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"Sensitivity Analyses"

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SENSITIVITY ANALYSES

Sensitivity Analysis studies the rate at which two or more variables change in relation to one another.

1. As an example consider the simple primary productivity equation (extend model in Figure):

$$dB/dt = K_1*I*N*B - K_2*B - (K_3 + K_4)*B*G/B_0$$

where B is the phytoplankton biomass, K_1 is the Light, I, and nutrient, N, coefficient, K_2 is the respiration coefficient, K_3 and K_4 & are the mortality and grazing coefficients. Note, this equation assumes a constant population of grazers.

2. Since the coefficients are all empirical constants, we might like to know to which is the Phytoplankton (B) the most sensitive, i.e. which could create the most error.

3.. We might like to know if any of these constants can cause instabilities in the model. This is shown by a simulation exercise on the coefficient K_1.

- K_1 small leads to an overdamped solution in which growth is quickly balanced by loss.
- K_1 large leads to oscillations and no steady-state condition.
- K_1 very large leads to large oscillations and a crash, which represents a change of state, i.e. population extinction.

4. The Sensitivity Analysis may reveal unanticipated dependencies or errors by answering questions such as:

- To which inputs are the outputs most sensitive, which may suggest, for example, the need for better accuracy for the input data.
- Is there a process represented to which the output is very sensitive, or conversely insensitive, which may suggest a model re-design.

5. Sensitivity Analysis might be used to reveal where within a system it is most/least sensitive. These are often called information 'choke points' or 'nodes'. For example:

- Is an input signal damped or amplified in a compartment?
- Where should we monitor the an intermittent point source, i.e. how far downstream are pulsed signals integrated into the entire stream flow?
- How do these choke points depend on other factors, e.g. if the monitored parameter is Nitrogen would seasonal temperature changes make a difference as to where in the stream you monitored its concentration.

6. Extend offers a quick and inexpensive means to ask simple questions of the modelled system that would be more difficult or expensive on other types of model formulations. Here are some examples:

a) Intelligent option analysis, or evaluating management trade-offs.

- Does it make any difference to an Estuary or River, if the atmospheric input of acid rain decays in concentration during a rain event? Or, in other words, could we simply represent acid deposition as having a constant concentration during a rain event?

- Under what conditions of treatment and flux, would there be an economic advantage in creating a riparian wetland pond as tertiary treatment zone? Would this increase the local fish yield in the river and provide a bird sanctuary of more benefit than otherwise using the land?

b) Intelligent error analysis.

- To which inputs are the outputs most sensitive to error, e.g. is the model output sensitive to error in daily rain as long as the longer-term mean is preserved?
- What is the nature of the error created by incomplete monitoring of a system, e.g. when should data gaps be avoided, or how much would the accuracy be increased with two monitoring sites instead of one?

c) Evaluating Thresholds and Changes in state.

- In a non-linear system, what is the range of validity of the modelled output, e.g. when a modelled bio-mass crashes, does this represent a loss of species in our system or is it a numerical problem in our model?

- What are the levels of a limiting factor which, when transgressed, would lead to a change in state, e.g. when is there insufficient light, due to turbidity, to eliminate benthic algal production?

Extend Layout for simple Primary Production Model, see *SensitPri-Prod.mox*