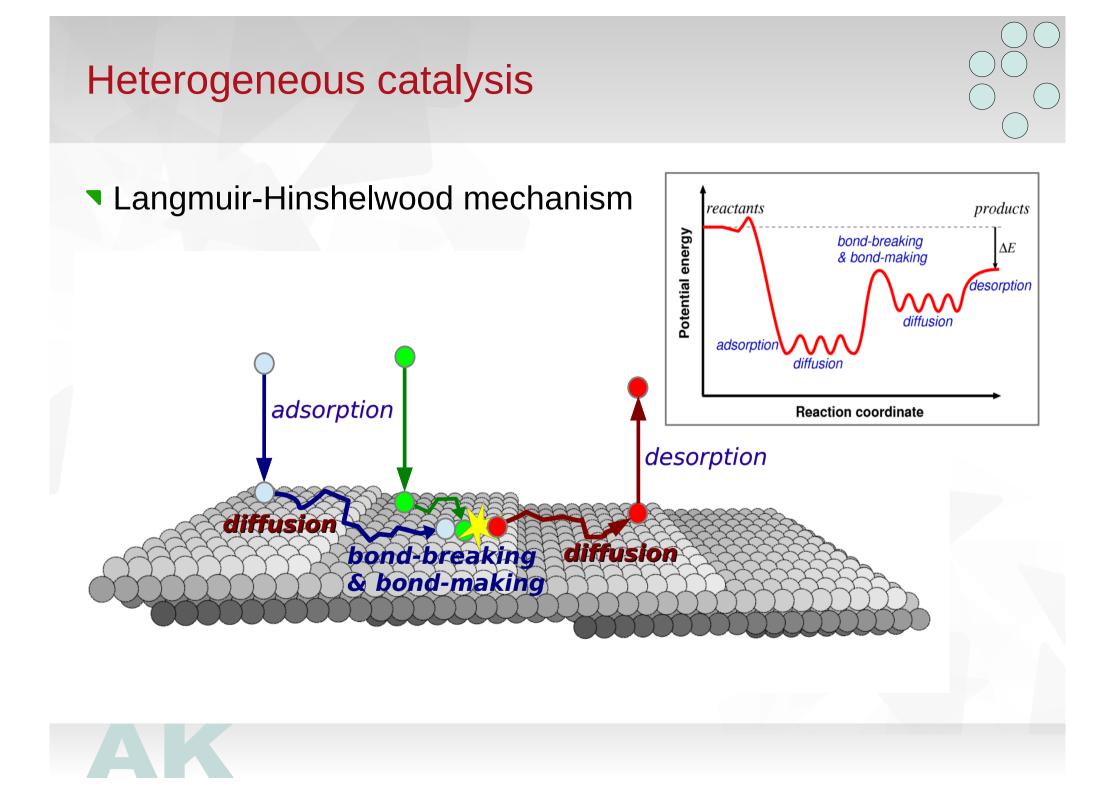
*cecam



Chemical reaction

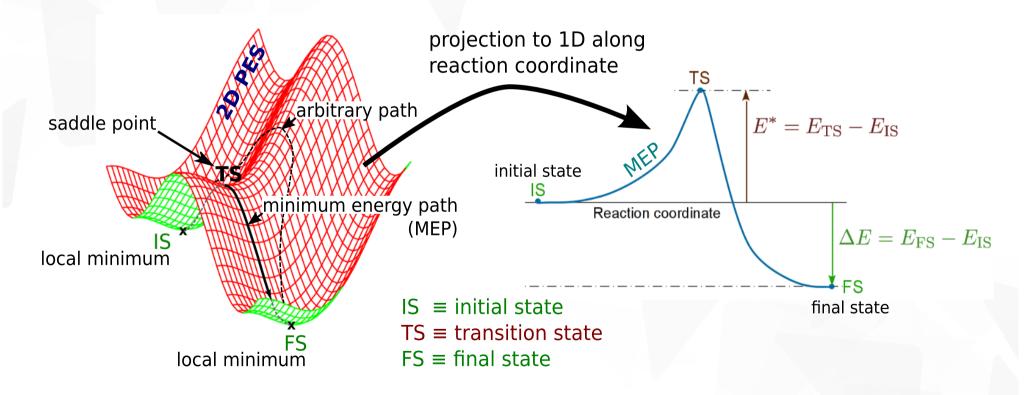






Elementary activated reaction step

potential energy surface (PES) is highly multidimensional (except for trivial examples)

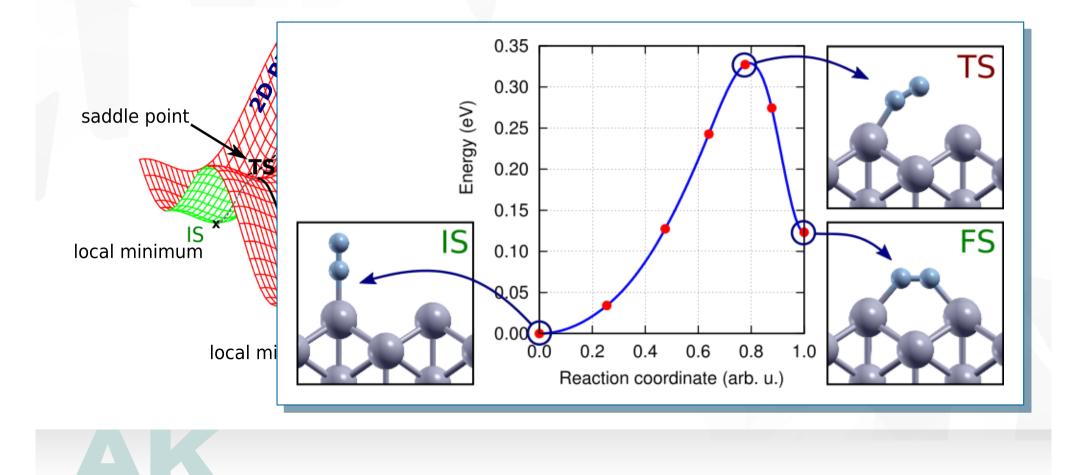


saddle points are unstable configurations and their location is a difficult talk



Elementary activated reaction step

potential energy surface (PES) is highly multidimensional (except for trivial examples)



Why is activation energy so important

- Activation energy is a very good criterion to tell if a given activated process (or chemical reaction) is kinetically feasible at given temperature or how fast is it ...
- Reaction rate constant: $k = \nu \exp(-E^*/kT)$ $\nu = \frac{\prod_{j=1}^{3N} \nu_j^{\text{IS}}}{\prod_{j=1}^{3N-1} \nu_j^{\text{TS}}} \approx 10^{13} \text{ s}^{-1}$
- "Frequency" prefactors v depend on reaction type (e.g. for desorption $v \approx 10^{16} \text{ s}^{-1}$) and were tabulated by V. P. Zhdanov (Surf. Sci. Rep. **12**, 183–242 (1991))



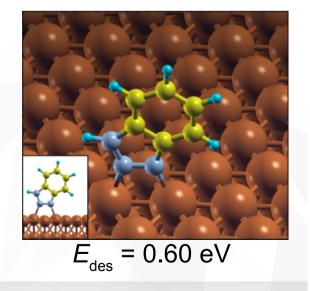
Why is activation energy so important

- Activation energy is a very good criterion to tell if a given activated process (or chemical reaction) is kinetically feasible at given temperature or how fast it is ...
- **Example:** typical residence time of a molecule with desorption energy (E_{des}) of 0.6 eV on the surface at T = 300 K

$$\tau = \nu^{-1} \exp\left(\frac{E_{\text{des}}}{kT}\right) \approx 10^{-6} \text{ s}$$

• For $E_{des} = 1 \text{ eV}$: $\tau = 10 \text{ s}$

molecule must adsorb stronger than 1 eV to persist on the surface at room T !



Jargon terminology – "image"

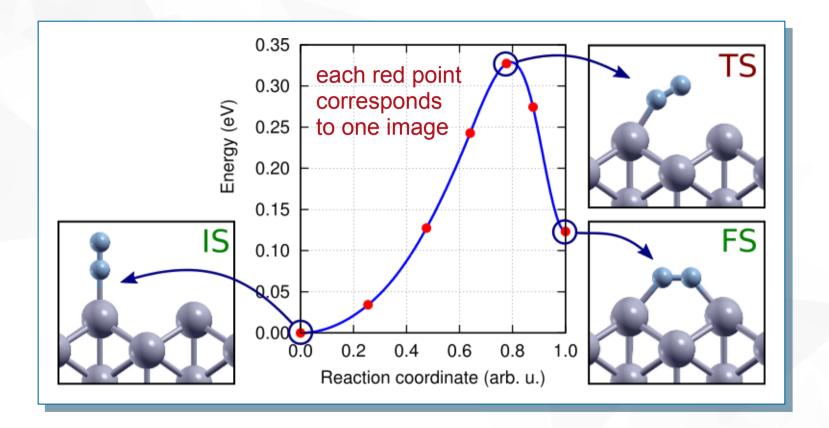


image is a given snapshot (configuration) of the whole system



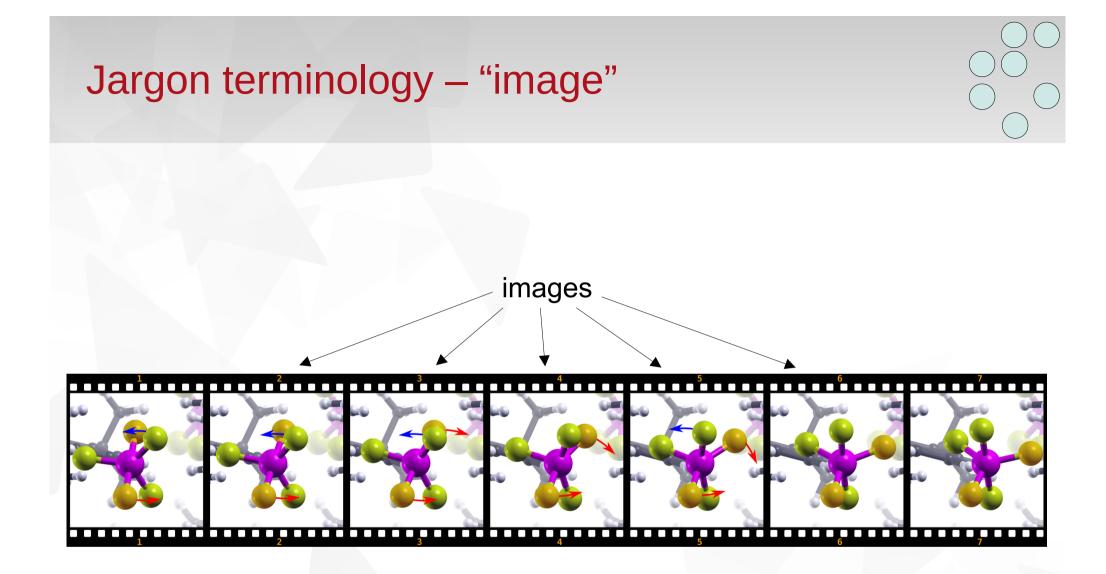
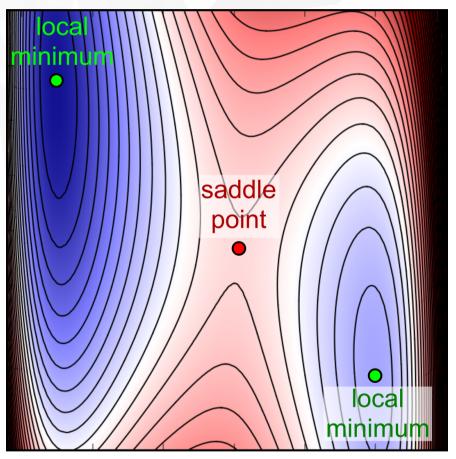


image is a given snapshot (configuration) of the whole system



PES – potential energy surface E(R)

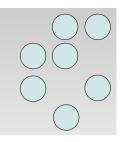


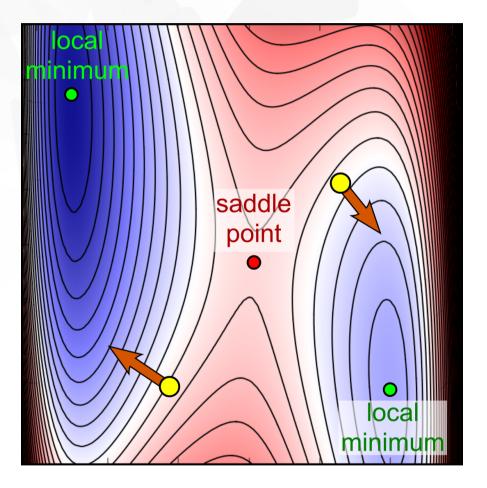
each point on this graph is an image

Remarks:

- for obvious reasons, only
 2D PES can be visualized
- For real examples, PES is highly multidimensional: 3N dimensional for N atoms
 - $R \equiv 3N$ dimensional vector (coordinates of all atoms)

Finding local minima





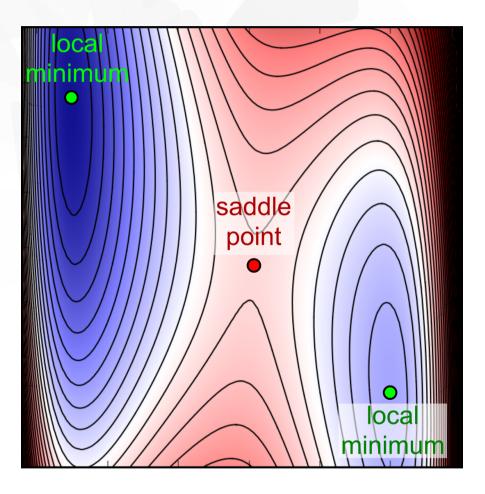
- Finding local minima is routine (provided the code can calculate first derivatives – forces)
 - ... by steepest descent

$$\boldsymbol{R}^{(n+1)} = \boldsymbol{R}^{(n)} - \lambda \nabla E(\boldsymbol{R}^{(n)})$$

or any other more efficient method (see the preceding talk)

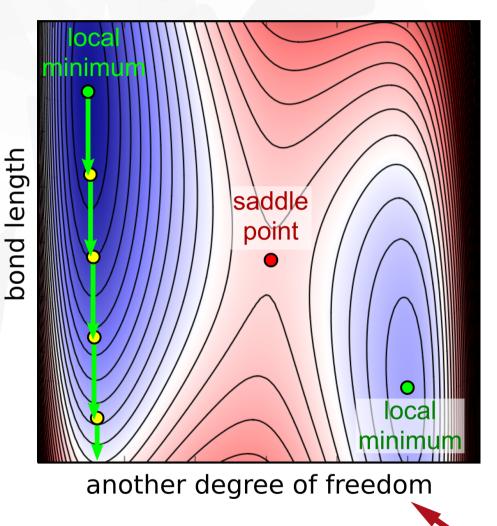
 $\mathbf{R}^{(n)} \equiv$ coordinates at iteration n

How to find saddle points?



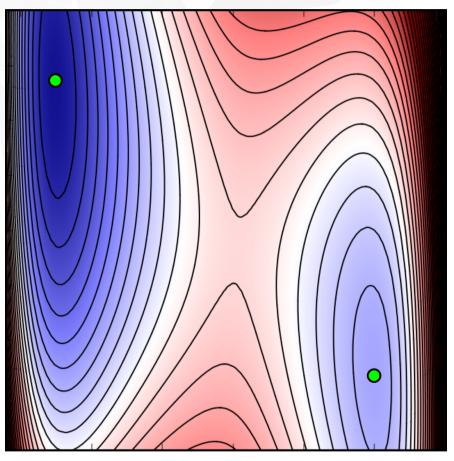
- Finding a saddle point is much more difficult than finding a local minimum
- An old, easy to realize method ("slowest ascend"):
 - step-wise stretch the bond
 - at each step perform constrained optimization

How to find saddle points?



- Finding a saddle point is much more difficult than finding a local minimum
- An old, easy to realize method ("slowest ascend"):
 - step-wise stretch the bond
 - at each step perform constrained optimization

May work well for dissociation reactions, but may also **fail badly**.



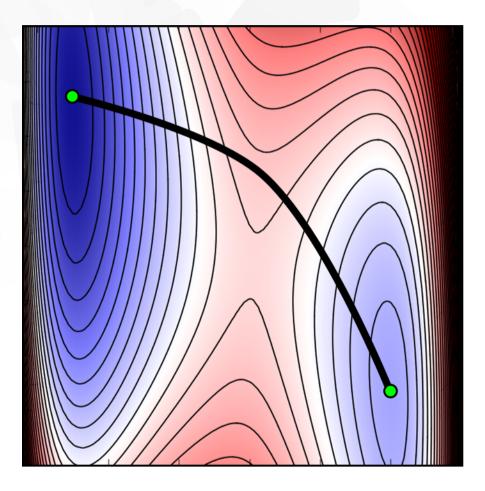
NEB = nudged elastic band

NEB references:

G. Henkelman, H. Jonsson, J. Chem Phys. **113**, 9978–9985 (2000)

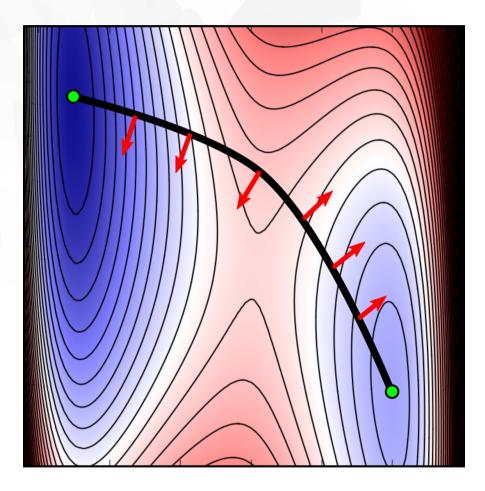
G. Henkelman, B. P. Uberuaga, H. Jonsson, J. Chem Phys. 113, 9901–9904 (2000)





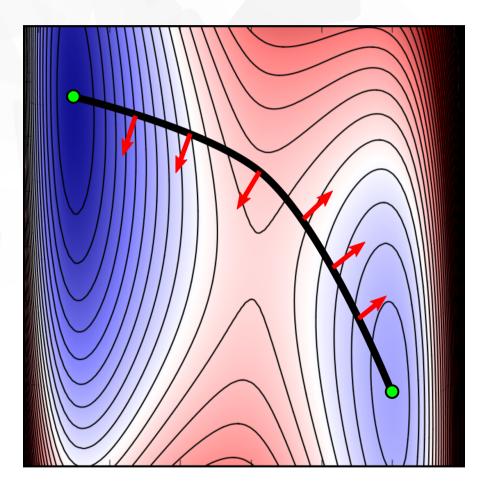
1. connect two minima with an elastic band





- 1. connect two minima with an elastic band
- 2. relax the band with orthogonal forces (F₁)





1. connect two minima with an elastic band

E.

 $\mathbf{F}_{\perp} = \mathbf{F} - \mathbf{F}_{\parallel}$

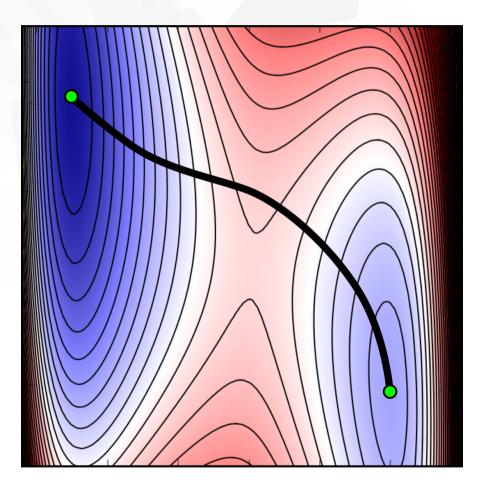
 $=\mathbf{F}-(\mathbf{F}\cdot\mathbf{ au})\mathbf{ au}$

and $\mathbf{F} = -\nabla E(\mathbf{R})$

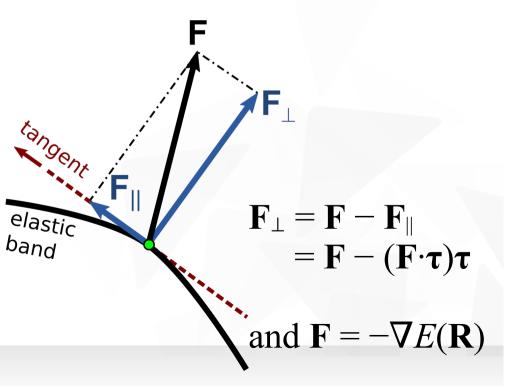
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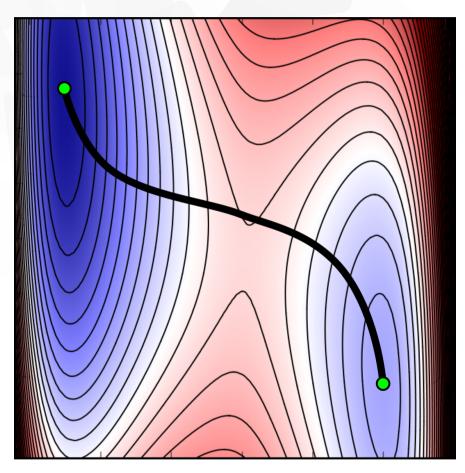
elastic

band

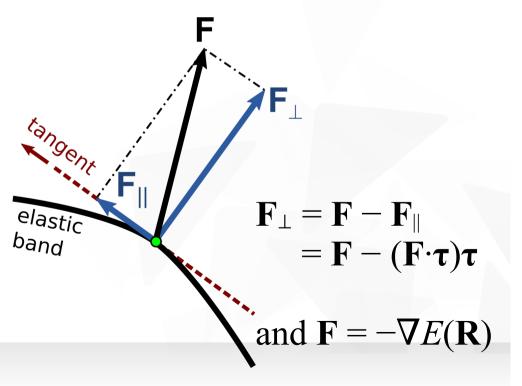


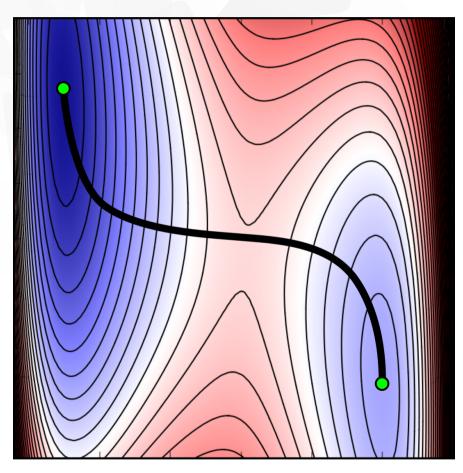
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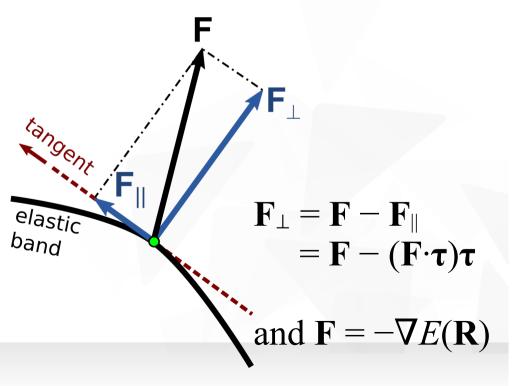


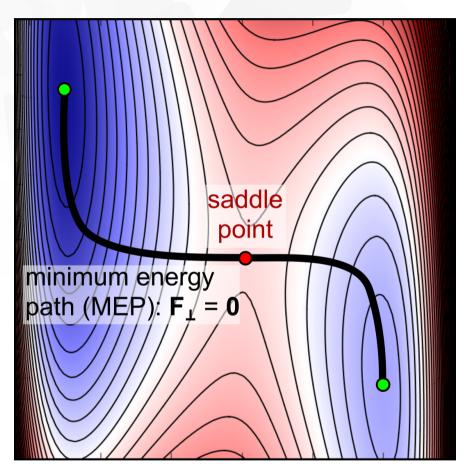
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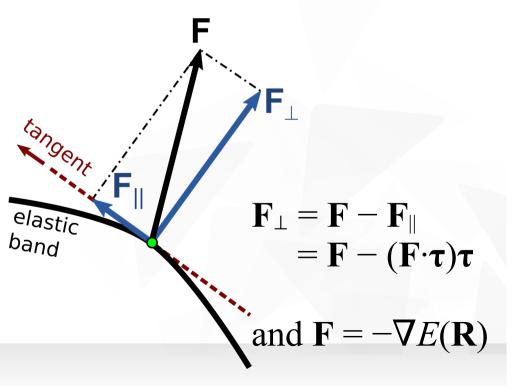


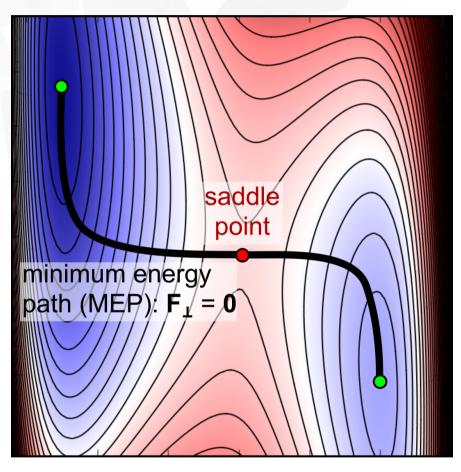
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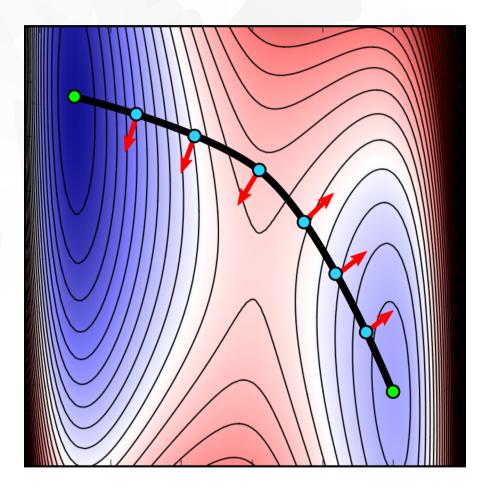
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- 1. connect two minima with an elastic band
- 2. relax the band with orthogonal forces (F₁)
- 3. but this was only thought experiment!



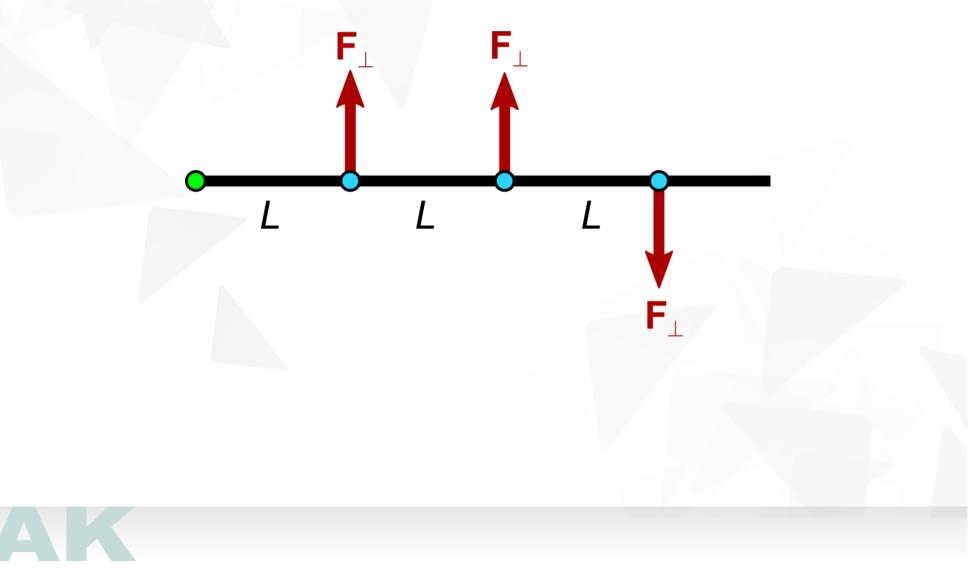


- 1. connect two minima with an elastic band
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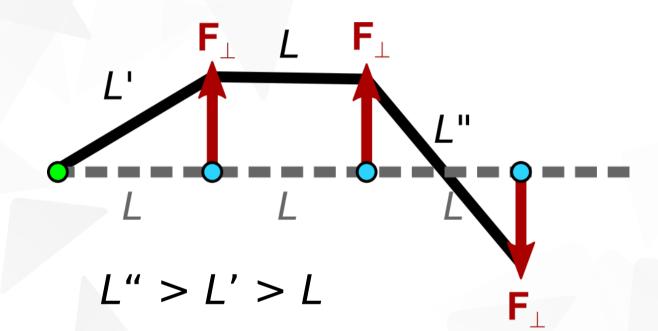
In computer, for obvious reasons, the elastic band is discretized into several "**images**"!



A problem with path discretization ...



A problem with path discretization ...



Path dynamics does not preserve inter-image distances (after many iterations severe problems can emerge)

Solution: connect images by springs

A problem with path discretization ...

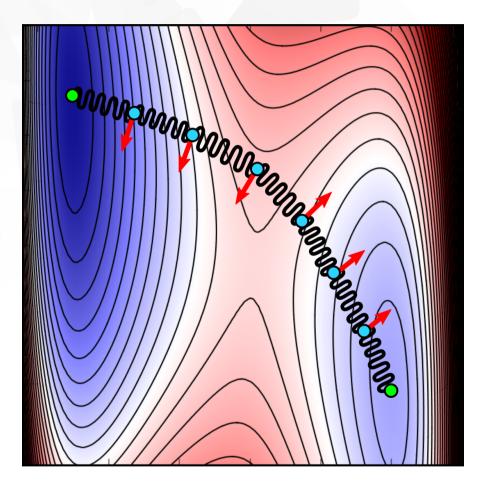
Path dynamics does not preserves inter-image distance (after many iterations severe problems can emerge)

Solution: connect images by springs that act only along reaction path

$$\frac{\mathbf{R}_{i-1} \quad \mathbf{R}_i \quad \mathbf{R}_{i+1}}{\boldsymbol{\tau}_i}$$

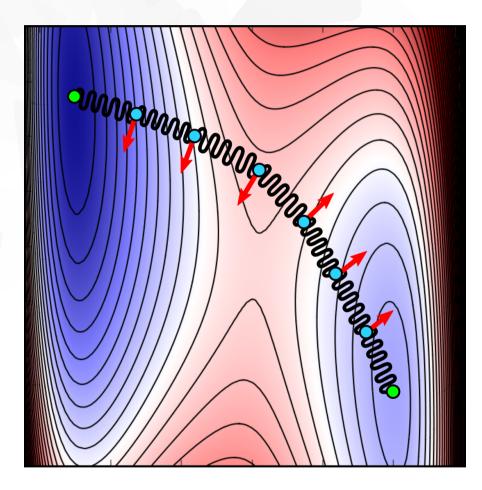
 \mathbf{R}_i = atomic coordinates of image *i*

 τ_i = normalized tangent to reaction path for image *i*



- 1. connect two minima with a "guessed" reaction path (can be linear)
- 2. discretize reaction path into several images: R₁, R₂, R₃...
- 3. connect images by springs
- 4. minimize reaction path using the NEB recipe





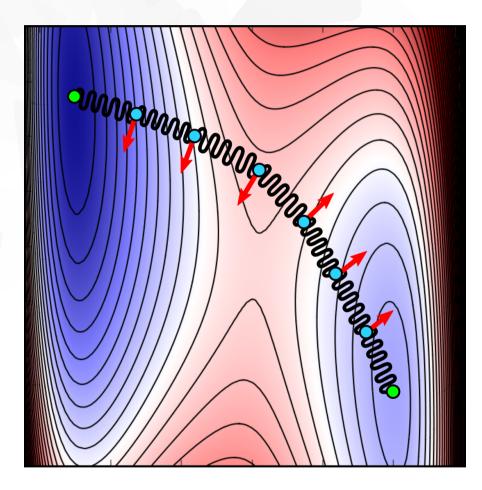
- 4. minimize reaction path using the NEB recipe:
 - True forces, $-∇E(\mathbf{R}_i)$, act only perpendicular to the reaction path
 - spring forces act only along the reaction path

$$\mathbf{F}_i = -\nabla E(\mathbf{R}_i)|_{\perp} + \mathbf{F}_i^{\text{spring}}|_{\parallel}$$

where $\nabla E(\mathbf{R}_i)|_{\perp} = \nabla E(\mathbf{R}_i) - (\nabla E(\mathbf{R}_i) \cdot \boldsymbol{\tau}_i)\boldsymbol{\tau}_i$ $F_i^{\text{spring}}|_{\parallel} = k(|\mathbf{R}_{i+1} - \mathbf{R}_i| - |\mathbf{R}_i - \mathbf{R}_{i-1}|)\boldsymbol{\tau}_i$



 τ_i = normalized tangent to reaction path for image *i*



- 4. minimize reaction path using the NEB recipe:
 - True forces, -∇E(R_i), act only perpendicular to the reaction path
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»nudging«

$$\mathbf{F}_i = -\nabla E(\mathbf{R}_i)|_{\perp} + \mathbf{F}_i^{\text{spring}}|_{\parallel}$$

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 τ_i = normalized tangent to reaction path for image *i*

Choice of the tangent $\boldsymbol{\tau}_i$ is crucial for convergence!

A good choice is a *local* tangent taken only with respect to the adjacent image with the higher energy:

$$T_{i} = \begin{cases} T_{i}^{+} = R_{i+1} - R_{i} & \text{if } E_{i+1} > E_{i} > E_{i-1} \\ T_{i}^{-} = R_{i} - R_{i-1} & \text{if } E_{i+1} < E_{i} < E_{i-1} \end{cases} \text{ and } \tau_{i} = \frac{T_{i}}{|T_{i}|}$$

(if $E_{i+1} > E_i < E_{i-1}$ or $E_{i+1} < E_i > E_{i-1}$ then T_i is an energy-weighted sum of T_i^+ and T_i^-)



- Transition state (TS) the saddle point on the MEP is the most interesting point
- **Hence:** higher resolution of points around TS is preferable
- Variable spring constants (between kmin & kmax)

$$k_{i} = \begin{cases} k_{\max} - \Delta k \frac{E_{\max} - \tilde{E}_{i}}{E_{\max} - E_{ref}} & \text{if } \tilde{E}_{i} > E_{ref} \\ k_{\min} & \text{if } \tilde{E}_{i} < E_{ref} \end{cases}$$

where $E_{ref} = \max(E_{IS}, E_{FS}) \checkmark IS = \text{initial state} FS = \text{final state}$

 $k_i \equiv \text{spring constant between images } i - 1 \text{ and } i$ $\tilde{E}_i \equiv \max(E_{i-1}, E_i) \qquad \Delta k = k_{\max} - k_{\min}$



Climbing image NEB (CI NEB)

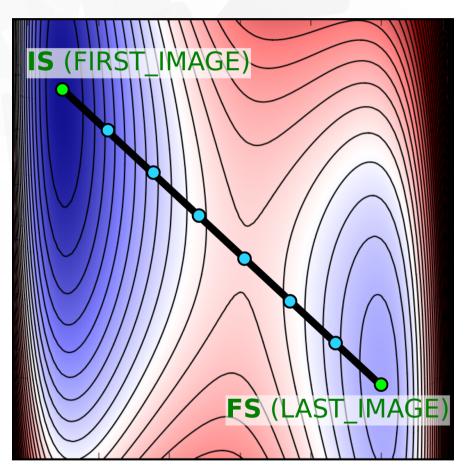
- Even with many images and variable spring constants no image may lie sufficiently close to the saddle point
- Hence: let the image with the highest energy climb up (this image is decoupled from the springs)
- Climbing-up means that F_{\parallel} is reverted —

$$egin{aligned} m{F}_{i_{ ext{max}}} &= m{F}_{ot} - m{F}_{ot} \ &= m{F} - 2m{F}_{ot} \ &= -
abla E(m{R}_{i_{ ext{max}}}) + 2(
abla E(m{R}_{i_{ ext{max}}}) \cdot m{ au}_{i_{ ext{max}}}) m{ au}_{i_{ ext{max}}}) \end{aligned}$$

 $oldsymbol{F}=oldsymbol{F}_{\perp}+oldsymbol{F}_{\parallel}$



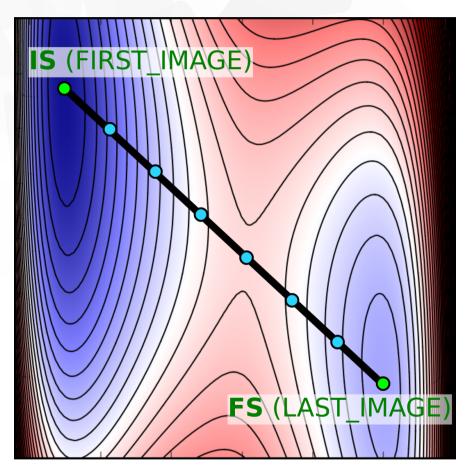
NEB calculations – practical aspects



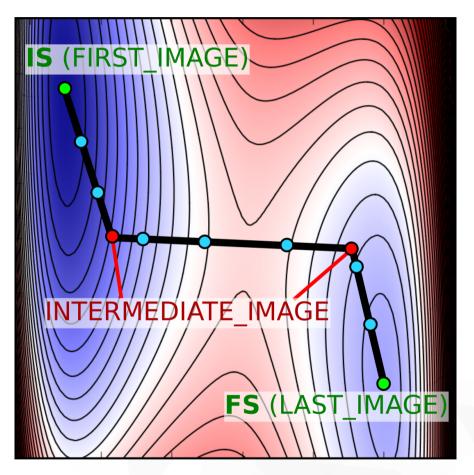
here linearly interpolated reaction path passes very close to the saddle point, but in real cases this is seldom the case

- IS ≡ initial state
 FS ≡ initial state
- default initial guess for the reaction path:
 linear interpolation

NEB calculations – intermediate images

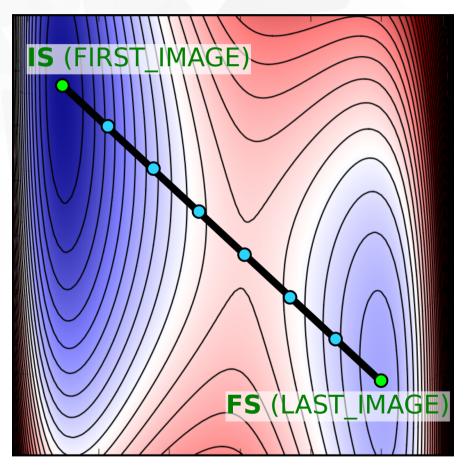


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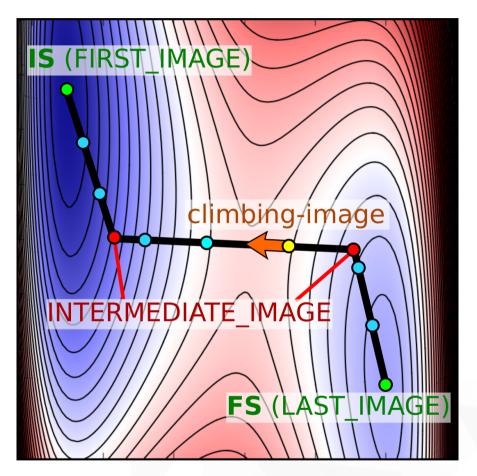


intermediate images are used to steer initial reaction path (they are not actually used); exploting chemical intuition is a plus

NEB calculations – climbing image

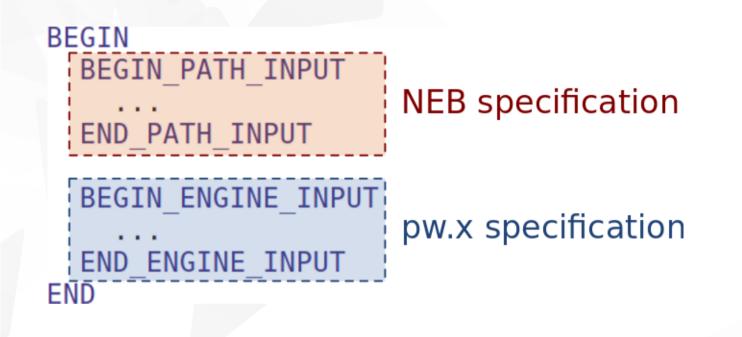


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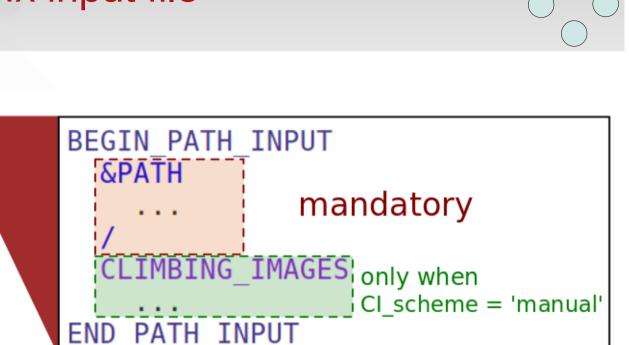
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Structure of the neb.x input file





Structure of the neb.x input file





BEGIN

END

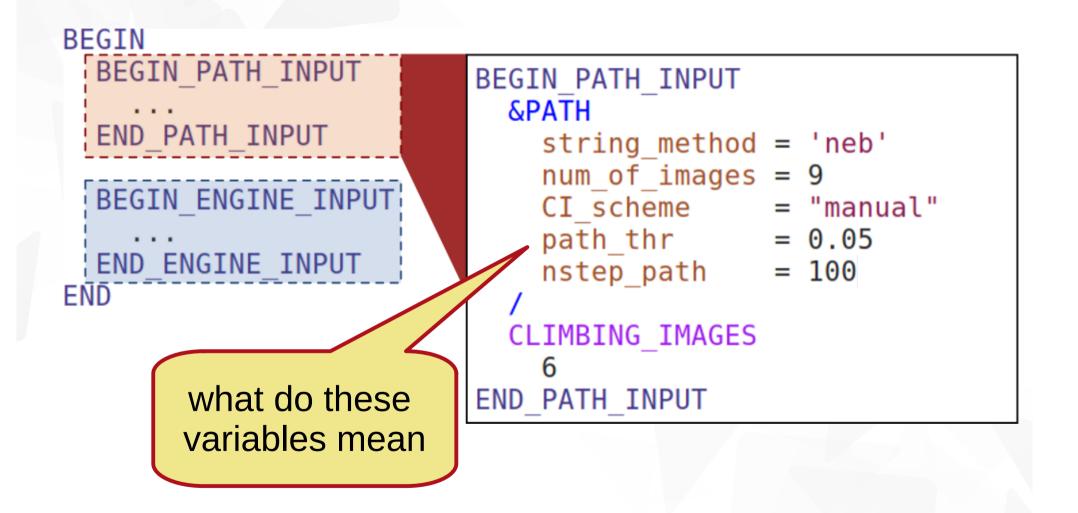
BEGIN PATH INPUT

END PATH INPUT

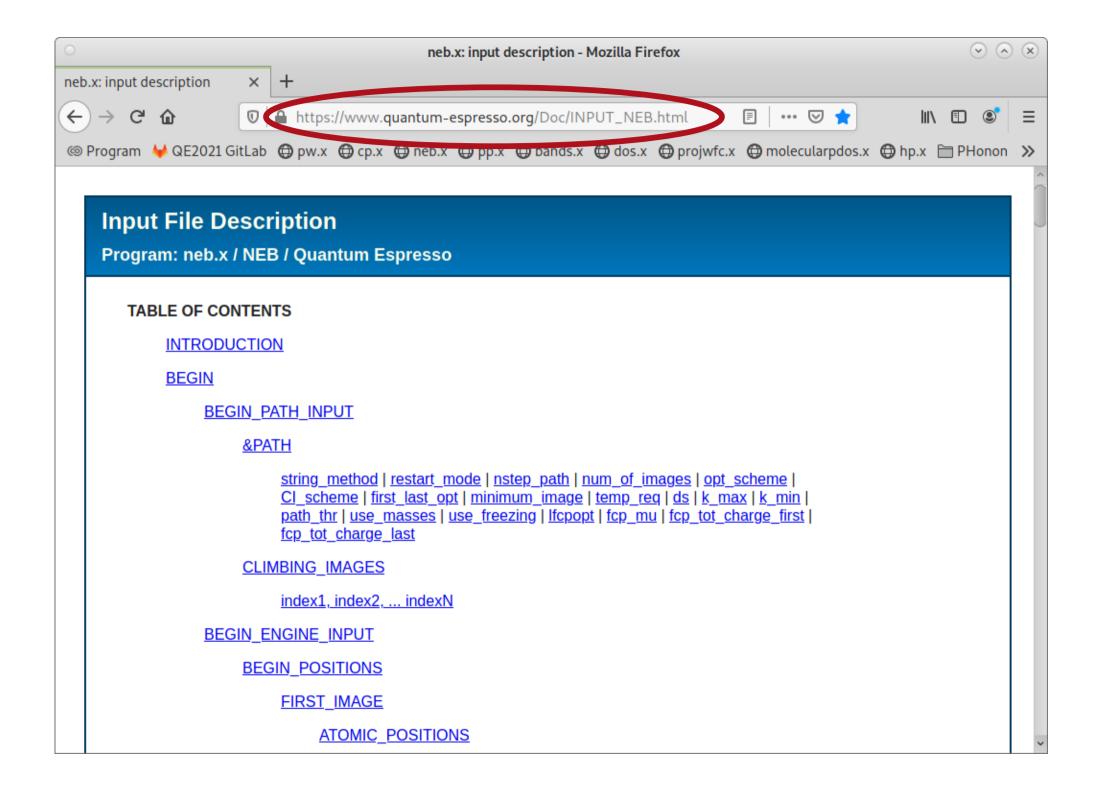
BEGIN ENGINE INPUT

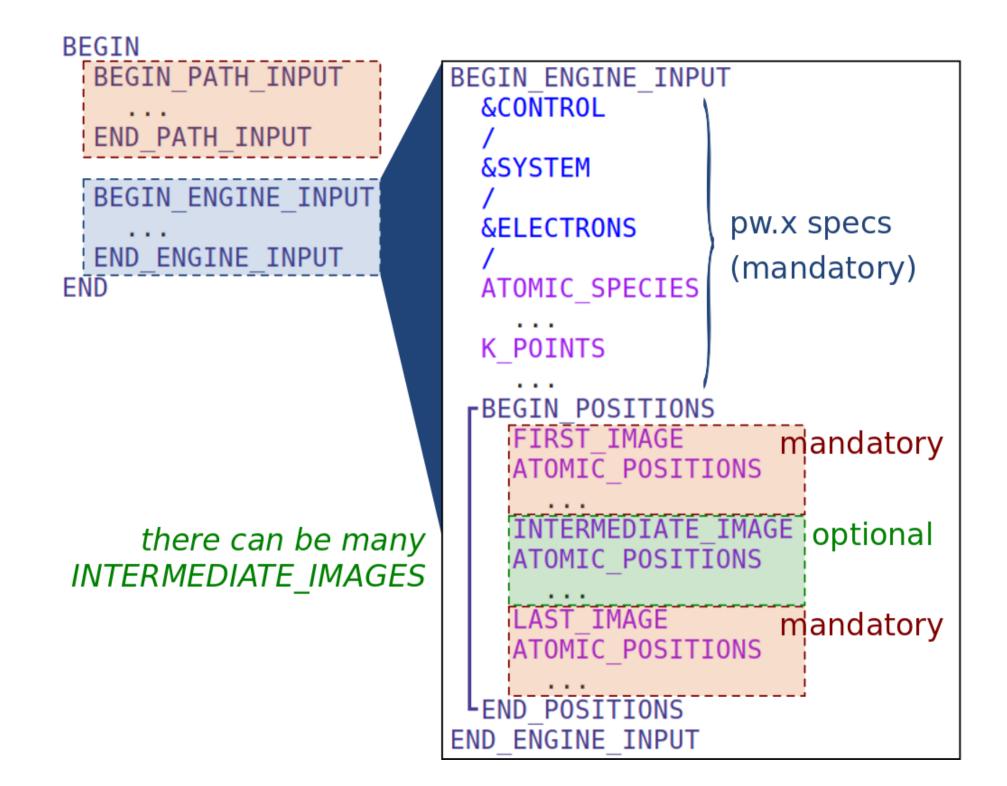
END ENGINE INPUT

Structure of the neb.x input file

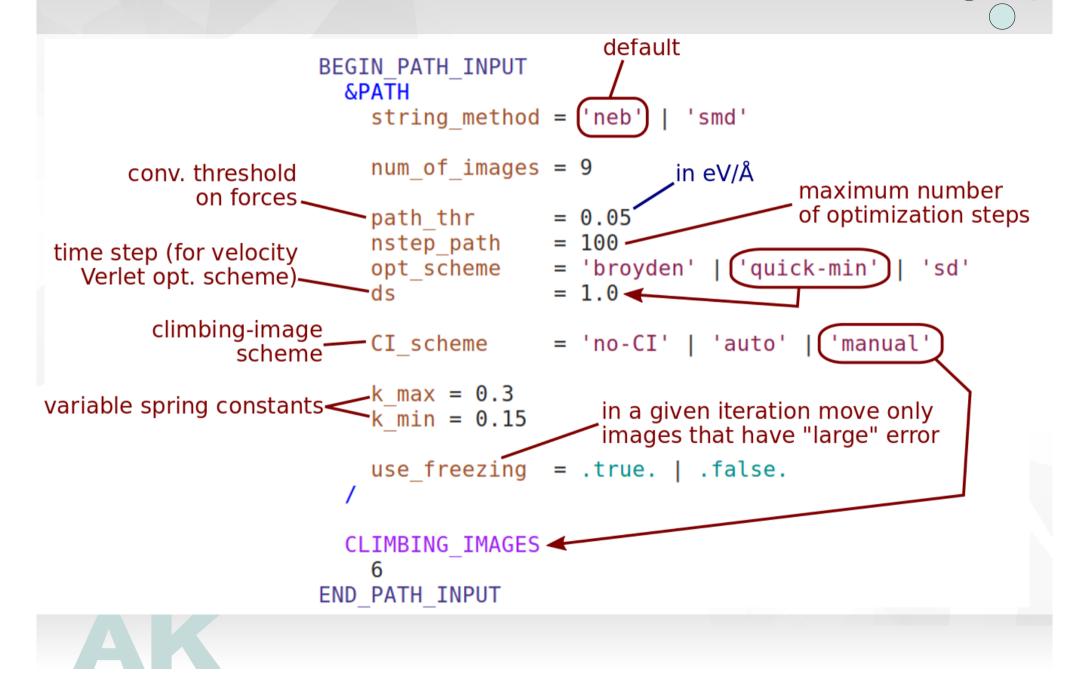


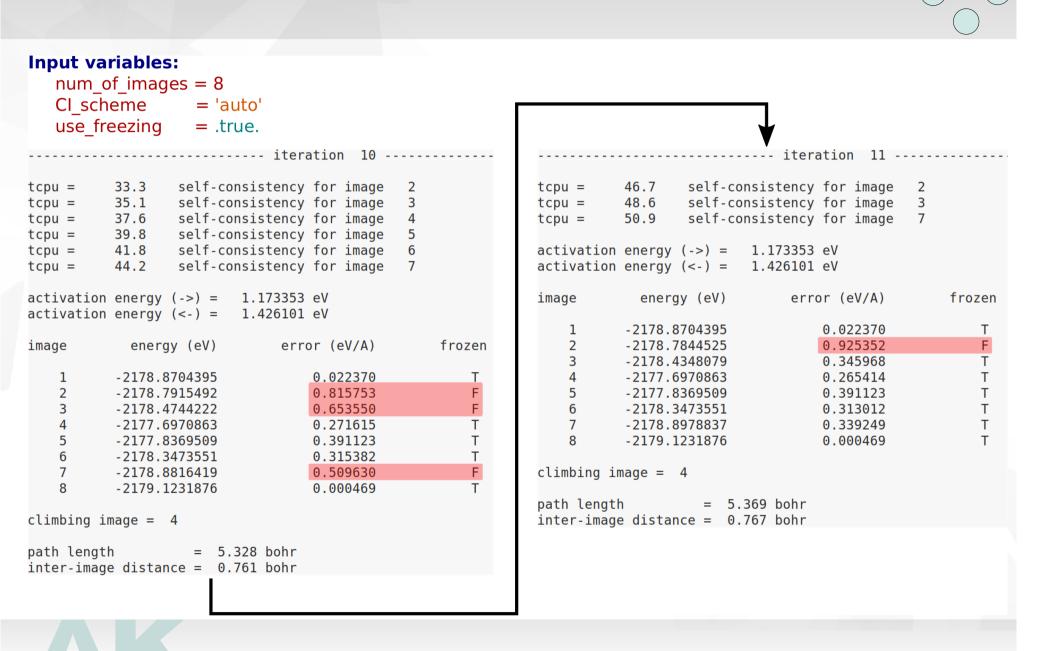






Structure of the neb.x input file: PATH input





Structure of the neb.x output file

NEB calculations: practical aspects

NEB calculation may be relatively difficult to converge (experience is a plus !)

- **Some practical guidelines** (based on experience):
 - use chemical intuition to guess a "reasonable" initial reaction path:
 - start from relaxed initial and final states
 (FIRST_IMAGE & LAST_IMAGE)
 - use INTERMEDIATE_IMAGES

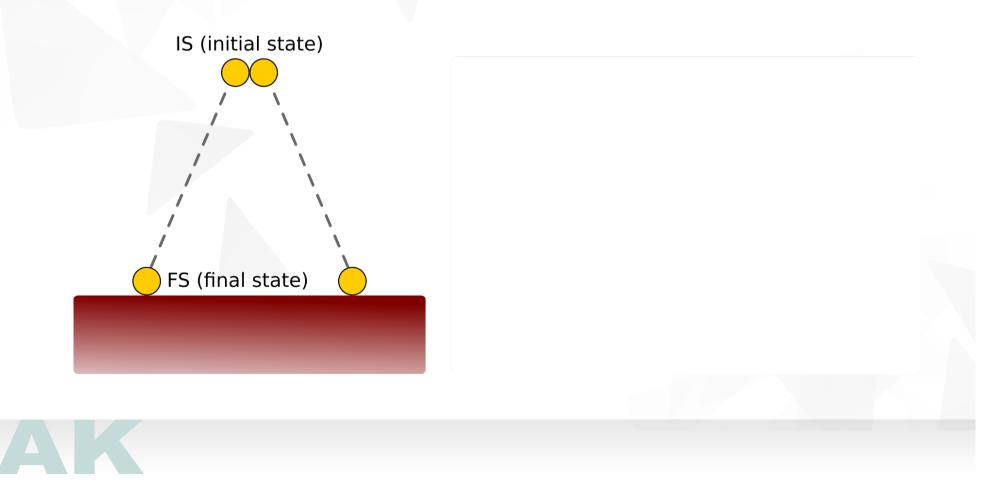
(use of first_last_opt = .true. is discouraged, i.e., optimization of FIRST_IMAGE and LAST_IMAGE during NEB run itself)



Practical aspects: use chemical intuition

Use INTERMEDIATE_IMAGES

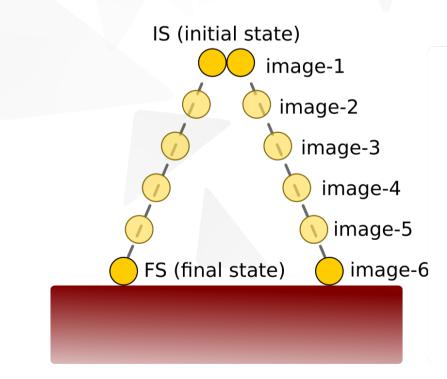
dissociation of a molecule at a surafce



Practical aspects: use chemical intuition

Use INTERMEDIATE_IMAGES

dissociation of a molecule at a surafce

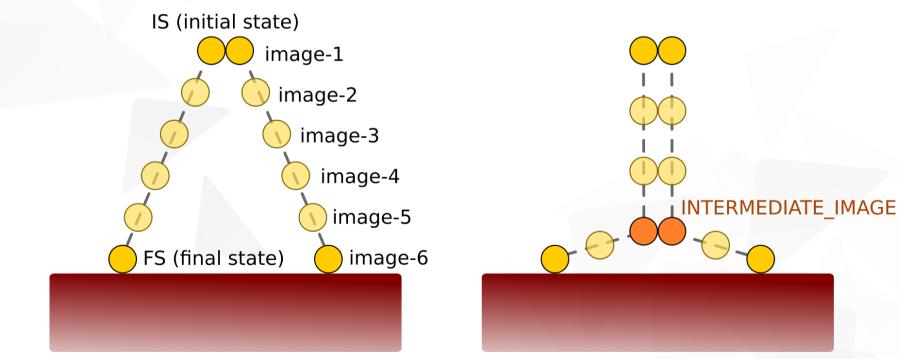




Practical aspects: use chemical intuition

Use INTERMEDIATE_IMAGES

dissociation of a molecule at a surafce



much better initial guess !



NEB calculations: number of images

Question: How many images should I use?

Answer: It depends, but usually inter-image distance in range of 1 to 2 Bohr should be OK (inter-image distance is printed in the output; make a neb.x dryrun by setting $nstep_path = 0$)

activation e activation e						
image	energy	(eV)	err	or (eV/A)	froze	en
1 2 3 4 5 6 7 8	0.00 0.00 0.00 0.00 0.00 0.00 0.00			$\begin{array}{c} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 &$	F F F F	
climbing ima	age = 0					
path length inter-image			247 bohr 750 bohr			
NEB	:	0.39s (CPU	0.41s WALL		

NEB calculations: number of images

Question: How many images should I use?

Answer: It depends, but usually inter-image distance in range of 1 to 2 Bohr should be OK (inter-image distance is printed in the output; make a neb.x dryrun by setting $nstep_path = 0$)

What is the inter-image distance & how is it measured? It is a distance in a 3N dimensional space, i.e.:

$$d_{i,i+1} = |\mathbf{R}_i - \mathbf{R}_{i+1}| = \sqrt{\sum_{j=1}^{3N} (x_{i,j} - x_{i+1,j})^2}$$

coordinates of all atoms of the image i

NEB calculations: number of images

Question: How many images should I use?

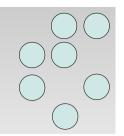
Answer: It depends, but usually inter-image distance in range of 1 to 2 Bohr should be OK (inter-image distance is printed in the output; make a neb.x dryrun by setting $nstep_path = 0$)

Visualize the initial reaction path before doing a calculation, e.g.:

xcrysden --pwi neb.in



Various hints ...



- Calculate one elementary reaction step (one saddle point) at a time (note that Cl_scheme = 'manual' allows for more saddle points simultaneously)
- Question: Which value of spring constants (k_min, k_max) should I use?

Answer: spring constant values are not that important, default works just fine. Otherwise, suggested values are printed in the neb.x output.



Do not activate climbing-image NEB from the start !

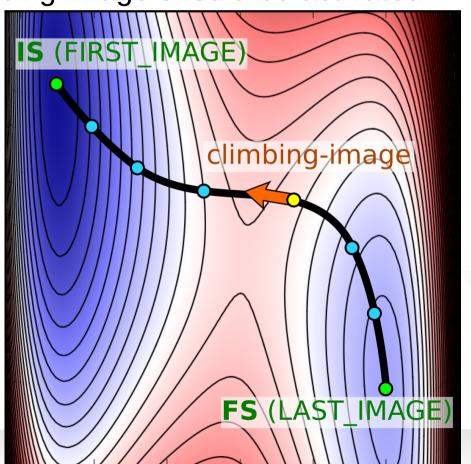
The reaction path should be first relaxed to some extent as to stabilize it and only then the climbing-image should be activated

(Cl_scheme = 'auto')

The problem:

CI-image # can oscilate from iteration to iteration

iteration *#n*: image-5 is climbing



Do not activate climbing-image NEB from the start !

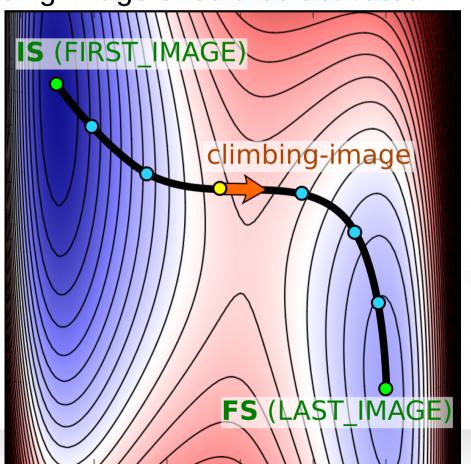
The reaction path should be first relaxed to some extent as to stabilize it and only then the climbing-image should be activated

(Cl_scheme = 'auto')

The problem:

CI-image # can oscilate from iteration to iteration

iteration *#n*+1: image-4 is climbing



Do not activate climbing-image NEB from the start !

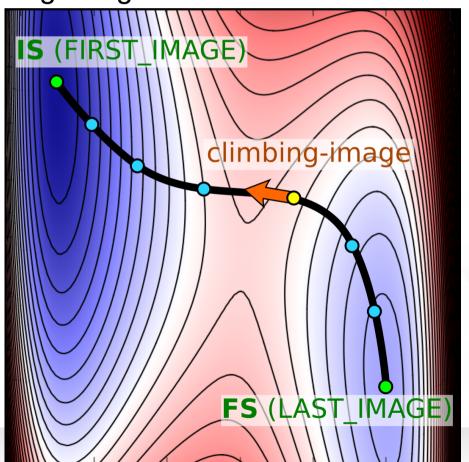
The reaction path should be first relaxed to some extent as to stabilize it and only then the climbing-image should be activated

(Cl_scheme = 'auto')

The problem:

CI-image # can oscilate from iteration to iteration

iteration *#n*+2: image-5 is climbing



Do not activate climbing-image NEB from the start !

The reaction path should be first relaxed to some extent as to stabilize it and only then the climbing-image should be activated ($Cl_scheme = 'auto'$)

PWTK snippet for 'no-Cl' + 'auto' NEB workflow:

```
load_fromNEBI neb.in
PATH {
    CI_scheme = 'no-CI'
    path_thr = 0.8
}
runNEB neb.noCI.in
PATH {
    restart_mode = 'restart'
    CI_scheme = 'auto'
    path_thr = 0.05
}
runNEB neb.auto.in
```



Specification of IMAGES

BEWARE: do not swap atoms !

(keep atom indices the same for all specified images)





It is time to try neb.x calculations during hands-on !

