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(SUMMARIES)

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NON-CANONICAL BEHAVIOUR IN CANONICAL THEORIES

R. JACKIW

MIT, Cambridge, Mass., USA.

A summary of recent work concerning the nature of commutators in model field theories is presented. It is shown that commutators defined via the Bjorken-Johnson-Low theorem [1] in perturbation theory will differ in the general case from their canonical value. These modified commutators are relevant, by construction to high-energy theorems such as those of Preparata and Weisberger [2] or Callan and Gross [3]. These high-energy theorems are accordingly modified. Modification of low-energy theorems, such as the one of Sutherland and Veltman for $\pi^0 \to 2\gamma$ decay [4], are shown to follow from the fact that the non-canonical commutators can be of a form which makes it impossible for Feynman's conjecture to hold, i.e., Schwinger terms do not cancel against divergences of seagulls [5)-7)]. The solution of the general problem of constructing Lorentz-covariant and gauge-invariant T^* products from a knowledge of the T product and of the commutators is indicated [7]. The dependence of the commutators on the dynamics is exhibited. A summary of results is presented in the Table.

REFERENCES

1) J.D. Bjorken, Phys. Rev. 148, 1467 (1966);
K. Johnson and F.E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. No. 37-38, 74 (1966).

2) G. Preparata and W.I. Weisberger, Phys. Rev. 175, 1965 (1968).

3) C. Callan and D.J. Gross, Phys. Rev. Letters 22, 156 (1969).

4) D.G. Sutherland, Nucl. Phys. B2, 433 (1967).

5) R. Jackiw and K. Johnson, to appear in Phys. Rev.

6) S.L. Adler and D.G. Boulware, to appear in Phys. Rev.

7) D.J. Gross and R. Jackiw, CERN preprint, 1969.

R E M A R K S
(for the table)

a) The solid line is a Fermion.

b) The wavy line is a vector boson in the Landau gauge (so that $Z_1 = Z_2$ is finite) coupled with strength g to $\bar{\psi}\gamma^{\mu}\psi$.

c) $G(p)$ is the unrenormalized Fermion propagator.

d) $\Gamma^{\mu}(p,q)$ is the unrenormalized vertex function.

e) x in the diagram represents the vector current.

f) The state $|\psi\rangle$ is a Fermion state with momentum p normalized so that $\langle 0|\psi|\psi\rangle = 1$.

g) Crossed diagrams must also be included.

h) Schwinger [7] showed that positivity and Lorentz covariance force the commutator to be non-zero. Previously, Goto and Imamura [8] derived a representation for this object which involved <u>one</u> derivative of the delta function. Their result is not verified by calculation.

i) $\tilde{x}$ in the diagram is the axial vector current.

j) $|\gamma\rangle$ is a one-photon state.

k) $\tilde{F}^{\mu\nu}$ is the dual electromagnetic tensor.

l) c is a constant.

m) The commutator has been written in explicitly covariant notation, with the help of a unit time-like vector n , and $P_{\alpha\beta} \equiv g_{\alpha\beta} - n_{\alpha} n_{\beta}$. Only the anomalous portion of the commutator is explicitly indicated.

n) α , β are bosons.

o) φ is a scalar or pseudoscalar field.

p) Some derivations of Weinberg's first sum rule assume a c-number Schwinger term [14]. In spite of the presence of q-number Schwinger terms this theorem remains true.

REFERENCES

(for the table)

1) H. Lehmann, Nuovo Cimento 11, 342 (1954).

2) R. Jackiw and G. Preparata, to appear in Phys. Rev.

3) G. Preparata and W.I. Weisberger, Phys. Rev. 175, 1965 (1968).

4) C. Callan and D.J. Gross, Phys. Rev. Letters 22, 156 (1969).

5) R. Jackiw and G. Preparata, Phys. Rev. Letters 22, 975 (1969).

6) S.L. Adler and Wu-Ki Tung, Phys. Rev. Letters 22, 978 (1969).

7) J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

8) T. Goto and I. Imamura, Progr. Theoret. Phys. (Kyoto) 14, 396 (1955).

9) D.G. Boulware and R. Jackiw, preprint 1969.

10) D.G. Sutherland, Nucl. Phys. B2, 433 (1967).

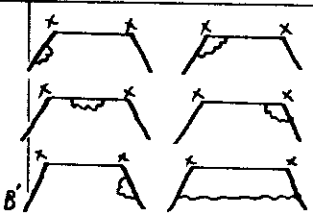
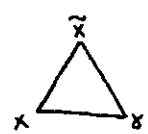
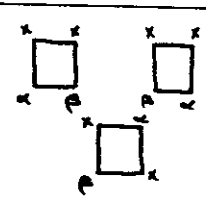
11) K. Johnson and F.E. Low, Progr. Theoret. Phys. (Kyoto), Suppl. No. 37-38, 74 (1966).

12) R. Jackiw and K. Johnson, to appear in Phys. Rev.

13) S.L. Adler and D.G. Boulware, Inst. of Advanced Studies, Princeton, preprint, 1969.

14) S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

15) T. Nagylaki, Phys. Rev. 158, 1534 (1967).

Canonical commutator	False theorem	Effective commutator	Offending diagrams	Remarks	References
$\langle 0\|[\psi(\underline{x},t), \bar{\psi}(\underline{y},t)]_+\|0\rangle = \gamma^0 \delta(\underline{x}-\underline{y})$	$G(p) \underset{p\to\infty}{\longrightarrow} i/\not{p}$ (Lehmann)	$\left(1-\frac{3g^2}{32\pi^2}\right)\gamma^0\delta(\underline{x}-\underline{y})$		a,b,c	1,2
$\langle 0\|[\psi(\underline{x},t), J^0(\underline{y},t)]\|\psi\rangle = \delta(\underline{x}-\underline{y})$ $\langle 0\|[\psi(\underline{x},t), J^i(\underline{y},t)]\|\psi\rangle = \gamma^0\gamma^i\delta(\underline{x}-\underline{y})$	$\Gamma^\mu(p,q) \underset{q_0\to\infty}{\longrightarrow} \gamma^\mu$	$\delta(\underline{x}-\underline{y})$ $\left(1-\frac{g^2}{8\pi^2}\right)\gamma^0\gamma^i\delta(\underline{x}-\underline{y})$		a,b,d,e,f	2
$\langle\psi\|[j^i(\underline{x},t), j^j(\underline{y},t)]\|\psi\rangle = \delta(\underline{x}-\underline{y})\, i\varepsilon^{ijk}\langle\psi\| J^5_k(x)\|\psi\rangle$ $\int d^3x \langle\psi\|[j^i(\underline{x},t), j^j(\underline{y},t)]\|\psi\rangle = (\delta^{ij}\underline{p}^2 - p^ip^j)A + \delta^{ij}B$	Preparata-Weisberger theorem about finite radiative corrections Callan-Gross sum rule for electroproduction	$\delta(\underline{x}-\underline{y})\, i\varepsilon^{ijk}\left(1-\frac{3g^2}{16\pi^2}\right)\times \langle\psi\| J^5_k(x)\|\psi\rangle$ $(\delta^{ij}\underline{p}^2 - p^ip^j)A + \delta^{ij}B + \delta^{ij}\underline{p}^2 B'$		a,b,e,f,g	3,4,5,6
$\langle 0\|[j^0(\underline{x},t), j^i(\underline{y},t)]\|0\rangle = 0$	See remark h	$S\partial^i\delta(\underline{x}-\underline{y}) - \frac{1}{24\pi^2}\nabla^2\partial^i\delta(\underline{x}-\underline{y})$		a,e,h	7,8,9
$\langle 0\|[j^0(\underline{x},t), J^\mu_5(\underline{y},t)]\|\gamma\rangle = 0$ $\langle 0\|[j^\mu(\underline{x},t), J^0_5(\underline{y},t)]\|\gamma\rangle = 0$	Sutherland theorem about $\pi^0 \to 2\gamma$ decay	$\delta([x-y]\cdot n)\langle 0\|[j^\mu(\underline{x},t), J^\nu_5(\underline{y},t)]\|\gamma\rangle$ $= -c\, n^\nu P_{\alpha\beta}\delta^\alpha(x-y)\partial^\alpha\langle 0\|\tilde{F}^{\mu\beta}(x)\|\gamma\rangle$ $+c\left(g^\mu_\delta[g^\alpha_\gamma n^\nu + g^{\alpha\nu}n_\gamma] + g^\nu_\delta[g^\alpha_\gamma n^\mu + g^{\alpha\mu}n_\gamma]\right)\times$ $P_{\alpha\beta}\partial^\beta\delta^\nu(x-y)\langle 0\|\tilde{F}^{\delta\gamma}(y)\|\gamma\rangle$		a,e,g,i,j,k,l,m	10,11,12,13
$\langle\alpha\|[j^0(\underline{x},t), j^i(\underline{y},t)]\|\beta\rangle = 0$	See remark p	= 0 if α and β are vector or pseudovector particles; $= c\,\langle\alpha\|\varphi^2(y)\|\beta\rangle\partial^i\delta(\underline{x}-\underline{y})$ if α and β are scalar or pseudoscalar particles		a,e,l,n,o,p	9,14,15

In addition to the entries of this Table, there exists the large number of commutators from the triangle graph by Johnson and Low[11)]

TABLE