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HERDING BEHAVIOR OF FINANCIAL ANALYSTS: A MODEL OF SELF-ORGANIZED CRITICALITY

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ABSTRACT. We investigate a simple model of investment recommendations issued by financial analysts in different industries. Financial analysts are influenced by the recommendations of some of their colleagues in the same industry as well as their own recommendations in certain other industries. Describing the relations with other financial analysts and among industries as a graph, we derive the complex dynamics of investment recommendations as relations change over time according to a simple rule, resulting in self-organized criticality. We show that financial analysts are more likely to be optimistic about industries that are unrelated to other industries than about industries that are related with others.

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1. INTRODUCTION

It is a widely accepted empirical fact that recommendations issued by financial analysts are overly optimistic, i.e. buy recommendations exceed sell recommendations by far, although the fundamental value does not necessarily support this view. This finding is usually attributed to the conflict of interest between financial analysts and other areas of investment banking, e.g. brokerage, Initial Public Offerings or Mergers & Acquisitions advisory, see e.g. Lin/McNichols (1998) or Michaely/Womack (1999). This observation became especially apparent in the recent internet bubble, where the behavior of financial analysts is currently being investigated by the U.S. House of Representatives.
A complementary result of empirical investigations is that financial analysts follow similar trends, e.g. by being overly optimistic on the prospects of certain industries or countries, see Trueman (1994), Graham (1999), Welch (2000), Hong et al. (2000) and Cooper et al. (2001). An identical result has been found for macroeconomic forecasts as shown in Lamont (1995), Prendergast/Stole (1996), Avery/Chevalier (1999) and Ashiya/Doi (2001). This observation is in the literature known as herding behavior.

Although several models have been suggested to explain herding behavior, e.g. Banerjee (1992) and Morris (2000), the mechanism causing herding and the sudden emergence and disappearance of these trends has thus far not been explained satisfactory.

The basis for our model will be the motivations of financial analysts in issuing their recommendations rather than the fundamental value of the assets, which we neglect for simplicity here. Financial analysts are much concerned about their future careers. The performance of a financial analyst is usually evaluated relative to the performance of their colleagues, where more senior financial analysts and colleagues covering the same or similar securities serve as reference points. The performance, i.e. precision of forecasts, forms the basis for promotions as well as remuneration.

Issuing recommendations that deviate substantially from those of their colleagues expose them to the risk of showing a low performance. When issuing recommendations similar to those of their colleagues, they will show an average performance. For risk averse financial analysts this situation is usually preferred. Hence the financial analyst has strong incentives not only to evaluate the fundamental value of the assets, but also take

\footnote{We could easily include the fundamental value into our model without changing the general properties of our result. We could redefine a recommendation as optimistic if it exceeds the fundamental value.}
the recommendations issued by his colleagues into account. This tendency to conform with
the majority view is further reinforced from investors following these recommendations.
Thus recommendations become self-enforcing.

The framework used in this paper will be the Bak-Sneppen model, Bak/Sneppen (1993),
whose properties have been extensively analyzed in recent years, e.g. Jain/Krishna (1998),
Lee et al. (2001), Kulkarni et al. (1999) and Ormerod/Johns (2001).

We proceed by describing in detail the model used in this paper in the next section.
The third section then describes the simulations conducted and section four evaluates the
properties of this model. Section five concludes the findings.

2. The model

We consider a market with $s$ financial analysts covering $n$ different industries, where
$n \ll s$. Let us further assume that all financial analysts cover every industry and issue
appropriate recommendations. Each financial analyst observes the recommendations of
all other financial analysts in every industry.

As has been mentioned above, financial analysts will take into account the recommenda-
tions made by other financial analysts. Naturally there will be financial analysts whose
recommendations are more important than those of others. If the importance is large,
we note this as the existence of an influence from this financial analyst, otherwise there
does not exist an influence. Influences do not have to be symmetric, i.e. while the recommendation issued by financial analyst $j$ is important for analyst $i$, the recommendation issued by financial analyst $i$ may be of no concern for financial analyst $j$, e.g. due to their different positions as senior and junior financial analysts. Let us finally assume that influences from other financial analysts are confined to a single industry, hence there are no (direct) influences from the recommendations issued by other financial analysts in different industries.

It is therefore possible to interpret financial analysts as nodes and any interactions among them as vertices of a directed digraph. Define $A^k = (a^k_{ij})$ as the adjacency matrix of this graph in sector $k$, where $a^k_{ij} = 1$ for $i \neq j$ if there exists an influence from financial analyst $j$ towards financial analyst $i$, and zero otherwise.

The individual industries are also more or less closely related to each other, e.g. through the existence of common factors. Hence the recommendations in two related industries issued by the same financial analyst should influence each other. These relations among industries can again be interpreted as a digraph, which has to be symmetric. We define the adjacency matrix of this industry graph by $D = (d_{ij})$ with $d_{ij} = 1$ if industries $i$ and $j$ are related and zero otherwise. Furthermore, $d_{ii} = 0$ and $d_{ij} = d_{ji}$.

The Bak-Sneppen model assigns a measure of fitness to each node of the graph. In our model this fitness can be interpreted as the strengths of the recommendations issued by the financial analysts for a specific industry, the higher this value the more optimistic the financial analyst is for this sector. The strength of the recommendation issued by
financial analyst \(i\) for sector \(k\) is denoted by \(y^k_i\). Following the Bak-Sneppen model we assume the dynamics of these beliefs to evolve as follows:

\[
\dot{y}_i^k = \sum_{j=1}^{s} a_{ij}^k y_j^k + \gamma \sum_{l=1}^{n} d_{kl} y_l^i,
\]

where \(\gamma \geq 0\) denotes the importance of the influence generated by the recommendations of other financial analysts in the same industry and the recommendations by the same financial analyst in other industries.

For \(\gamma \to 0\) the recommendations in each industry are issued independently and the properties of our model would be identical to the Bak-Sneppen model, while for \(\gamma \to \infty\) the financial analysts would issue recommendations independently of each other, only taking into account their recommendations in other sectors. We are here concerned about the influences of both, other financial analysts as well as other industries, and thus will choose an intermediate value for \(\gamma\).

We can define \(A = \text{diag}\{A^k\}\) such that

\[
C = A + \gamma D \otimes I_s
\]

is the adjacency matrix of the supergraph including influences arising from other financial analysts as well as other industries. The nodes of this supergraph would be the financial analysts in a specific sector, while the vertices still represent the influences between the nodes.
With \( y_k = (y_k^1, \ldots, y_k^s)' \) and \( y = (y_1, \ldots, y_n)' \) equation (1) can be rewritten as

\[
(3) \quad \dot{y} = Cy.
\]

As in the Bak-Sneppen model it is assumed that the equilibrium in the recommendations is reached immediately, such that only equilibrium values for \( y \) are considered in the analysis. The equilibrium values of \( y \) will be normalized by the following transformation:

\[
(4) \quad y_i^k \longrightarrow \frac{y_i^k}{\sum_{j=1}^{s} y_j^k}.
\]

This normalization ensures that the sum of the recommendation strengths in each industry is unity. We will say that a financial analyst \( i \) is optimistic for industry \( k \) if \( y_i^k \) is above the average, i.e. \( y_i^k \geq \frac{1}{s} \), otherwise he is realistic. Given the pressure imposed on financial analysts to issue buy recommendations, i.e. be optimistic, we do not include pessimistic financial analysts.

While the industry graph does not change over time, the graph capturing the interactions of financial analysts within a single industry does evolve over time according to a fixed rule.

Due to the pressure on financial analysts to issue buy recommendations, we suggest here that in each sector the financial analyst that is the least optimistic, i.e. has the lowest value for \( y_i^k \), is replaced by a new financial analyst using different reference points. All vertices emerging or ending at the node of financial analyst \( i \) in sector \( k \), which is
the least optimistic in that sector, are eliminated and replaced by new vertices according to the following rule: \( \forall j = 1, \ldots, s; j \neq i : a^k_{ij} = 1 \) with probability \( p \) and \( a^k_{ij} = 0 \) with probability \( 1 - p \). As before it is \( a^k_{ii} = 0 \). Based on these new adjacency matrices \( A^k \) the new equilibrium is determined in the coming time period.

Given the structure of this business, we could identify one time period in our model with approximately one week in reality. The developments in markets are followed continuously and any updates on recommendations regularly issued, especially when significant event trigger a change.

The analysis will now investigate the dynamics of the number of financial analysts being optimistic in each time period arising from this model.

3. Numerical simulation

The model is investigated using a simulation study based on specific parameter constellations using 500 simulations.

For each simulation an industry graph is determined randomly, where \( d_{ij} = 1 \) with probability \( q \) for \( i \neq j \), \( d_{ij} = 0 \) with probability \( 1 - q \) for \( i \neq j \) and \( d_{ii} = 0 \) for all \( i = 1, \ldots, n \).

The initial adjacency matrix \( A^k \) of the interactions of financial analysts in a sector \( k \) is chosen randomly and independent of each other such that for every \( i \) and \( j \) \( a^k_{ij} = 1 \) with probability \( p \) for \( i \neq j \), \( a^k_{ij} = 0 \) with probability \( 1 - p \) for \( i \neq j \) and \( a^k_{ii} = 0 \). The probability \( p \) is the same as that chosen for updating the graph in subsequent periods. The initial
values for $y_i^k$ are taken from independent uniform distributions and renormalized accordingly.

In every simulation we used 15,000 time periods, of which the first 5,000 time periods are not further analyzed to eliminate any influences from the choice of the initial matrices. The parameter constellation investigated has been as follows: $s = 100$, $n = 10$, $p = 0.0025$, $q = 0.1$, and $\gamma = 2$.

Were the issue of recommendations purely random, the model would suggest that the number of optimistic financial analysts should fluctuate around 50 with no further structure to be found.

4. Results

We define an industry to be in a group if all other members of the group are reachable from this industry. The number of industries in a group is called the group size. An industry belonging to a group of size two or more is called integrated, and isolated otherwise. We furthermore call a situation with more than 50 financial analysts being optimistic a bias. The number of financial analysts being optimistic, we define as the size of the bias.

We can use the group size to which an industry belongs as the explanatory variable for the average time length of a bias in each industry, the number of such events in the 10,000
time periods considered, the fraction of time spent with a bias and the average size of the bias. These relations are shown in figure 1.²³

For integrated industries, the group size has no statistically significant influence on these variables, while for isolated industries the differences are statistically significant. The time length of individual biases is longer for isolated industries and biases are more frequently observed. We also see that the average size of the bias is smaller for isolated industries. Table 1 shows the descriptive statistics for isolated and integrated industries. All differences are significant at any reasonable level.

²We excluded biases with a length of less than 10 time periods. The reason for this is that in cases where the beliefs of financial analysts are very homogeneous, i.e. $y_i^k \approx \frac{1}{2}$, small variations in $y_i^k$ can cause large fluctuations in the number of financial analysts issuing positive recommendations. These fluctuations are erratic and have no further meaning. We also conducted investigations by excluding all biases with a length below 100 and did not exclude any biases. In both cases the results were qualitatively identical.

³Using the number of direct links from an industry or the average distance to other industries as the explanatory variables does not change the results significantly.
Table 1. Descriptive statistics of biases

<table>
<thead>
<tr>
<th></th>
<th>Isolated industries</th>
<th>Integrated industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average time length of a bias</td>
<td>154.78</td>
<td>43.60</td>
</tr>
<tr>
<td>Number of biases</td>
<td>17.89</td>
<td>7.94</td>
</tr>
<tr>
<td>Fraction of time spent with bias</td>
<td>0.294</td>
<td>0.063</td>
</tr>
<tr>
<td>Average size of bias</td>
<td>73.64</td>
<td>85.14</td>
</tr>
</tbody>
</table>

The intuition for this result is straightforward. As we know from the Bak-Sneppen model, it happens from time to time that keystone financial analysts issue the least optimistic recommendation and hence are replaced by another financial analyst in the same industry. This can cause the entire network structure established among financial analysts in that industry to break down, causing the bias to fall suddenly as financial analysts become less optimistic (self-organized criticality). In our model, this mechanism works for all industries, hence also isolated industries see these sudden changes in the bias. If industries are integrated, however, any such development in one industry can be further propagated into other industries and thus makes it more likely for the bias to disappear, although the network structure in those industries are not affected. This causes the time it takes for the bias to vanish to be reduced. The structure of the networks is however not that vulnerable to these disturbances in other industries that the group size has a significant impact.

In order to establish a positive bias, isolated industries only have to develop an appropriate structure within their industry. Integrated industries, however, have to overcome the feedback arising from other industries, too. This makes the emergence of positive biases much less likely.
Taking together these two effects it is apparent that the time spent with a bias is longer for isolated than for integrated industries. When observing the recommendations of financial analysts at a certain point of time, we are therefore more likely to observe a bias in isolated than in integrated industries.

On the other hand, once these obstacles are overcome and a bias has been established, the integrated industries enforce the bias such that it is larger than for isolated industries.

5. Conclusions

The main result that can be derived from our model is that it is more likely that a substantial proportion of financial analysts are optimistic about isolated industries than about integrated industries. We can interpret this result as arising from the herding behavior of financial analysts that results from the interactions with other financial analysts. Additionally we saw that biases in the recommendations of financial analysts vanish as quickly as they appear.

If we assume that investors tend to follow the recommendations of analysts, we should observe bubbles, i.e. excessive valuations, more frequently in isolated industries, while bubbles in integrated industries should be much rarer as should be a bubble in the entire market.

When looking at historic bubbles, this picture is confirmed. There is a large number of bubbles occurring in isolated industries: Tulipmania 1634-1637, Mississippi-Bubble 1716-1720, South-Sea Bubble 1717-1720, English Canals 1792, Railways 1847, Automobiles
1922, Internet 1998-2000, to name only the best known examples. Similarly there are a large number of examples of (stock) markets in not well integrated countries, usually developing countries, to experience the sudden influx of speculative capital that causes the stock market to increase significantly or the currency to be overvalued. Bubbles in integrated industries as well as countries are much rarer found and usually less pronounced. Most market wide bubbles, e.g. before the stock market crashes 1929 and 1987, or the stock market bubble 1997-2000, were fueled by an isolated industry, like the internet stocks in the most recent events. Hence we have a first indication that our model produces some realistic features.

It is worth stressing that our model represents the dynamics of opinions (investment recommendations issued by financial analysts) rather than trading decisions or stock price dynamics. In order to model actual trading decisions, much more complex considerations have to be included, like the risk associated with the decisions for investors.

Besides an empirical verification of the model, future research could extend the model in various ways, e.g. by enabling financial analysts to concentrate on certain sectors rather than covering all sectors, by allowing the graphs of financial analysts to be correlated across industries or generalizing the industry and analyst graph by including different degrees of relations, i.e. $d_{ij}, a^k_{ij} \in [0; 1]$ instead of $d_{ij}, a^k_{ij} \in \{0; 1\}$ in our model. Since similar variations in the Bak-Sneppen model did not change the results significantly, we cannot expect significantly new insights from any of these changes.
References


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