

Design and operation of accelerator chain and storage rings

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- 1. How does a synchrotron light source look like ?
- 2. What are the main physical processes ?
- 3. How does it work ?

Synchrotron radiation – recap Single particle linear motion

- Longitudinal phase stability, synchrotron oscillations
- Transverse dispersion, betatron tunes, emittance

Perturbations to linear dynamics

- Equilibrium distribution
- Chromaticity, resonances, dynamic aperture
- Beam lifetime

Operation

- Beam injection and storage.
- Brilliance, diffraction limit.





Why do we need x-ray sources?



X-rays are ideal probes of chemical bonds, where most of science is rooted.

They can be used to visualize proteins structure, molecular dynamics, atomic levels and orbitals...







Serving as a Microscope







Synchrotron radiation

Synchrotron radiation is e.m. energy de-coupled from a charge by centripetal acceleration. For example, an ultra-relativistic electron in a magnetic dipole field.







v << c:
$$P'_{SR} \propto |\mathbf{S}'| \propto |E'_{x}|^{2} \propto \left(\frac{F'_{x}}{m_{e}}\right)^{2} \propto a'_{x}^{2}$$

Lorentz-transform to Lab frame:

Strong increase with particle total energy



 $\begin{cases} t' \to t/\gamma \\ x' \to x \end{cases} \Longrightarrow a'_{x}^{2} \to \gamma^{4} a_{x}^{2}$

"Light" particles (e.g., leptons) radiate more *than "heavy"* ones (e.g., hadrons).







frame:

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Angular distribution





Spectrum



Dipole emission is a *short* flash light of duration Δt_{sr} .

The spectral bandwidth, of the order of $\Delta \omega \approx 1/\Delta t_{sr}$, is **broad** $\Delta \omega / \omega_c \approx 1$.

$$\Rightarrow \omega_c \approx \Delta \omega \approx \frac{1}{\Delta t_{sr}} \approx \frac{c\gamma^3}{R}$$





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Evolution of SRLS

• First observation:

1947, General Electric, 70 MeV synchrotron

First user experiments:

1956, Cornell, 320 MeV synchrotron

• 1st generation light sources: machine built for High Energy Physics or other purposes used parasitically for synchrotron radiation

• 2nd generation light sources: purpose built synchrotron light sources, SRS at Daresbury was the first dedicated machine (1981 – 2008)

• **3rd generation light sources**: optimised for high brilliance with low emittance and Insertion Devices; ESRF, Diamond,







Dipole magnet









Energy-dispersion function powerful collimated

High energy electrons on a circular path



$$\frac{mv_z^2}{R} = F_{L,x} = ev_z B_y;$$
$$p_z = eB_y R$$
$$E \longrightarrow E - E_{sr} + \dots$$









RF cavities replenish the beam by the energy lost every turn \Rightarrow beam energy per turn is constant (on average).



Longitudinal electric field:

$$E_z \approx E_{z,0}\cos(\omega t + \phi_0)$$





E.M. field in a Pill-box

$$\begin{pmatrix} \vec{\nabla} \wedge \vec{E} \end{pmatrix}_{\phi} = -\frac{\partial B_{\phi}}{\partial t} \\ \left(\vec{\nabla} \wedge \vec{B} \right)_{z} = -\frac{1}{c^{2}} \frac{\partial E_{z}}{\partial t}$$

$$\begin{cases} \frac{\partial E_r}{\partial 2} - \frac{\partial E_z}{\partial r} = -\frac{\partial B_{\phi}}{\partial t} \quad \cdot \partial_r \\ \frac{1}{r} \left[\frac{\partial (rB_{\phi})}{\partial r} - \frac{\partial F_r}{\partial \phi} \right] = -\frac{1}{c^2} \frac{\partial E_z}{\partial t} \quad \cdot \partial_r \end{cases}$$

$$\frac{\partial^2 B_{\phi}}{\partial r \partial t} \implies \frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r}$$
$$\frac{\partial^2 B_{\phi}}{\partial r \partial t} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

$$A(r) = a_0 J_0\left(\frac{\omega r}{c}\right)$$

$$J_0(r \approx 0) \approx 1 \implies E_z \approx E_{z,0} \cos(\omega t + \phi_0)$$



-1

0

-2



 $\partial^2 E_z$

 $_1\,\partial B_\phi$

____ ωr/c 2

1



Synchronization

accelerating Cu chamber beam field

Synchronization of particle arrival time and RF field:

$$\omega = h\omega_{riv}, \ h \in \dot{\mathbb{N}} (\gg 1)$$

• How many consecutive bunches can be stored in a ring ("train")?







$$dR = \frac{C_2 - C_1}{\theta_b} = \frac{1}{\theta_b} \left(\oint ds_2 - \oint ds_1 \right) = \frac{1}{\theta_b} \oint d\theta \left[(R_1 + x) - R_1 \right] = \frac{1}{\theta_b} \oint x d\theta = \langle x \rangle_\theta$$

Orbit difference per unit of energy deviation ("momentum compaction"):

$$\alpha_{c} = \frac{dR/R}{\delta} = \frac{1}{R} \frac{\langle x \rangle_{\theta}}{\delta} = \frac{\langle D_{x} \rangle_{\theta}}{R} = \frac{1}{R\theta_{b}} \int d\theta D_{x} = \frac{1}{C} \int ds \frac{D_{x}(s)}{R(s)}$$

Slip factor

Revolution frequency difference per unit of energy deviation ("slip factor"):

$$\eta := \frac{d\omega/\omega_s}{dp_z/p_{z,s}} = \frac{1}{\gamma^2} - \alpha_c \xrightarrow{} - \alpha_c$$

GeV energies





Phase stability



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Bending and Focusing



Vacuum chamber (Al, Cu, Steel) at ultra-low pressure (< 10⁻⁹ mbar), to avoid gas-scattering



Particle beam must be kept in! ---> *external focusing* 300 MV/m !

$$rac{q|ec{E}|}{q|ec{v}\wedgeec{B}|} = rac{E}{vB} \equiv 1 \ \Rightarrow rac{|ec{E}|}{|ec{B}|} = eta c$$



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 $\frac{F_{e}}{\vec{F_{m}}}$

1 Tesla ...



Quadrupole magnet

 $(\mu_r \gg \mu_0)$:

$$\oint \vec{H} d\vec{s} = \int_1 \vec{H} d\vec{s} + \int_2 \vec{H} d\vec{s} \int_3 \vec{H} d\vec{s} = \int_0^{R_b} H(r) dr + \int_1^2 \vec{H} d\vec{s} + \int_2^3 H_y dx =$$

Normalized quadrupole strength:

$$k[m^{-2}] = 0.2998 \frac{g[T/m]}{p_z[GeV/c]}$$



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Transverse motion (linear approx.)







$$x(s) = x_{\beta}(s) + x_{\epsilon}(s) =$$

$$= x_{\beta}(s) + D_{\chi}(s)\delta$$

Alternated gradient strengths (Hill's eq. assumes linear motion & no frictional forces):

$$\ddot{y}(s) - k(s)(1 - \delta)y(s) = 0$$

$$\ddot{x}(s) + \begin{bmatrix} k(s)(1 - \delta) \\ 0 \end{bmatrix} x(s) = 0$$

"Strong" Relative "Weak"
focusing energy focusing
deviation





Phase space

$$u(s) = \sqrt{2J_u\beta_u} \cos \Delta \mu_u$$
$$u'(s) = -\sqrt{\frac{2J_u}{\beta_u}} \left(\alpha_u \cos \Delta \mu_u + \sin \Delta \mu\right)$$

Quasi-harmonic oscillator in (x,x') and (y,y') ----> the oscillation amplitude depends on s: $\beta_u(s), \alpha_u(s)$







Oscillations:

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$$\begin{split} \varepsilon(t) &= A_{\varepsilon}(t)\cos(\Omega_{s}\frac{\Delta z}{c} + \phi_{0}) \equiv A_{\varepsilon}(t)\cos\phi \\ \tau(t) &= -\left(\frac{\alpha_{c}}{E_{0}\Omega_{s}}\right)A_{\varepsilon}(t)\sin\phi \end{split} \Rightarrow \begin{cases} A_{\varepsilon}^{2} &= \varepsilon^{2} + \tau^{2}\left(\frac{E_{0}\Omega_{s}}{\alpha_{c}}\right) \\ \langle \varepsilon^{2}(t) \rangle_{\phi} &= \frac{A_{\varepsilon}^{2}(t)}{2} \end{cases}$$

$$\langle \delta A_{\epsilon}^2 \rangle_{\phi} = \langle \delta \epsilon^2 \rangle + \langle \delta \epsilon \rangle \left(\frac{E_0 \Omega_s}{\alpha_c} \right)^2 = 2 \langle \epsilon \delta \epsilon \rangle + \frac{1}{2} \langle (2\delta \epsilon) \delta \epsilon \rangle = -2 \langle \epsilon u \rangle + \langle u^2 \rangle \cong$$

$$\approx -2\langle \epsilon \frac{du}{d\epsilon} \epsilon \rangle + \langle u^2 \rangle = -A \frac{du}{\epsilon d\epsilon} + \langle u^2 \rangle$$

"damping" "excitation"

dl K R H H

Characteristic **damping time** to reach **equilibrium** Gaussian

distribution:

Equilibrium distribution

$$\frac{d}{dt}\langle \delta A_{\epsilon}^2 \rangle_{\phi} \rangle_R \simeq \frac{d\langle A_{\epsilon}^2 \rangle_R}{dt} = -\langle A_{\epsilon}^2 \rangle_R \langle \frac{d}{dt} \frac{du}{d\epsilon} \rangle_R + \langle \frac{d}{dt} \langle u^2 \rangle_{\phi} \rangle_R = 0$$

 $\tau \approx T_0 \frac{E_0}{U_0}$



Beam size and emittance

Remind:
$$u(s) = \sqrt{2J_u\beta_u} \cos \Delta\mu_u$$

 $\sigma_x = \sqrt{\varepsilon_x\beta_x}$ depends or strength or

depends on quads strength only; varies through the lattice

constant through the lattice ("equilibrium")

$$\varepsilon_{x,eq} = C_e \frac{\gamma^2}{L_e} \frac{\langle H_x \rangle_R}{R}$$



$$\frac{\langle H_x \rangle_R}{R} \approx \frac{1}{R} \left(\frac{1}{\beta_x} \langle D_x^2 \rangle + \beta_x \langle D_x'^2 \rangle \right) \propto \frac{\theta_b}{l_b} \left[\frac{l_b^2 \theta_b^2}{4\beta_x} + \beta_x \theta_b^2 \right] \propto \theta_b^3 \left(\frac{l_b}{\beta_x} + \frac{\beta_x}{l_b} \right) \propto \left(\frac{2\pi}{N_b} \right)^3$$



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This is driving world-wide upgrades to multi-bend lattices. Radiation is far more collimated and more intense – higher "brilliance"!



Scaling laws





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Resonances

Remind:



The error sum coherently if the particles goes back to it with same amplitude and phase (position and angle) every r-turns







Chromaticity

Particles at (slightly) different energies are focused differently:



• Would more quads help to "zeroing" chromaticity?

- 1. Phase advance $\tan(\Delta \mu_u) \approx -\beta_u \frac{u'}{u}$ 2. Small phase variation by error kick: $d(\tan(\Delta \mu_u)) \approx d(\Delta \mu_u) \approx -\beta_u \frac{du'}{u}$
- 3. Quad error kick: $\Delta u' \approx \mathbf{k} \boldsymbol{\delta} \cdot ds \cdot u$
- 4. Local tune change:

$$\mathrm{d} Q_u = \frac{d(\Delta \mu_u)}{2\pi} \approx -\frac{1}{2\pi} \beta_u \; k \delta ds$$

5. Global tune change (chromaticity): $\xi_{u}^{nat} := \frac{\Delta Q_{u}}{\delta} = -\frac{1}{4\pi} \oint ds \beta_{u}(s) k(s)$





Sextupole magnet

$$\xi_x^{cor} = \frac{\Delta Q_x}{\delta} = -\frac{1}{4\pi} \oint \beta_x(s) \left[k(s) + m(s)\eta_x(s)\right] ds$$
$$\xi_y^{cor} = \frac{\Delta Q_y}{\delta} = -\frac{1}{4\pi} \oint \beta_y(s) \left[-k(s) + m(s)\eta_x(s)\right] ds$$







Sextupoles acts as a quadrupole of normalized gradient proportional to the dispersion function.

• How many sextupole "families" would we need to correct the chromaticity in both x- and y- plane?





Dynamic aperture





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Nonlinear dynamics





Lifetime

$$\left(\frac{dN}{dt}\right)_{W_c} = \left(\frac{dN}{dW}\frac{dW}{dt}\right)_{W_c}$$
 , where

$$\left(\frac{dN}{dt}\right)_{W_c} = -\frac{2N}{\tau} \frac{W_c}{\langle W \rangle} e^{-\frac{W_c}{\langle W \rangle}} \Rightarrow \begin{cases} N(t) = N_0 e^{-\frac{t}{\tau_q}} \\ \\ \tau_q = \frac{\tau}{2} \frac{\langle W \rangle}{W_c} e^{\frac{W_c}{\langle W \rangle}} = \frac{\tau}{2} \frac{e^{\xi}}{\xi} \end{cases}$$

Due to physical (apertures) or dynamic boundaries (transverse and longitudinal acceptance), the beam current decreases exponentially.





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Particle Scattering

Touschek scattering:

single, large angle event

If two particles collide in the c.m. frame transferring their (transverse) momentum $\vec{p}'_i = (p'_x, 0)$ into (longitudinal) momentum $\vec{p}'_f = (0, p'_z) = (0, p'_x)$, $\Delta p_z = p_{z,f} - p_{z,i} = \gamma (p'_{z,f} + \frac{\beta}{c}E'_f) - \gamma (p'_{z,i} + \frac{\beta}{c}E'_i) = \gamma \Delta p'_z + \gamma \frac{\beta}{c}\Delta E' =$

$$=\gamma(p'_{z,f}-p'_{z,i})=\gamma p'_{x}=\gamma p_{x}=\gamma p_{z}\sigma_{u'}$$

$$\Rightarrow \frac{\Delta p_z}{p_z} \approx \gamma \sqrt{\frac{\epsilon_u}{\beta_u}}, \ u = x, y$$
 must be << long. acceptance



Intrabeam scattering:

multiple small angle events (diffusion)







Injection chain - LINAC

DC thermo-ionic Gun + "buncher" + RF











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Injection chain - BOOSTER



Energy ramp ---> magnetic field ramp, frequency shift

$$(\Delta E)_{turn} = (\Delta p_z)_{turn} \beta c = e \dot{B}_y r T_0 \beta c = 2\pi R_s r q \dot{B}_y \equiv q V_0 cos(\psi_s - \psi_0)$$

$$\psi_s(t) = \psi_0 + \arccos\left(2\pi R_s r \frac{B_y}{V_0}\right)$$





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Injection chain - BUMP

Multi-turn beam accumulation (e.g., off-axis injection)

Injection efficiency.

Transparency to stored beam (users)

New schemes: single-kicker, on-axis (swapout), longitudinal injection, linac,...





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Insertion devices





 $B_{\gamma} =$

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Brilliance

Brilliance = 6-D photon density:

 $\frac{dN_{\gamma}/dt}{4\pi^{2}\Sigma_{x}\Sigma_{x'}\Sigma_{y}\Sigma_{y'}\Delta\omega/\omega}$

Effective radiation size (at the source)

$$\Sigma_{u} = \sqrt{\sigma_{u,s}^{2} + \sigma_{u,R}^{2}} \cong \sqrt{(\beta \varepsilon)_{u,s} + (\beta \varepsilon)_{u,R}}$$
$$\Sigma_{u'} = \sqrt{\sigma_{u',s}^{2} + \sigma_{u',R}^{2}} \cong \sqrt{(\varepsilon/\beta)_{u,s} + (\varepsilon/\beta)_{u,R}}$$

• It is maximized by **source-radiation matching:** $\beta_{u.s} = \beta_{u.R}$

$$B_{\gamma} = \frac{dN_{\gamma}/dt}{4\pi^{2} \Delta \omega/\omega} \frac{1}{(\varepsilon_{x,s} + \varepsilon_{R})(\varepsilon_{y,s} + \varepsilon_{R})}$$
• and by a **diffraction limited source:**

$$B_{\gamma} = \frac{dN_{\gamma}/dt}{\Delta \omega/\omega} \frac{1}{(\lambda^{2}/2)(\kappa + 1)}$$

$$\varepsilon_{x,s} = \varepsilon_{R} = \frac{\lambda}{4\pi}$$

$$\kappa = \frac{\varepsilon_{y,s}}{\varepsilon_{x,s}}$$
Coupling coefficient
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$$\kappa = \frac{\delta N_{\gamma}}{\delta \omega}$$



 $\frac{dN_{\gamma}/dt}{\Sigma_{x}\Sigma_{x},\Sigma_{y}\Sigma_{y'}}$ is a **conserved quantity** in a *perfect* optical system. However, a **real**

beamline includes slits, mirrors, gratings, etc. for manipulation of the pulse. They show geometrical and surface **imperfections** ---> **optical aberrations, wavefront**

distortion, absorption, scattering.





Flux is maximized. Smaller and higher quality mirrors. Higher degree of transverse coherence.

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- Synchrotrons provide light up to tens of beamlines simultaneously, each beamline receiving light from its own insertion device (undulators allow independent tuning).
- Large flexibility in tuning or selecting radiation wavelength and intensity. Spectrum from IR to hard x-rays.
- High average radiation power at the expense of low peak power (incoherent emission) and long pulses (several 10's ps).

Extremely stable.





Reduction in the **source emittance**, thus **increase in brilliance**, will lead to:

- significant gain in the emitted or transmitted signals from the samples;
- reduced acquisition time for all types of spectroscopies and x-ray scattering techniques;
- implementation of *photon-hungry techniques* such as: high pressure experiments with anvil cells and dilute samples, and spin-resolved ARPES;
- improvement of the *lateral resolution* with focusing optics down to a few-nm scale (e.g. nano-PES, nano-ARPES)

Higher degree of transverse **coherence** will open unique opportunities for:

- Coherent Diffraction Imaging (CDI) with chemical specificity
- Ptychography



X-ray photon correlation spectroscopy (XPCS)



EU initiative

A new consortium of excellence in Europe devising a transformative level of coordination and integration

13 European Synchrotron Radiation and 6 FEL Facilities are joining forces to master the challenges of the next decades.







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