# Social Phase Transitions

Moshe Levy[*]

April 2002

[*]Hebrew University of Jerusalem, Jerusalem 91905, Israel. Tel: 972 2 5883219. Fax: 972 2 5881341. email: mslm@mscc.huji.ac.il.

# Social Phase Transitions

## Abstract

What do the market crash of 1987, the collapse of the Berlin wall, the outbreak of social cooperation and the instantaneous emergence of traffic jams have in common with boiling water at 100ºC ? In all of the above systems a small event (or parameter change) has triggered a dramatic transition in the system. We show that social systems in which individuals have an inclination to conform with each other may undergo "phase transitions" very similarly to physical systems such as water starting to boil or spins aligning in a magnet. We develop a general criterion which determines whether a given social system is susceptible to undergo a phase transition, and we show that phase transitions may occur in a wide range of social systems. The heterogeneity of the agents comprising the system is shown to play a crucial role in determining both the possibility of a transition and its magnitude. Transitions may occur only if the system is not "too heterogeneous". The more homogeneous the system, the more dramatic the transition will typically be. We further show that at the transition point the system is unstable and thus transitions may reverse (as in the unsuccessful 1989 uprising in China, or the Nasdaq crashing 13.5% and then climbing back 11.8% on the same day in April 2000). Knowledge about the distribution of individuals comprising a social system may allow prediction and even induction or prevention of a social phase transition.

Keywords: *Social Transition, Market Crash, Heterogeneity.*

JEL Classification: A12, A14, G14.

## I. Introduction

Most social systems are usually continuous: a small change in one of the system's parameters leads to a small change in the system's properties. For example, a small increase in marketing effort usually leads to a small increase in sales; a small increase of the fine for speeding on the freeway usually leads to a small reduction of speed limit violations; when an oppressive government slightly worsens the living conditions of its citizens this usually leads to a slight increase of the opposition to the government, etc. While this is *usually* true, many social systems sometimes behave very differently – in reaction to a small parameter change they may undergo a dramatic "discontinuous" transition. In many cases the triggering parameter change may be so small that it is even unobservable. Such dramatic discontinuous transitions which are due to a small or even an unnoticeable change in the conditions are observed in a wide variety of social systems ranging from stock markets to political systems, norms of drug abuse, and even traffic flow.

The stock market crash of 1987 was a very dramatic event which shook the world, with markets around the globe dropping by tens of percents in just a few days. The puzzling part about the 1987 crash is that it was not triggered by any significant new economic information. Why did the market crash on October 1987 and not a month earlier or a year later? For that matter, why did it crash at all? To this day, we lack a satisfactory answer to these questions (see Roll [1991]).

The thundering collapse of the East European block took the world by complete surprise. This discontinuous political earthquake was totally unanticipated. Even with hindsight it is difficult to explain the timing of this collapse, or to justify it as a consequence of a significant triggering event. This is true not only of the East European revolution of 1989, but also of many other political revolutions such as the

French revolution of 1789, the Russian revolution of 1917, and the Iranian revolution of 1978-9 (see Kuran [1989], [1991]). Spontaneous discontinuous transitions, which occur without any significant external trigger are also observed in the flow of traffic on the freeway (Kerner and Rehborn [1997]), the emergence of social cooperation (Glance and Huberman [1993], [1994]), the outbreak of drug abuse among teenagers (Kraw [2000]), racial segregation (Schelling [1978]), and many other social systems.

Dramatic transitions in social systems are typically analyzed in the context of the specific system considered. Kuran [1989, 1991] develops a model of political revolution which is based on the concept of preference falsification, which is the difference between an individual's private preference and his public preference (a person may secretly hate the government but publicly support it for fear of persecution). Welch [1992] and Bikhchandani, Hirshleifer and Welch [1992] show that the fragility of mass behavior and fads can be explained in terms of informational cascades, which occur when it is optimal for an individual to follow the behavior of the preceding individual without regard to his own information. Glance and Huberman [1993, 1994] model social cooperation and show that the degree of cooperation in a society can abruptly and drastically change without warning. Kerner and Rehborn [1997] analyze and model traffic flow on freeways. They show that dramatic transitions between free traffic flow and traffic jams often occur spontaneously, without any noticeable change in traffic volume to trigger them. Topol [1991], Gennotte and Leland [1990], and Levy, Levy and Solomon [1994] suggest models which can explain spontaneous stock market crashes. Kirman [1993] shows that a simple tandem-recruitment model can explain "herding" behavior and sharp changes in the aggregate choice between two identical sources.

Physical systems with many interacting elements are also known to undergo discontinuous transitions. Perhaps the most well-known example is that of a system of many interacting $H_2O$ molecules – water. When water is heated from, say 87.1ºC to 87.2ºC nothing dramatic happens. The small temperature increase leads to small changes in the properties of the water (volume, pressure). The same is true for small temperature changes up to a temperature of 99.9ºC. However, when the water is heated from 99.9ºC to 100ºC a very dramatic transition occurs as a result of the small temperature increase - the water begins to boil, and the properties of the system change discontinuously. In the context of statistical mechanics this discontinuous transition is known as a "phase transition". Indeed, several researchers have applied statistical mechanics models for the investigation of social and economic systems (for some of the first examples, see Föllmer [1974], and Haken [1977]). In a recent series of innovative papers, Brock [1993], Brock and Durlauf [1995], and Durlauf [1999] employ the statistical mechanics Ising model[1] to investigate various economic systems. Some researches caution of such analogies between statistical mechanics and economics, and point to their limitations (see Hors and Lordon [1997]). One major difficulty with these analogies is that one has to "tailor" the economic model so that it is mathematically identical to the statistical mechanics model. Obviously, this could be problematic.

In this paper we suggest that dramatic transitions in a wide variety of social systems can be explained by a single basic mechanism which is similar to the mechanism responsible for phase transitions in physical systems. Although the social phase transition mechanism suggested here is analogous to the statistical mechanics phase transition mechanism, the analogy does not depend on any specific assumptions

[1]The Ising model is one of the fundamental models in statistical physics for describing systems with many interacting elements. For a description of this model, see, for example, Stanley [1971].

regarding the underlying model, and it is therefore very general, as will become evident. In this analogy the degree of heterogeneity in the social system plays the role of temperature in physical systems, and just as temperature plays a key role in physical phase transitions we show that heterogeneity plays a key role in social phase transitions[2]. Specifically, the degree of heterogeneity determines whether a phase transition may occur, and it is closely related to the magnitude of the transition. We show that any social system in which individuals have some inclination to conform with their peers, and in which the population is not very heterogeneous, may undergo a phase transition.

The structure of this paper is as follows. In section II we provide the framework of the analysis and show how phase transitions may occur. In section III we provide a criterion which determines whether a given social system is susceptible to undergo a phase transition. In section IV we develop tools to estimate the magnitude of possible transitions for general systems, and we show that this magnitude is related to the heterogeneity of the system. Section V analyzes the stability of the system at the transition point and discusses the possibility of reverse transitions. Section VI suggests practical applications and points to several possible directions for further investigation.

## II. Framework for Analysis of Phase Transitions

Consider a system of individuals who are faced with a binary decision, such as the decision whether to join a revolution or not, whether to use drugs or not, or

[2]In the previously mentioned example of water beginning to boil, temperature plays both the role of "heterogeneity" and that of the parameter which is slightly changed to trigger the transition. In the analysis of social systems we take the degree of heterogeneity as given, while a small change in a *different* parameter triggers the transition. This is analogous to the phase transition which occurs when the water is kept at a constant temperature and a different parameter (for example pressure) is slightly changed.

whether to invest in the stock market or not[3]. The decision of an individual is a function of some personal characteristics and some global parameters. Formally, the utility function of individual i can be written as[4]:

$$U_i = U_i(\underline{h}_i, g, x, s_i), \tag{1}$$

where $\underline{h}_i$ is a vector of personal characteristics, $\underline{g}$ is a vector of global parameters, $s_i$ is the choice of the individual (in our case of a binary choice $s_i$ can be either 0 or 1), and x is a global parameter which we wish to distinguish from the other global parameters in g – it is the percentage of individuals in the population who choose $s_i = 1$ (i.e. $x = \frac{\sum_{i=1}^{N} s_i}{N}$, where N is the total number of individuals). Thus, by definition x is in the range [0,1]. For a concrete example consider the choice of a teenager deciding whether to use drugs ($s_i = 1$) or not ($s_i = 0$). Her decision will be influenced by some personal characteristics $\underline{h}_i$ (such as her psychological state of mind, the effect drugs have on her, the availability of money to buy drugs, etc.), some global parameters (such as the sanctions if caught using drugs, the health hazards of drugs, etc.), and by peer pressure, x, which is the percentage of drug users in the population. Given the values of the parameters $\underline{h}_i$, g, and x, the individual makes a choice between $s_i$=0 and $s_i$=1 by comparing his utility in the following two alternatives states

$$U_i(\underline{h}_i, g, x, 0) \text{ with } U_i(\underline{h}_i, g, x, 1),$$

and choosing $s_i$ such as to maximize her utility.

We wish to focus here on systems in which individuals have some inclination to conform with their peers. For example, a teenager observing many peers using

[3]In this framework we consider only binary choices. However, the model can be extended to the case where there are more than two alternative choices. This point is discussed in Section VI.

[4]The utility function U can be generally thought of as a multidimensional utility function (as in Kihlstrom and Mirman [1974], and Levy and Paroush [1974], for example), and not necessarily as the classical von Neuman–Morgenstern utility function defined only on wealth.

drugs may be more inclined to use drugs herself. An oppressed citizen contemplating demonstrating against the government will be encouraged to do so if she sees many other people demonstrating, etc. The motivation to conform can arise from technological reasons (Arthur [1989]), informational reasons (Welch [1992]), reputational reasons (Kuran [1998]), or a combination of several of the above (Kuran [1999]). Thus, we discuss systems in which the higher x, (i.e. the higher the proportion of people choosing s=1), the higher $U_i(\underline{h}_i, \underline{g}, x, 1)$ becomes (and perhaps the lower $U_i(\underline{h}_i, \underline{g}, x, 0)$ becomes), and the more inclined the individual becomes to choose $s_i=1$.[5] Let us denote the level of x which makes individual i indifferent between choosing $s_i=0$ and $s_i=1$ by $x_i^T$.[6] This is individual i's threshold x: if x is below it she will choose $s_i=0$; if x is above it she will choose $s_i=1$. Of course, $x_i^T$ will generally vary across individuals depending on their utility functions and their personal characteristics: one person may decide to use drugs if she observes 20% or more of her peers using drugs, while another may cave in only if she observes at least 70% of her peers using drugs. Thus, there will be some distribution of the threshold $x^T$ in the population. The heterogeneity of the population with respect to $x^T$ turns out to play a crucial role in social phase transitions. For simplicity, when no misunderstanding can arise we simply denote $x^T$ by x, omitting the superscript T. We denote the probability density of the threshold x in the population by f(x), and the cumulative distribution of x by F(x). Figure 1 depicts a typical distribution F(x) [7]. According to Figure 1, 20% of the individuals would choose s=1 even if they

---

[5]Formally, the inclination to conform means that $\frac{\partial[U_i(h_i, g, x, 1) - U_i(h_i, g, x, 0)]}{\partial x} > 0$.

[6]The effect of individual i's decision on x is neglected in his decision-making, as the population is assumed to be large. Also, it is possible that some individuals prefer $s_i=0$ even if x=1. Others may prefer $s_i=1$ even if x=0.

observed everybody else choosing s=0 (see point A in Figure 1). 40% of the individuals would choose s=1 if they observed x=0.2 (see point B in Figure 1). 90% of the individuals would choose s=1 if they observed x=1.0 (which means that 10% would choose s=0 even if they observed everybody else choosing s=1, see point C; Thus, F(1) does not necessarily equal to 1 in the general case, since a proportion 1-F(1) of the individuals do not choose s=1 even if they observe x=1)[8].

(Insert Figure 1 About Here)

The unique equilibrium point in Figure 1 is point E. At this point F(x) crosses the 45º line and $F(x^*)=x^*$, where $x^*$ is the equilibrium percentage of the population choosing s=1. This is an equilibrium because if $x^*$ is observed, a proportion $F(x^*)$ of the population chooses s=1, and since $F(x^*)=x^*$ point E is indeed a self-consistent steady-state equilibrium. Note that the equilibrium E is globally stable. To see this, suppose that the system is initially out of equilibrium, say at x=0.2. The proportion of individuals choosing s=1 given that x=0.2 is F(0.2)=0.4. This leads to x=0.4 . The proportion of individuals choosing s=1 given that x=0.4 is F(0.4)=0.6 which leads to x=0.6, and so forth. Eventually the system converges to the equilibrium E along the dotted line in Figure 1 (of course, the same is true if the system starts at an out of equilibrium state with $x> x^*$).

**Phase Transitions**

A small change in one of the global parameters, g, leads to a small change in F(x). For example, a small increase in policing efforts can lead to a small increase in the perceived probability of being caught using drugs, therefore inducing a downward shift of F(x)- at each level of given x less people will choose to use drugs. Figure 2

[7]This is similar to the framework suggested by Schelling [1978].

[8]Of course, F(1) shown in Figure 1 plus 1-F(1) which is not shown in the Figure add up to 1, hence F(x) constitutes a probability distribution.

depicts such a shift in F(x).[9] This usually leads to a small shift in the equilibrium point (from $E_1$ to $E_2$ in Figure 2). Thus, small parameter changes *usually* lead to small changes of the equilibrium point.

(Insert Figure 2 About Here)

The above is true most of the time. However, if F(x) is shifted further downwards (see the lowest curve in Figure 2), at some stage a small parameter change may lead to a new crossing point (or points) of F(x) and the diagonal, and therefore to a new possible equilibrium point(s) which may be located far away from $E_2$ (see, for example, point $E_5$ in Figure 2).[10] When this occurs a phase transition may take place: a small parameter change may cause the system to jump discontinuously from $E_2$ to $E_5$, with dramatic changes in the properties of the system[11]. The transition can generally be from an equilibrium with large x to an equilibrium with small x, as in Figure 2 ($E_2$ to $E_5$), or from small x to large x, as in Figure 3 ($E_1$ to $E_2$). The phase transition mechanism described here is analogous to phase transitions in statistical mechanics systems, with heterogeneity playing the role of temperature (see, for example, Stanley [1971]). However, while the statistical mechanics phase

[9] F(x) can be either simply shifted downwards, or it can also be slightly deformed as a consequence of the small parameter change. In the present analysis for the sake of simplicity we ignore the effect of possible deformations.

[10] Notice that in Figure 2 two new equilibria are created, $E_4$ and $E_5$. However $E_4$ is a non-stable equilibrium, because small deviations from $E_4$ lead to convergence either to $E_3$ (if the starting point is $E_4 + \varepsilon$) or to $E_5$ (if the starting point is $E_4 - \varepsilon$). $E_5$, on the other hand, is stable. In general, an equilibrium point $x^*$ is locally stable if $|F'(x^*)|<1$, and unstable if $|F'(x^*)|>1$. For a general analysis of the stability of equilibria, see, for example, Azariadis [1993].

[11] When the system is as described by the lower curve in Figure 2 a transition *may* occur but it does not *necessarily* have to occur since $E_3$ is also a valid equilibrium. Generally we would expect the transition to occur if most individuals are better of at $E_5$. In the present analysis we do not model the dynamics of the transition itself. For example, it may be the case that most individuals are better off at $E_5$ and when this equilibrium becomes possible individuals rationally switch their choices and the system "jumps" to $E_5$. Alternatively, the transition may involve coordination problems and information frictions and may be more complex than a single jump to the new equilibrium (see Cooper and John [1988]). Nevertheless, in both cases if a transition occurs it is expected to be sharp and dramatic. For dynamic modeling of transition dynamics see, for example, Arthur [1989] or Chamley [1999].

transition theory holds for very specific systems (see Hors and Lordon [1997]), in what follows we show that this mechanism is quite general, and may explain phase transitions in a wide variety of different social systems.

(Insert Figure 3 About Here)

## III. Conditions for Phase Transitions

This section develops a criterion for determining whether a given system is susceptible to undergo a phase transition or not. We show below that most social systems in which agents have some inclination for conformity and in which agents are not "too heterogeneous" (as specified below) may undergo phase transitions.

A phase transition may occur when there is more than one equilibrium, i.e. when F(x) crosses the 45° line twice or more. In order for this to happen the slope of F(x) at some point *must* be larger than (or equal to) the slope of the 45º line (otherwise F(x) can cross the diagonal at most once). Thus, for a system to be susceptible to a phase transition we require:

$$\frac{dF(x)}{dx} \geq 1 \qquad \text{for some } x_0. \qquad (2)$$

This condition does not necessarily mean that the system has two equilibria (see for example the solid line in Figure 2, for which the slope of F(x) is greater than 1 in some region, but there is only a single point where F(x) crosses the diagonal. For a transition to occur it is also necessary to have more than one equilibrium point.) If condition (2) holds then at some stage a small parameter change *may* cause a new equilibrium to emerge and a phase transition *may* occur (dotted line in Figure 2).

Since $\frac{dF(x)}{dx}$ is simply the density f(x), eq(2) can also be rewritten as:

$$f(x) \geq 1 \text{ for some } x_0. \qquad (3)$$

Intuitively, condition (3) means that the system has a range of self-reinforcing positive feedback. To see this, suppose that initially the proportion of individuals choosing s=1 is $x_0$. Now suppose that this proportion is slightly increased to $x_0+\Delta x$ (due to a random fluctuation, or to individuals leaving or entering the system). Following this slight increase in the observed x, some individuals will change their choice (from s=0 to s=1, if $\Delta x$ is positive). Since $\Delta x$ is small, the number of individuals switching from s=0 to s=1 can be approximated by $\left.\frac{dF(x)}{dx}\right|_{x_0} \cdot \Delta x = f(x_0) \cdot \Delta x$. If $f(x_0)>1$ this means that following the observation of $x_0+\Delta x$, the new proportion of individuals choosing s=1 will be *greater* than $x_0+\Delta x$, and x will thus further increase. This increase in x will make even more individuals switch to s=1, and so on. This amplification effect goes on as long as f(x)>1. Thus, when f(x)>1 small perturbations are amplified by a positive-feedback effect. This positive feedback "snowball" effect is essential in order for phase transitions to occur[12].

Another way to view condition (3) is as a limit on the heterogeneity of the population. A phase transition may occur as long as the system is not "too heterogeneous". While condition (3) is not a conventional measure of the heterogeneity of individuals in the system (such as the standard deviation), it does impose a limit on this heterogeneity. Roughly speaking, condition (3) means that the distribution of the threshold in the population is concentrated around some threshold value $x_0$, and that it is not spread "too much". If all individuals have some threshold

[12]f(x)=1 is a rather special case for which the slope of F(x) is equal to the slope of the diagonal. In this case the two lines may overlap over some range, and many equilibria may therefore be possible.

value in the range [0,1][13], then $\int_0^1 f(x)dx = 1$. In this case, condition (3) always holds. Even in the extremely heterogeneous case, where every threshold has the same frequency in the population, f(x)=1 everywhere, and condition (3) holds (see solid line in Figure 4). In this case F(x) coincides with the diagonal, and every x is a possible equilibrium. Obviously, in this situation large jumps may occur. For any other distribution with a smaller degree of heterogeneity f(x) will be greater than 1 for some values, and condition (3) will again hold (see dotted line in Figure 4). In the case where some of the individuals do not have any threshold values (i.e. they stick to their choice no matter what everybody else does), $\int_0^1 f(x)dx < 1$ and it is generally possible to have f(x)<1 for all x. If f(x)<1 for all x, then a phase transition will never occur. However, for f(x)<1 to hold in the whole range, the population must be quite heterogeneous. To illustrate this point consider the case where the threshold x is normally distributed in the population (where x<0 and x>1 correspond to individuals with no thresholds). In this case condition (3) is translated to an upper bound on the standard deviation of the distribution. Namely, condition (3) is violated only if the standard deviation of x is larger than $\frac{1}{\sqrt{2\pi}}$ or larger than about 0.4.[14] This is quite a large degree of heterogeneity. For lower values of the standard deviation, condition (3) holds, and the system is susceptible to phase transitions.

(Insert Figure 4 About Here)

---

[13]This may not always be the case, because some individuals may choose s=1 even if they observe x=0 (which means that F(0)>0), while others may choose s=0 even if they observe x=1 (F(1)<1).

[14]The density of f(x) is given by: $f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(x-\mu)^2}{2\sigma^2})$ and it is maximal at x=$\mu$ where $f(x) = \frac{1}{\sqrt{2\pi}\sigma}$. Thus, f(x)<1 implies that $\sigma > \frac{1}{\sqrt{2\pi}} \approx 0.4$.

## IV. Transition Magnitude

While condition (3) determines whether a given system is susceptible to a phase transition or not, it does not indicate the magnitude of the possible transition. Obviously, this magnitude is of great interest, as one would like to know how dramatic of a transition the system might undergo. In this section we develop a general relation between the properties of F(x) and the magnitude of possible transitions[15]. We show that the heterogeneity of agents in the system (with respect to their thresholds) is key in determining the transition magnitude.

Consider a system for which f(x)>1 in some range $a \le x \le b$. Then, it can be inferred that the minimal phase transition will be of magnitude b-a. This is because regardless of the specific shape of F(x), if f(x)>1 in the range [a,b], then the slope of F(x) is greater than the slope of the 45º line in this range, and at the transition (when F(x) is lowered until it is tangent to the diagonal) the equilibrium points will be one to the left of a and the other to the right of b, as in Figure 5.[16]

If more information is available about F(x), the lower limit on the magnitude of the transition can be increased. For instance, if F(a) and F(b) are known (where again [a,b] is a range in which f(x)>1), it can be shown that the minimal magnitude of the transition is F(b)-F(a).[17] To see this, note that F(x) is non-decreasing, so that a segment of at least the size of segment m in Figure 5 must be added to the lower bound estimate of the transition magnitude. Thus, the transition is at least of

---

[15]Of course, if full information about F(x) is available, one can calculate the magnitude of the transition precisely. Here we would like to develop a general rule which does not depend on the specific functional form of F(x).

[16]This analysis assumes that F(x) is not deformed as it is translated downwards. If such deformations occur their secondary effects must be taken into account.

[17]Which is greater than b-a since f(x)>1 in the range [a,b] and F(b)-F(a)= $\int_a^b f(x)dx > \int_a^b 1 \cdot dx = b - a$.

magnitude (b-a)+m. However, segment m is equal to segment n, and segment (b-a) is equal to segment k (both triangles in Figure 5 are isosceles). Thus, the minimal transition magnitude is given by k+n which is exactly F(b)-F(a). This result holds regardless of the specific functional form of F(x).

(Insert Figure 5 About Here)

Figure 6 graphically depicts the magnitude F(b)-F(a). Recall that a and b are the endpoints of the range in which f(x)>1. F(b)-F(a) is simply the area below f(x) between a and b, which is the percentage of individuals in the population with thresholds in the range [a,b] (see Figure 6). The intuition for why F(b)-F(a) is the minimal transition magnitude is fairly straightforward. As mentioned previously, the range in which f(x)>1 is a positive-feedback range in which small changes in x are amplified. Thus, if some individuals with thresholds between a and b switch from s=0 to s=1 (or vice versa) this effect will be magnified until all the rest of the individuals with thresholds in the range [a,b] follow. Therefore, if a transition occurs, it will involve at least a proportion F(b)-F(a) of the population[18].

(Insert Figure 6 About Here)

The minimal transition magnitude, F(b)-F(a), is closely related to the degree of heterogeneity in the system. For example, if the system is very homogeneous then F(b)-F(a) will be close to 1 (see Figure 6A, in which the area below f(x) between a and b is close to 1). In this case, if a transition occurs it will involve a dramatic switch of almost the entire population. This result is very intuitive: if the population is very homogeneous, almost everybody will make the same choice. Thus, if a transition

[18]Notice that this is just a lower bound on the transition magnitude. In most cases the transition will be even larger, as more people from outside the positive-feedback range (with thresholds outside the range [a,b]) may also switch. (However, the effect of their switching is attenuated rather than being further magnified).

occurs, it will involve almost everybody in the system. In contrast, in a very heterogeneous population, even if a transition does occur, it will *typically* be rather small (see Figure 6B). While various assumptions regarding the specific functional form of F(x) may lead to various dependencies of the transition magnitude on the heterogeneity of the system, in general, the more homogeneous the system the larger one would expect the transition to be[19]. A specific case of great interest is that of a normal threshold distribution. Consider a normal threshold distribution with mean $\mu$ and standard deviation $\sigma$. In this case, the points a and b are given as the solutions to:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = 1,$$

which yields:

$$a = \mu - \sigma\sqrt{\ln\left(\frac{1}{2\pi\sigma^2}\right)} \quad \text{and} \quad b = \mu + \sigma\sqrt{\ln\left(\frac{1}{2\pi\sigma^2}\right)}.$$

Figure 7 shows b-a and F(b)-F(a) as a function of $\sigma$, the standard deviation of the threshold distribution[20]. Notice that even though b-a is not monotonic (as $\sigma$ increases the distribution becomes "wider", which tends to increase b-a, but it also becomes "lower", which tends to decrease b-a), the minimal transition magnitude, F(b)-F(a), is shown to be monotonically decreasing with $\sigma$. Thus, the more heterogeneous the population, the smaller the lower bound on the magnitude of the transition. As $\sigma$ reaches $\frac{1}{\sqrt{2\pi}} \approx 0.4$ transitions are no longer possible, as shown in section III. This

[19] Yin [1998] investigates multiple equilibria and implications for strategy in a model of political revolution with various assumptions regarding the threshold distribution.

[20] This analysis assumes that a>0 and b<1, in which case the magnitude of the transition does not depend on $\mu$. As b-a is always smaller than 0.25 (see Figure 7), and a and b are symmetrical around $\mu$, the above assumption holds as long as $0.125 \le \mu \le 0.875$. For $\mu$ outside this range a (or b) reaches 0 (or 1) and the analysis should be slightly modified, however, the same general result holds.

relationship between heterogeneity and the transition magnitude conforms with empirically observed phenomena, as discussed in section VI.

(Insert Figure 7 About Here)

## V. Stability and Reverse Phase Transitions

A phase transition may occur either immediately when a new equilibrium is feasible (see solid line in Figure 8) or afterwards when F(x) is slightly further translated (dashed line in Figure 8). Notice that while $E_3$ in Figure 8 is a stable equilibrium, $E_2$ is not. Namely, if the situation is as depicted by the solid line in Figure 8 a slight increase from $E_2$ to $E_2+\varepsilon$ will lead to a dramatic reversal back to the original equilibrium $E_1$.

(Insert Figure 8 About Here)

Thus, if the transition occurs immediately when the alternative equilibrium emerges, this transition may still be reversible and a reverse transition may occur. Indeed, such reversals have been empirically observed. For example, some revolutions such as the Prague Spring of 1968 and the 1989 uprising in China have been overturned with return to the old equilibrium. The recent 13.5% crash of the NASDAQ index and its amazing revival by 11.8% on the same day of April 4, 2000 may be thought of as another example of a transition followed by a quick reversal.

## VI. Concluding Remarks

Many social systems undergo dramatic and often surprising transitions which do not seem to be triggered by any significant cause. In this paper we suggest a “social phase transition” mechanism, analogous to the statistical mechanics phase transition mechanism, which may explain many of these dramatic events such as the

1987 stock market crash (and the more recent April 2000 Nasdaq crash), the 1989 East European revolution (and many other political revolutions), the outbreak of social cooperation, the instantaneous emergence of traffic jams, and the dramatic outbreak of teenage drug abuse. The social phase transition mechanism is very general, and may take place in a wide variety of social systems in which individuals have some inclination to conform with each other. The role of agents' heterogeneity (with respect to their thresholds) in social phase transitions is analogous to the key role of temperature in statistical mechanics phase transitions. The heterogeneity in a social system determines whether the system is susceptible to a phase transition. In systems in which transitions are possible, the heterogeneity of the system is closely related to the transition magnitude.

Social phase transitions may occur as long as the population is not "too heterogeneous". Furthermore, the more homogeneous the system, the more dramatic the transition will typically be. Thus, homogeneous systems are more susceptible to dramatic phase transitions. Indeed, one of the reasons suggested to explain the 1987 stock market crash is the homogeneity of investors which came about because many investors employed similar program-trading rules[21]. Levy and Levy [1996] also find that in their stock market model crashes are more frequent and more dramatic when investors are more homogeneous. Glance and Huberman [1993,1994] find that transitions in social cooperation systems are also more dramatic when the system is more homogeneous.

Thus, heterogeneity makes the system more immune to dramatic phase transitions. This has implications for regulators and strategists. For example, in stock markets, where dramatic transitions are generally undesirable, regulations can limit

[21]See "The Report of the Presidential Task Force on Market Mechanisms", Fed. Sec. L. Rep. (CCH), special report no. 1267, January 12, 1988 (the Brady Commission Report).

homogeneity by putting various restrictions on program trading and by encouraging competition between many diverse players, rather than letting a few homogeneous players dominate the market. In contrast, when aiming to induce a transition it would seem wise target initial efforts at a homogeneous sub-population. Indeed, many political revolutions have started with an initial transition among a homogeneous sub-population, usually students (see Kuran [1989, 1991]).

In this paper we consider systems where individual's choices are effected by the average behavior of *all other individuals,* as captured by the variable x. This is analogous to the "mean-field" approximation in statistical physics (see Stanley [1971]). The phase transition approach could be extended to consider systems in which various investors have a different effect on the decision making of a certain person (for example, in some systems a person may be affected by the behavior of individuals in his neighborhood more than he is affected by the behavior of individuals on the other side of town). In such systems geometry plays an important role and may significantly complicate the analysis. Another possible extension would be to consider systems in which there are more than two alternatives to choose from. It seems reasonable to expect phase transitions in such systems as well, because in statistical mechanics systems with more than two alternative states for each element (such as the plane rotator or the Potts model) phase transitions have indeed been found (see, for example, Yeomans [1993]).

There is no doubt that the mutual influence of individuals on one another plays an important role in many social systems. The social phase transition framework allows one to analyze the effects of this mutual influence, and to predict when it might lead to dramatic discontinuous events. This theory could prove an invaluable tool for policy making, as public polls could tell the policymaker whether a system may undergo a phase transition, and if so, how close the system is to the transition (is the system like the solid line or the dashed line in Figure 2?). This is very important information, which can help predict the transition, and perhaps act to avoid it or induce it, depending on its social desirability.  Analysis of models with local interactions (as mentioned above) could also provide answers to questions regarding the optimal way to exert forces in order to bring about social change. For example, in fighting drug abuse, is it optimal to spread policing efforts uniformly across problematic neighborhoods, or is it better to focus efforts on one neighborhood at a time? We believe that this research avenue holds great promise both for advancing the theoretical understanding of the dynamics of social systems, and as a powerful practical policy-making tool.

## References


Arthur, B., "Competeing Technologies, Increasing Returns, and Lock-In by Historical Events", *The Economic Journal*, 1989, 99, 116-131.

Azariadis, C., *Intertemporal Macroeconomics*, Blackwell, Cambridge, Mass. 1993.

Bikhchandani, S., Hirshleifer, D., and I. Welch, "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades", *Journal of Political Economy*, 1992, 100(5), 992-1026.

Brock, W., "Pathways to Randomness in the Economy: Emergent Nonlinearity and Chaos in Economics and Finance", *Estudios Economicos*, 1993, 8(1), 3-55.

Brock, W. and S. Durlauf, "Discrete Choice With Social Interaction I: Theory", Mimeo, University of Wisconsin, 1995.

Chamley, C., "Coordinating Regime Switches", *Quarterly Journal of Economics*, 1999, 869-905.

Cooper, R. and A. John, "Coordinating Coordination Failures in Keynesian Models", *Quarterly Journal of Economics*, 1988, 441-463.

Durlauf, S. N., "How Can Statistical Mechanics Contribute to social Science?", *Proceedings of the National Academy of Science*, 1999, 96, 10582-10584.

Föllmer, H., "Random Economies With Many Interacting Agents", *Journal of Mathematical Economics*, 1974, 1, 51-62.

Gennotte, G., and H. Leland, "Market Liquidity, Hedging, and Crashes", *American Economic Review*, 1990, 999-1021.

Glance, N. S. and Bernardo A. Huberman, "The Outbreak of Cooperation", *Journal of Mathematical Sociology*, 1993, 17(4), 281-302.

Glance, N. S. and Bernardo A. Huberman, "The Dynamics of Social Dilemmas", *Scientific American*, March 1994, 76-81.

Haken, H., *Synergetics: An Introduction*. Springer, New York. 1977.

Hors, I., and F. Lordon, "About Some Formalisms of Interaction: Phase Transition Models in Economics?", *Evolutionary Economics*, 1997, 7, 355-373.

Kerner, B.S, and H. Rehborn, "Experimental Properties of Phase Transitions in Traffic Flow", *Physical Review Letters*, 1997, 79, 4030.

Kihlstrom, R., and L. Mirman, "Risk Aversion with Many Commodities", *Journal of Economic Theory*, 1974, 8(3), 361-88.

Kirman, A., “Ants, Rationality, and Recruitment”, *The Quarterly Journal of Economics*, 1993, 108, 137-156.

Kraw, N., “A 700% Increase in Juvenile Drug Abuse Cases”, *Ha’aretz*, May 18, 2000.

Kuran, T., “Sparks and Prairie Fires: A Theory of Unanticipated Political Revolution”, *Public Choice*, 1989, 61, 41-74.

Kuran, T., “The East European Revolution of 1989: Is It Surprising That We Were Surprised?”, *American Economic Review*, 1991, 81(2), 121-125.

Kuran, T., “Ethnic Norms and Their Transformation Through Reputational Cascades”, 1998, 27, 623-660.

Kuran, T., and Cass R. Sunstein, “Availability Cascades and Risk Regulation”, *Stanford Law Review*, 1999, 51, 683-768.

Levy, H., and J. Paroush, “Toward Multivariate Efficiency Criteria”, *Journal of Economic Theory*, 1974, 7, 129-42.

Levy, M., Levy H. and S. Solomon, "A Microscopic Model of the Stock Market : Cycles, Booms, and Crashes," *Economics Letters*, 1994, 45.

Levy, M. and H. Levy, “The Danger of Assuming Homogeneous Expectations”, *Financial Analyst Journal*, 1996, 52, 3.

Roll, R., “The International Crash of October 1987”, in Kolb, ed. *The International Finance Reader*. Miami: Kolb, 1991.

Schelling, T. C., *Micromotives and Macrobehavior*, W.W. Norton & Company, New York, 1978.

Stanley, E. H., *Introduction to Phase Transitions and Critical Phenomena*, Clarendon Press, Oxford, 1971.

Topol, R., “Bubbles and Volatility of Stock Prices: Effect of Mimetic Contagion”, 1990, Financial Market Group discussion Series No. 101, London School of Economics.

Welch, I., “Sequential Sales, Learning and Cascades”, *Journal of Finance*, 1992, 47.

Yeomans, J.M., *Statistical Mechanics of Phase Transitions*, Clarendon Press, Oxford, 1993.

Yin, C.C., “Equilibria of Collective Action in Different Distributions of Protest Thresholds”, *Public Choice*, 1998, 97, 535-567.

Figure 1: Convergence to the Single Equilibrium E

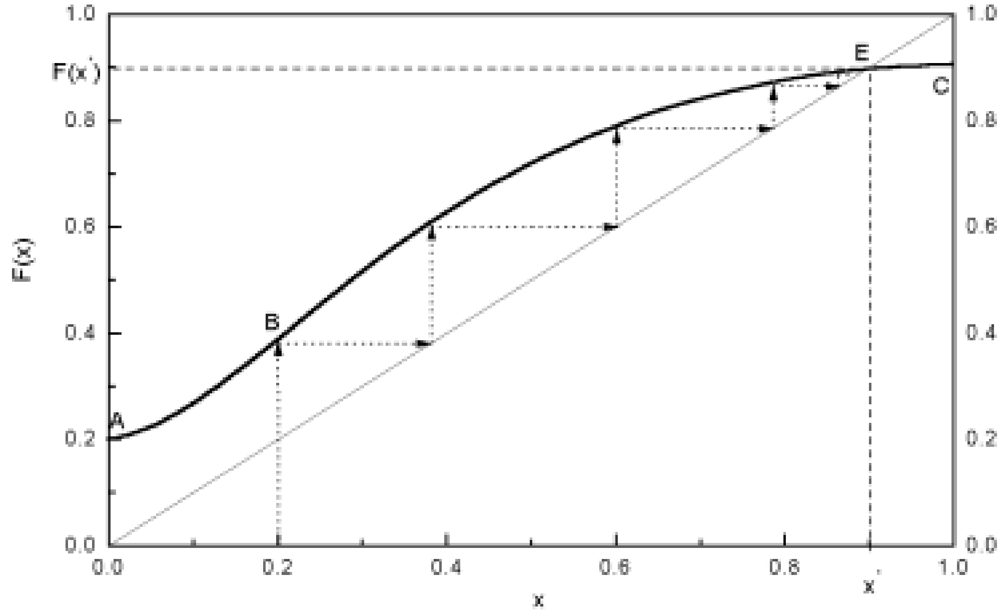
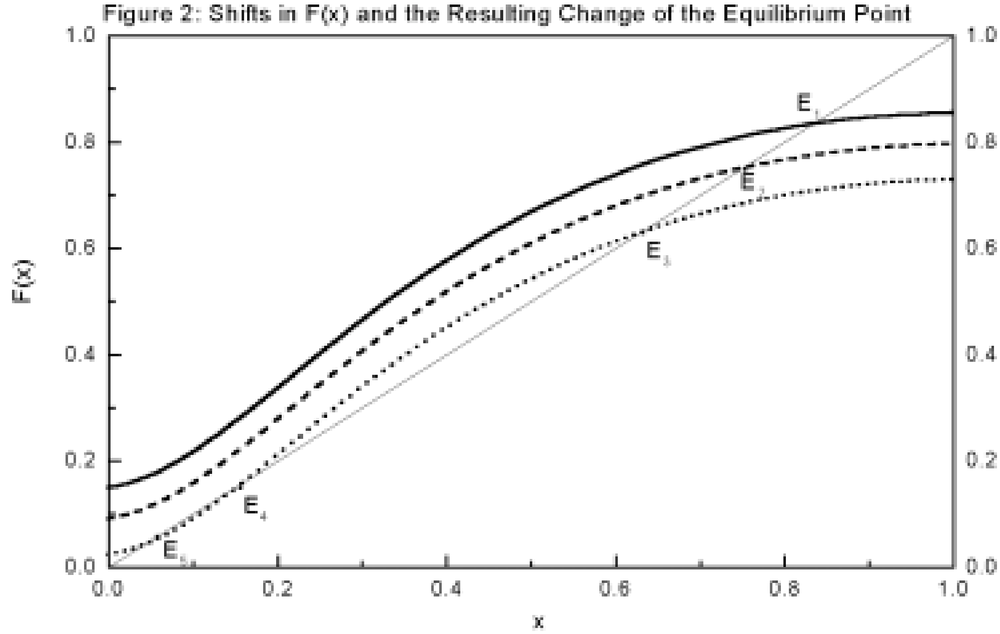



Figure 2: Shifts in F(x) and the Resulting Change of the Equilibrium Point



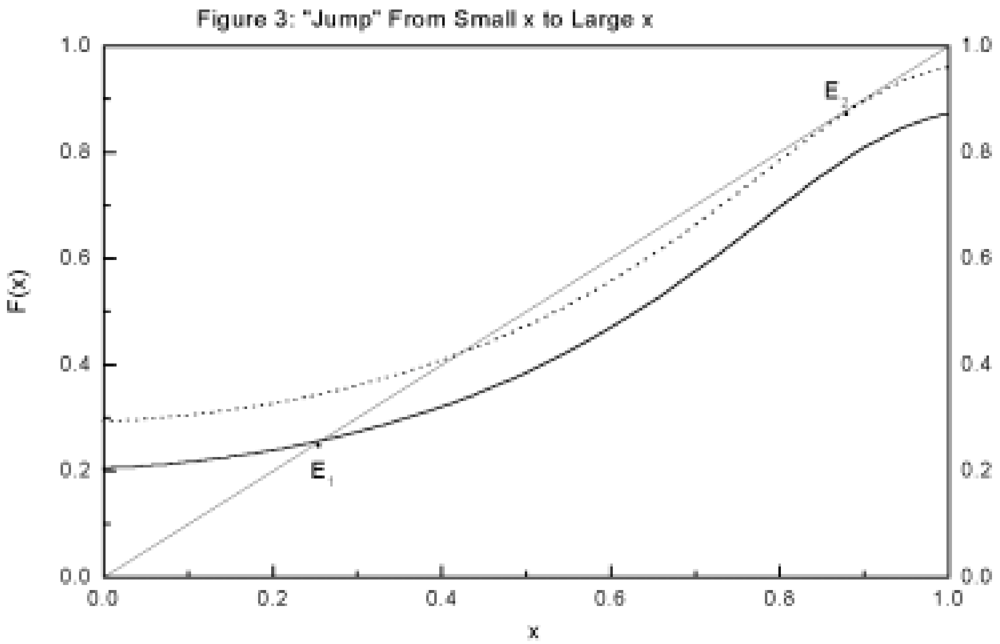


Figure 3: "Jump" From Small x to Large x
F(x)
x
E1
E2
0.0
0.2
0.4
0.6
0.8
1.0

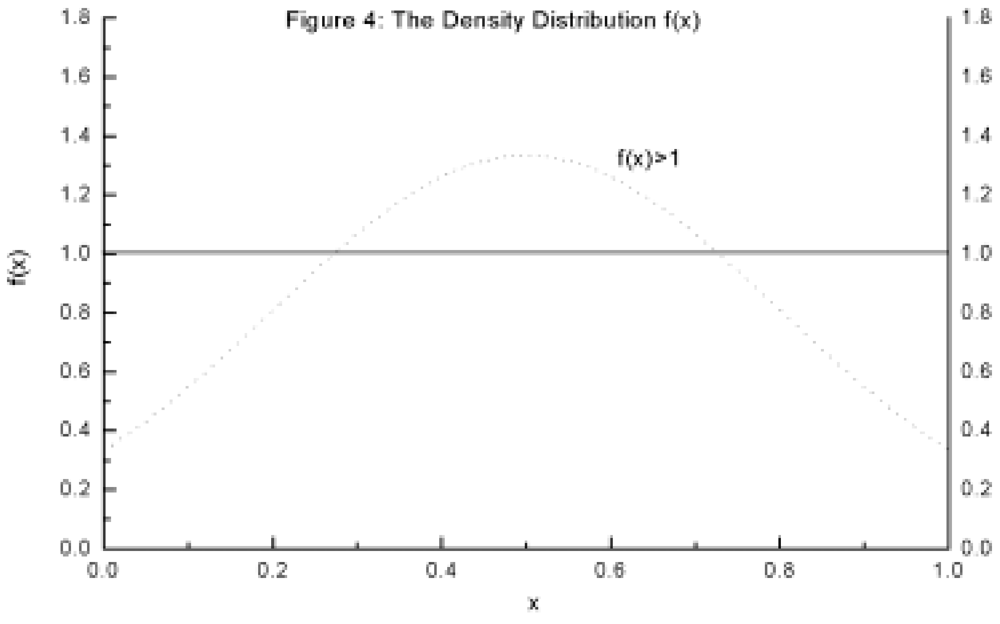


Figure 4: The Density Distribution f(x)
f(x)
x
f(x)>1
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

Figure 5: The Magnitude of the Transition

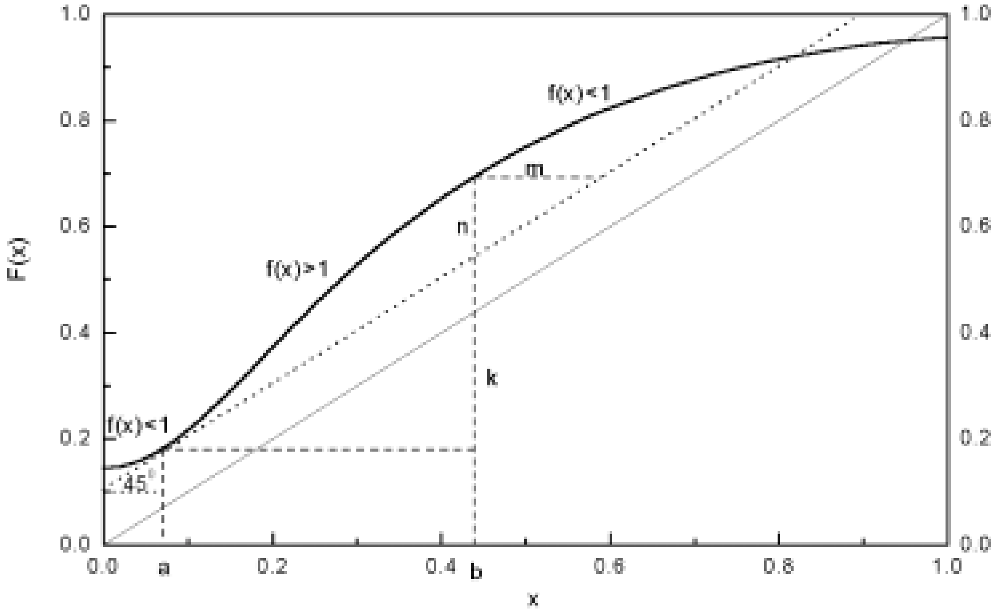


Figure 6: Heterogeneity and the Transition Magnitude

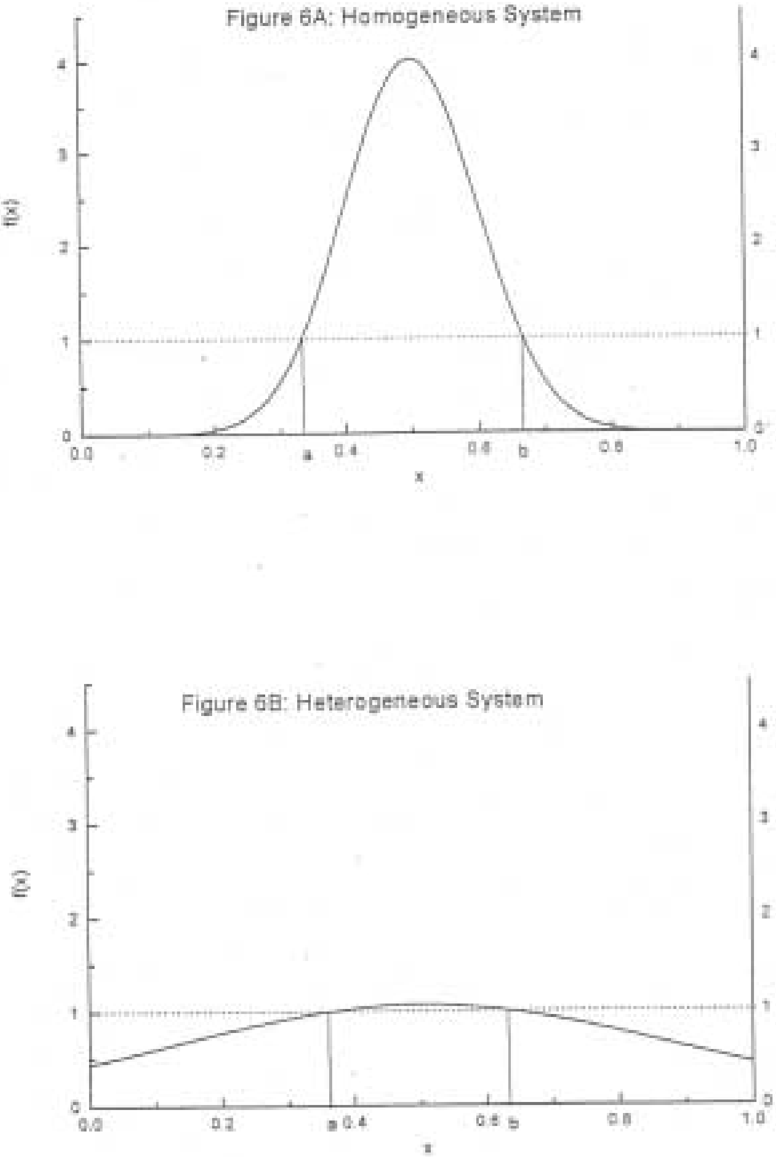


Figure 7: Minimal Transition Magnitude as a Function of the Heterogeneity

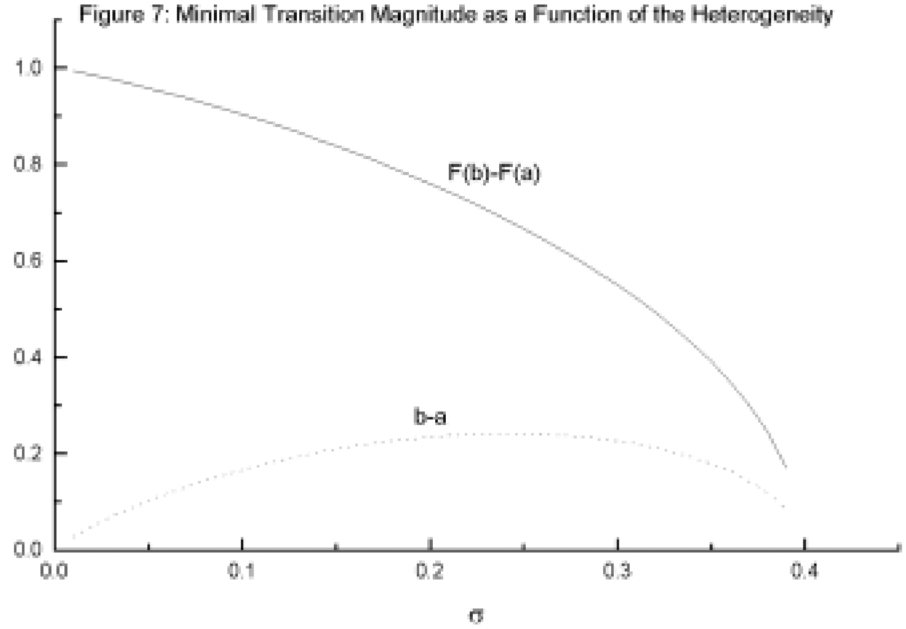



Figure 8: Reversal of Phase Transitions