

smr.1242-6

**Earth Systems Science Course in Watersheds &  
Coastal Zone Simulation Modeling  
2 - 13 October 2000**

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**"Coastal Watersheds"  
"Estuarine Circulation"  
"Water Quality Assessment"  
"Flushing"**

**T. S. HOPKINS  
North Carolina State University  
MEAS  
Raleigh, NC  
USA**

These notes are intended for internal circulation only.



**EARTH SYSTEMS SIMULATION MODELLING  
MEA/ES 400**

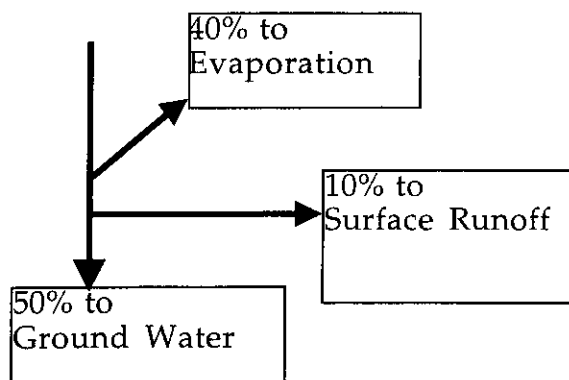
**COASTAL WATERSHED**

**A. Scope.** The Watershed Compartment traces the systems variables from atmospheric and internal sources to the river. Because our central issue concerns how land-use alters the functioning of the Watershed with respect to fluxes of the systems variables to the Estuarine Compartment, we must simulate the spatial variability according to type of land use. To the first approximation this is done by dividing the Watershed area into 'virtual sub-compartments' a based on land-use (Forested, Urban, and Agricultural) with a mean prescribed hydrological characteristic and any offshore or instream flow controls.

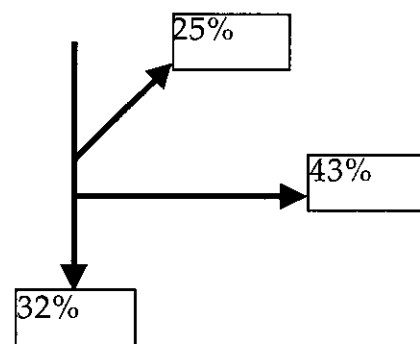
**B. Issues.** There are several major issues within the Watershed Compartment that result from our choice of systems variables.

1. How is the water pathway altered and partitioned within the watershed? For example, a typical conversion from 'natural' to 'urban' land use results in the following changes in water flux:

**Natural Land Use**



**Urban Land Use**



Most important is the bifurcation between surface runoff and ground water recharge. This relates to how much water is stored in the watershed and how much is shunted directly to the river. Since by definition the surface runoff is in contact with the land surface, without the cleansing process of filtration by the soil, it carries to the river a much greater load of particulate and dissolved loads than does the ground-water input to the river. Most common urban alterations involve buildings and pavement that prevent infiltration, chemicals deposited on urban surfaces that increase the pollutant loading, channelization of tributary flow that decreases the travel time to the river, and construction that exposes bare soil to erosive losses.

The increases in surface runoff volume cause the response of the river to change (hydrographic curve) and enhance the probability of flooding and/or of disturbing the estuarine habitat (by organic overload, depressed salinities, hypoxia, etc.).

2. How is the nitrogen cycle altered? Intensive agriculture increases the nitrogen loading. Excess or improper application of fertilizers either

contributes to ground water accumulation (Blue-baby syndrome) or to high concentration of nitrogen in the surface runoff. The use of buffer zones (strip of vegetated land along the riparian boundary) is often recommended for the latter. However, their effectiveness depends on the width, type of vegetation, slope and, importantly, whether or not drainage ditches or eroded channels exist that circumvent the buffer zone.

The increased use of intensive animal farming is becoming the primary source of nitrogen pollution in many areas. This practice is driven by the

- Market economy that rewards those who can get their goods to the market first,
- Cheap energy that allows grain food to be grown and transported from long distances, and
- Expensive land prices that favor intensive farming by decreasing the initial capital cost of land.

The result of this practice is an animal-sewage problem of immense proportion. The sewage is typically held in a 'containment pond' to allow some denitrification before being released into the waterways, or it is sprayed onto fields where a good proportion of the ammonia volatilizes to the atmosphere only to return as acid rain with a fairly short length scale  $\sim O(100\text{km})$  – depending on the local meteorological conditions.

Note that simulations intending to provide information or 'preventive' solutions of this aspect would necessarily require a land-use distinction between traditional and intensive farming and money in the model design. Essential to the economic quantification would be the costing into the assessment the costs paid by the community for loss of benefits from the river/estuarine compartments.

3. How much nitrogen (and other pollutants) is accumulating in the ground water? Ground water residence times (volume/recharge rate) are often very long and bacterially driven cycles (that chemically transform substances) are very weak. These characteristics increase the probability of a long-term source of pollution directly through drinking water and indirectly through discharges to the waterways. Very important, also, is the effect of the continuous, dispersed source of nitrogen resulting from atmospheric deposition over the watershed.

4. How is the sediment loading of the waterways modified? The flow controls in most rivers have greatly lowered the coarse-grain fraction of the river-borne reaching the coast. Most is trapped behind the dams. The lack of sediment to the coast is upsetting the sediment balance of adjacent beaches and causing severe beach erosion. Note, this aspect would require an expansion of the system boundaries to the wave-dominated portion of the continental shelf.

The finer suspended load can remain in suspension through the dams with shorter residence times. Both in the river and in the estuary it causes a decrease in the light available for primary production (surface and bottom) and thereby decreases the rate of carbon-nitrogen cycling. On deposition in the slower moving areas of the estuary it can cause a smothering effect either physically (inorganic matter), biologically (organic component adding to BOD) or chemically (flocculated pollutants).

**C. Processes.** The major processes, very briefly, are as follows:

1. Interception is the interruption of precipitation by vegetation. It buffers the mechanical impact of rain on soil, provides a temporary storage of water on the leaves. Some direct absorption can occur. Depends on type of foliage and cover.

2. Evaporation is the loss of water from wetted surfaces. Evaporation from leaves cools them and leaves behind dissolved substances. Depends on the temperature, relative humidity and the wind speed.

3. Transpiration is the water loss of vegetation to the atmosphere through the microscopic openings (stomates) on the leaf surface (underside) that facilitate gas (CO<sub>2</sub> & H<sub>2</sub>O) exchange. The stomates control the exchange using the RH of the air: closed when RH low and open when RH is high. Remember that evaporation of water vapor is less when RH is high.

4. Infiltration is the process of water entering the soil, which is controlled by both gravitational (down) and osmotic (omnidirectional). The capacity of the soil to contain water is determined by its porosity (volume of pore space/volume of soil space). The permeability describes the capacity of the soil to transport water (also hydraulic conductance). When the soil is completely saturated (all pore space filled with water) then only gravity and permeability control its movement. When the soil is relatively dry the movement is controlled by the gradient in soil moisture – towards dryer soil. When rain strikes dry soil it is pulled in by a combination both gravity and osmotic pressure. The vertical distribution is determined by the rate of supply and the rate of movement dictated by the two forces. The layer of soil above the water table and generally in the root zone is called the Vadose Zone. Obviously, the soil types can vary considerable throughout a watershed and in the vertical making the infiltration a difficult process to model. See Hydrology text, e.g. Dingman (1994).

5. Ponding occurs when the top portion of the soil becomes saturated and when the rate of rainfall exceeds the permeability (conductivity at saturation). Under these conditions the water that can not infiltrate into the soil and the excess collects at the surface.

6. Surface runoff describes the dispersal of the ponded water as it moves along topographical fall lines to the stream. Most commonly the water erodes a course way so that the distribution of runoff is not uniform and the probability of additional filtration is decreased. In some cases, thick grass roots or pavement, the surface runoff occurs in sheets. In a model the surface runoff pertains only to this temporary excess and not to small tributaries that may have some ground water input (see below).

7. Water uptake by roots proceeds also by the osmotic process. The vegetation is a major storage for water, which acts to buffer the local climate, to shade the ground, reduce soil evaporation and inhibit erosion. The water not used in photosynthetic production is transpired

to the atmosphere. Thus, the net effect is to dry out the deeper portions of the soil and to keep the surface more moist.

8. Darcy's law provides the mathematical relationship between for the flux of water through saturated soils. Primarily it is used to calculate the flux of groundwater towards the river, which is taken as  $Q_{gr} = (\text{hydraulic conductivity}) \cdot (\text{topographic inclination}) \cdot (\text{cross-sectional area of the aquifer})$ . The underlying force balance is that of gravity acting down slope and the inhibiting force of friction within the soil. (cf. , Dingman (1994). In most watersheds the ground water volume exceeds the annual supply, such that it is not normally emptied during periods of no rain. In fact the ground water supply can be equated to the river flow (or base flow) observed during periods of no rain or surface runoff.

9. Deep aquifer loss occurs in a watershed when the shallow (or first aquifer) is not completely underlain by an impermeable surface.

10. Nitrogen Uptake by vegetation occurs in the dissolved form via the water uptake and internal to the water liquid by diffusion. Consequently, during periods of growth, greater fluxes of nitrogen are required. Because growth and vegetation type are highly variable in a watershed, the process is usually formulated as a bulk uptake dictated by the seasonal growth and maintenance of the plants.

11. Interception of atmospheric N is a recently documented process (Science Vol. 279: 988) by which leaves take up atmospheric nitrogen (acid deposition) directly instead of through the root system. This is a damaging process because the leaves can not regulate the uptake (as by roots) with the result that growth is stimulated but without the other minerals normally taken from the soil, a process which is aggravated by the concomitant leaching effect of acid rain on the soil minerals.

12. Land Denitrification constitutes another important process in the cycling of nitrogen in the terrestrial system. It is difficult to observe and integrate over diverse soil types and is normally estimated as an unknown in a nitrogen budget, i.e. in a steady state system it can be approximated by subtracting the via the deep ground water from the amount of nitrogen fixed from the destroying/fragmenting the host forest ecosystem and similarly the downstream aquatic ecosystems.

It is important to consider that the time scale and potential interaction with the biotic components of the aquatic systems varies markedly between the coarse and fine grain fraction of eroded sediment. The time scale for bed load transport is much longer than for suspended loads and bed load impacts primarily the benthic habitats while suspended loads impact both the surface and benthic biota.

**D. Inputs.** The following information is needed for modelling the fundamental processes of the watershed with respect to the systems variables: rainfall, temperature, relative humidity, wind, evaporation, land use, soil type, vegetation cover, vegetation characteristics, water-table height, water-table inclination, stream length, distance to stream, Vadose Zone porosity, shallow aquifer conductivity, ponding area, wet & dry nitrogen deposition;

fertilizer application rates, land-use characteristics relating to vegetation, soil types, slope per land-use type; urban development/ sediment containment ponds, agriculture practices and water removals with and without return.

**E. Outputs.** The information required as output from a coastal watershed model will depend on the model objectives (problem being quantified). Some examples would be: ground, surface runoff, storage in vegetation, N in ground water & surface runoff; N storage in vegetation & ground water; N lost to atmosphere, total suspended load, dissolved load, organic content.

## ESTUARINE CIRCULATION

**A. Scope.** Estuaries are bodies of water that act as physical, geological, chemical, biological buffers between river and coastal ocean systems. Much of their characteristics is determined by their geomorphology which, in turn, is a function of the geological history of the coast (e.g. depositional, tectonic, volcanic, etc.). In this course we are focussing on riverine estuaries of depositional coasts fed by freshwater runoff. In these cases, the abiotic environment is strongly determined by the degree of runoff, the supply of terrestrial material, the local meteorology, and the amount of ocean tidal energy in the estuary. In order that an estuary act as a buffer there must be a spatial length scale to the transition between the river/fresh-water and coastal/marine domains. This is usually provided by geomorphological features such as: deltas, sandbars and spits, drowned riverbeds, coastal embayments, etc.

While the river flow is forced by the inclination of its seabed, the circulation of the estuary is more complex and is forced by combinations of the amount of fresh water, the potential energy, the tidal energy and the wind. Time and space variations in these forces generate gradients in energy that both disperse (mix) and organize (sustain gradients) in the distributions of water-borne substances. Riverine estuaries are usually classified based on their vertical stratification between the degree of runoff that tends to stratify the estuarine waters and the amount of tidal energy that tends to mix them:

Vertically Mixed	Small runoff, strong tidal energy
Slightly Stratified	Intermediate runoff and tide
Salt Wedge	Large runoff, weak tidal energy

**B. Aspects.** Several aspects of the physical control of the estuarine environment are briefly explained in the following paragraphs.

Water as an Abiotic Medium. Water serves to provide a means of transport for abiotic substances. The conversion between the forcing of water and its response as movement is a dissipative process that occurs over a spectrum of scales from those associated with the estuary to the friction between molecules. However, the movement response, in terms of circulation, mixing, etc, is not a continuous spectrum but is organized in modes that represent energy storage and dissipation. For this reason the spatial/temporal gradients that result from water movements are also not continuous in a spectral sense. In general, these modes must have some stability in time and space in order that the in-situ biota is exposed to an abiotic environment stable with respect to the adaptive time scale of the biota. The richness of the physical energy in estuaries thus permits a large biotic diversity for its ecosystem.

Reference. Similar to the river system, we can characterize the environment with respect to a moving volume of water (as experienced by phytoplankton) or with respect to a fixed location (as by the benthos). When the reference point moves with the water it is referred to as "Lagrangian", and when it fixed in space, it is called "Eularian" after two famous, but not estuarine, mathematicians.



Conservative and Non Conservative Variables. We generally classify substances transported by water as two types depending on whether or not they can change their concentration in-situ by other than physical means. A “conservative” property indicates that it has no in-situ sources or sinks or that it is inert. A “non conservative” property indicates that it can change its concentration within the water by a number of processes, e.g. sedimentation, chemical precipitation, biological uptake, chemically reactions, etc.

Conservative substances are only affected by the physical processes of diffusion in a Lagrangian framework and diffusion and advection in an Eulerian framework. Diffusion is the net movement of a substance down gradient caused by small-scale turbulent motions in the water. Advection is the change in the concentration of a substance caused by non-zero gradients in the direction of a non-zero flow (see Figure).

As an example consider a point source discharge of fresh water and nitrogen into an estuary:

- The fresh water content is conservative and will be transported by the mean water movement. If you are observing it from a small boat (Lagrangian), then changes in its concentration will only be determined by diffusion, as evidenced by the spreading of the plume relative to the mean flow. If you are in an anchored boat then you will observe changes caused only by changes in the mean flow or in the mean concentration of fresh water coming from the source.
- The nitrogen content is non-conservative and will be transported by the water movement and will be taken up by the phytoplankton. This means the changes in nitrogen will differ in space and time as the nitrogen is taken up, in addition to being subjected to advection and diffusion.

Physical Forcing. The aspects of physical forcing that control the circulation of estuarine waters are explained briefly in the following paragraphs.

- Density is a water property that is conservative except at the surface. In a fresh water system only the temperature controls the density but in a marine system both the temperature and the salinity control it: less dense water is warmer and has less salt. The water’s density can be changed at the surface through
  - 1) Heat exchange with the atmosphere, which cools or warms the water,
  - 2) Water vapor exchange with the atmosphere which precipitates or evaporates water making the surface water less or more salty, respectively,
  - 3) The addition of fresh water at the boundaries, as with river discharge.
- Pressure. Horizontal differences in pressure move water towards the point with the less pressure. The pressure at depth can vary horizontally because of differences in sea level or differences in the internal weight of the water column between two locations. The undisturbed sea level is the horizontal surface that would exist if the waters were homogeneous and not in motion. The internal weight of a water column is the vertical integrated density between some depth and the undisturbed sea level.

When the sea level is up or down at one place relative to the other, a “barotropic” pressure gradient force is generated that accelerates the waters under the high

sea level location towards the waters of the low sea level. This is a quick response (dumping a bucket of water into a bathtub) and causes all the water columns to shift to bring the water level to a new equilibrium. On the other hand, when the waters are less dense at one part than at another (hot end of the bathtub), a "baroclinic" pressure gradient force is generated which increases with depth (weight difference is more) that accelerates the more dense waters to flow under the less dense waters. As a result the sea level at the more dense location drops and a barotropic pressure gradient develops accelerating the entire less dense water column toward the denser one. Combining these two forces results in a "two-layer flow" with the baroclinic force predominating at the bottom and the barotropic force at the surface. See Figure. This combined action defines the estuarine circulation where more dense seawater moves landward in the bottom layer and the less dense fresh water moves seaward in the surface layer. At the point in the vertical where the speed is zero is also the point where water properties (temperature, salinity, and oxygen) will have their strongest vertical gradients. Other forces tend to modify this circulation by opposing or favoring or retarding the flow in one of the layers or by mixing the waters to modify the pressure gradient force.

- Tides. The ocean tide is driven by cyclic changes in the local balance between the integrated centrifugal and gravitational equilibrium between the earth and the moon (or sun) which causes the coastal sea level to move up and down. A dislevel occurs between the ocean and the estuary due to the restricted connection (at the mouth of the estuary) because not enough water can enter to keep the internal sea level in phase with the rising (or lowering) ocean sea level. As a result a barotropic force accelerates water in (flooding) when the ocean tide is higher and out (ebbing) when the ocean tide is lower. The general outcome of this oscillating current is a vertical mixing of the estuarine waters as the kinetic energy of the tidal motion is dissipated by bottom friction (below).

- Winds. The local winds can have a significant (but variable) effect depending on the wind strength and its fetch (distance over which the wind blows). The effect of the wind is the result of three different dynamics that are activated by the wind:
  - 1) The wind stress (proportional to the square of the wind velocity) drags the surface layers of water with it creating a "surface-frictional flow". For example, the wind could be blowing up the estuary and opposing the surface estuarine circulation, or vice versa.
  - 2) The total transport of this surface frictional flow is directed anywhere from downwind to  $90^\circ$  to the right (Northern Hemisphere) of the wind. This transport converges against the boundaries of the estuary, raising the sea level and generating a barotropic pressure gradient force away from the boundary. When the estuary is shallow the wind transport will be downwind and the barotropic force will favor a return transport at the bottom. When the estuary is deep (>20 m) wind transport will be to the right of the wind and the baroclinic pressure force will generate a (geostrophic) circulation around the estuary with the coast on the right hand side of the current.
  - 3) The surface waves generated by the wind will cause mixing in the surface to a depth of approximately  $1/2$  the wavelength. This mixing will change

redistribute surface properties in the vertical and even affect the depth of the thermocline, halocline, oxycline, etc.

Bottom Stress. The movement of the water is retarded and dissipated by moving over the bottom. The frictional force generated by the bottom is similar to that generated by the wind over the surface and is proportional to the square of the bottom flow speed. This makes it a strongly controlling negative (controlling) feedback loop on the system (the more the flow the more the opposing force).

Salt Balance. The conservation of mass (salt) over long time scales is often utilized to estimate the estuarine circulation. The fact that the total salt of the estuary is in approximate steady state over long time periods implies that the exchange of salt with the ocean must be balanced. This requires that the salt in equal the salt out:

This salt balance can be expressed by the following relationship for mass continuity (or conservation of mass):

$$\rho_{in} S_{in} V_{in} = \rho_{out} S_{out} V_{out} \quad (1)$$

where  $S_{in}$  and  $S_{out}$  = salinities of the inflowing Ocean water and outflowing Estuarine water, respectively,

$$\begin{aligned} \rho_{in} \text{ and } \rho_{out} &= \text{respective densities (kgm}^{-3}\text{), and} \\ V_{in} \text{ and } V_{out} &= \text{respective volume transports (m}^3\text{d}^{-1}\text{).} \end{aligned}$$

In practice, the  $\rho_{no}$  and  $\rho_{ps}$  can be canceled since the two densities will be the same within 3% at the most (Pickard and Emery 1990). Therefore equation (1) becomes (2):

$$\begin{aligned} S_{in} V_{in} &= S_{out} V_{out} \\ \text{Salt In (grams)} &= \text{Salt Out (grams)} \end{aligned} \quad (2)$$

Volume continuity, on the other hand, gives the following relationship:

$$V_{in} + R + P = V_{out} + E \quad (3)$$

where  $R$ ,  $P$  and  $E$  are river runoff, precipitation, and evaporation, respectively. If we let  $(R + P) - E = WB$  then

$$V_{out} = V_{in} + WB \quad (5)$$

Combining equations (2) and (5), we get the Knudsen Relations for salt

$$V_{in} = \frac{WB \cdot S_{out}}{(S_{in} - S_{out})} \qquad V_{out} = \frac{WB \cdot S_{in}}{(S_{in} - S_{out})}$$

conservation through a cross section in an estuary.

## WATER QUALITY ASSESSMENT (*think rates and processes not amounts*)

**Qualitative Indicators** - Numerical values that give information on a system

**Use rate > Replenishment rate?** - i.e. ground water, etc.

**Use rate > Purification rate ?** - i.e. pollution accumulating in lake sediments, etc.

**Threshold values** e.g. DO < 2 ml/l, TSM > 100 mg/l, Pb < 15 ppb

**Diversity Indexes** measures the number of individual species and the distribution of individuals among species. Should weigh intolerant species more (as **Keystone Species**)

**Abiotic Habitat** - multivariate analyses of environmental parameters, synergistic damage, i.e. acid rain leaching of soil, freshening of estuarine nursery areas, etc.

**Pathways, man to nature to man**

**Transport from source**

- in gas or aqueous form within atmosphere or hydrosphere
- flocculation onto particles (clay) forming suspended colloidal aggregates
- bio-transport via uptake and movement differential to the water

**Return to man**

- bioaccumulation
- direct ingestion or contact

**Residence times** - as defined above **Remember**

- Importance of compounding the flushing time of the media and the reaction time of the pollutant
- Pathway may involve a sequence of residence times that must be considered in series
- A stalled pathway or very long residence times can imply relative irreversibility.

**Entropy Gamble**

**Dilution - original Adaptive approach to pollution** - OK, until reach threshold values at which time

- the dose curve changes
- more sensitivity to synergistic effects
- the cost of clean up skyrocket

**Containment or Export - modern Counteractive approach** - euphemistically called **Waste Management**

- Defers the problem in time or in space
- Often causes long-term environmental damage

**Source Elimination, Reduction, Benign Recycling - are Preventive approaches**

- Considered too expensive - up front costs
- Requires understanding the problems of the above approaches
- Long-term preservation of Natural Capital and economic gain

**Cost of Pollution**

Valid aspects to remember

- Law of diminishing returns
- Mixing substances increase Entropy enormously
- It takes Energy (\$) to decrease Entropy

**General functionality of pollution reduction:**

**Amount of pollution reduction = function of (amount, distribution, extraction, reactivity)**

Consider a very simple mixing case of dumping salt and sugar into the same container. Physical separating them using taste would be tedious. Such a method would be linear assuming that your taste buds didn't get confused. The result would be that cost and degree of un-mixing would be directly proportional to time spent.

Consider another case of spreading the salt on the floor and having to physically remove it. Then the rate at which you clean it up is proportional to the amount on the floor.

$$\Delta P/\Delta t = \kappa P \text{ or } P = P_0 \exp(-\kappa t) \text{ when } t=0 \text{ then } P = P_0 \text{ and when } t=1 \text{ then } P = P_0/2 \text{ or } P = P_0 \exp(-0.7t)$$

If you are lucky, you can get \$50/hr for this intriguing task, i.e.

<u>Hours</u>	<u>Removal</u>	<u>Cost</u>
1	50%	\$50
2	75%	\$100
4	93.75%	\$200
8	1ppt	\$500
20	1ppm	\$1000

Since this could go on until the last particle of flour was retrieved, your boss would have to accept some threshold level of flour pollution to live with.

## Natural Systems Health

**Food-web changes** - higher entropy state, less productive efficiency, loss diversity and stability.

**Extinctions** - loss of niches in the system, loss of species

**Resilience** - abiotic trends (as above), generally are a cause of stress that precedes the change of state for the system or weakens the immune system for individuals

**Size constraints** - system fragmentation and critical population size are important aspects need in considering natural system health. They are poorly defined since they need integrated assessments.

## STRATEGIES

**ADAPTIVE** - Continue to practice as before without trying to change the source of pollution (problem). Examples,

- stop drinking water and buy Pepsis
- continue to drink tap water
- move to another area

**COUNTERACTIVE** - Take some remedial action that mitigates the consequence but not the problem. Examples,

- drink bottle water
- buy a water purifier
- vote for use of a better water source
- vote for regulations

**PREVENTIVE** - Act to reduce or eliminate the cause of the problem. Examples,

- prevent pollutants from entering into the water source
- regulate the use and disposal of pollutants

**NOTES:**

- Often preventive measures require a larger scaled action (other political/economic entities) and more up-front costs (to invest in different practices) and are therefore blocked.
- The logic of taking preventive action escapes many persons who don't understand the indirect damages of pollutants (as above under assessments), it is often difficult to convince voters and politicians.
- Adaptive, especially, and Counteractive policies eventually lead to enormous costs.
- Preventive policies are self perpetuating, because the problem vanishes, and therefore in the long run are much less expensive.

**Handout and Homework on FLUSHING**

**Consider a simple tank situation - as with the Lake Pollution model.**

1. Consider an example where the Vol = 100 m<sup>3</sup> with an inflow and outflow of 10m<sup>3</sup>/day, then the "mean residence time" (RT) of the water would be computed as RT = Volume/Outflow or 10 days. This really tells us how long it would take to drain or fill the Lake with water.

2. However, if the lake were exposed to a pollutant spill along its shore, and we wanted to know how much would be left the above residence time would not give us a very good estimate. Note, we assume the inflow is clean and the outflow is not.

In fact, let's say the lake is mixed with respect to the pollutant, then we can assume the outflow ( $P_{out}$ ) will be proportional to the amount of the pollutant mixed in the lake ( $P$ ) then the rate of change of the volume  $\Delta P / \Delta t = \kappa P$  where  $\kappa =$  is the constant of proportionality. Let's take 10%. This has the solution  $P = P_0 \exp(-0.1t)$  where  $k = 0.1$ .

Thus, the amount of pollutant remaining in the lake would decrease exponentially, as the outflow takes out 10% each day, e.g.

<u>Time</u>	<u>% of original amount</u>
0	100
1	90
2	81
3	75
7	50
10	37
20	14
30	5

This means that for a "conservative" pollutant, the original amount would only be reduced to 37% after 10 days. Since this exponential continues on forever, the rate of change is usually specified by a "e-folding time" or "half life", which cites the time for the quantity to decrease by half of its original value. In this case it's 7 days.

Note, this occurs when the power of  $e$  is equal to 0.7. A useful rule of thumb is that if something is increasing or decreasing at a fixed percentage per time, you can divide the percentage into 70 and you will get the number of time units for the something to increase or decrease by a factor of 2; e.g. a stock increasing at 5% will take 14 years to double.



3. If we would like to ask about a "non-conservative" pollutant, we would have to know its half life, or decay time. Let's assume that it is 20 days. If we look at the combined effects of flushing and decay, we have to add the exponents.

Let  $P$  equal the amount of mass for the pollutant in the Lake, then

$$P = P_0 \exp(-0.035t), \text{ where } \kappa = 0.035 \text{ (from } \kappa 20 = 0.7)$$

Since we want to know the amount of mass remaining in the original water, we would multiply the two expressions

$$P = P_0 \exp(-0.135t) \text{ which would have a half-life of 5.2 days (} t_{1/2} = 0.7/.135).$$

### Questions:

1. How would you calculate the concentration in the Lake after 5.2 days?
2. Consider that the lake had 2 layers with only the surface layer being mixed, and with the non-conservative pollutant sinking out to the bottom layer at 10% per day. Consider negligible upwelling of the pollutant from the lower layer to the upper layer.
  - a. What would be the half life of the pollutant in the surface layer?
  - b. When would the bottom layer have a maximum?

