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# INFORMAL MEETING ON RENORMALIZATION THEORY

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(SUMMARIES)

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# SOME COMMENTS ON ANALYTIC RENORMALIZATION

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Analytic renormalization is one of the few methods which can be used to give meaning to a Lagrangian field theory in the framework of perturbation theory. Some problems which arise in dealing with this method in practical cases and some results which have been obtained will be reported in what follows.

A class of renormalization procedures, with which we have worked, arises from the prescription [1)] to replace every Feynman denominator of a general Feynman diagram

$$\frac{1}{m^2 - p^2 - i\epsilon} \quad \text{by} \quad \frac{f(\lambda)\, m^{2\lambda}}{(m^2 - p^2 - i\epsilon)^{1+\lambda}} \quad \text{where} \quad f(\lambda) = 1 + \lambda C_1 + \lambda^2 C_2 + \ldots$$

where $C_i$ are arbitrary constants and one has to take a different $\lambda$ for every internal line (but the same $C_i$ 's for every line belonging to a special sort of particle). The diagram is then evaluated along standard lines keeping $\lambda$ large enough. After the integrations have been performed a certain evaluator is applied, which essentially consists in symmetrizing with respect to the $\lambda$'s , continuing analytically towards $\lambda \to 0$ and omitting contributions which are singular in this limit. These contributions are of the type of a polynomial in $\Box$ times a $\delta$-function $\delta(z_i)$ in co-ordinate space where $z_i = 0$ is a point at which the T-product (which forms the diagram) is not defined. In the result obtained the constants $C_i$ occur and the interesting problem is how many of them are involved and how they can be fixed from physics. In renormalizable theories they have to be accommodated in renormalization constants.

In theories in which this is relevant, it turns out that the prescription given above does not lead to gauge-invariant results. As long as $\lambda$ is finite this is clear, since the modified propagator does not fulfill Ward's identity. As can be shown by considering a special example [2], the invariance is not regained after application of the evaluation procedure. One could apply as a remedy a procedure described by Kroll [3] which ensures gauge-invariant results for an arbitrary propagator modification by introducing appropriate vertex modifications (additional one- and multiphoton-vertices). This procedure is, however, very impractical for the modification studied here. A much simpler remedy has been shown to work in Ref. 2. It consists in modifying only photon propagators if there are no closed loops (this does not spoil gauge invariance). For closed loops a procedure is described which is, in effect, a modification of the whole loop (only one $\lambda$ for the whole loop). The procedure is given in Ref. 2 for a general closed loop with $n + 1$ corners, at one of which there is a $\gamma_\mu$ vertex (the other vertices are arbitrary). The procedure reads complicated, but the calculation turns out to be very simple (much simpler than with Pauli-Villars regulators; this is in general the case if one uses analytic regularization!). Some applications have been discussed in Ref. 2, a study of the anomaly of the axial vector divergence (i.e., the triangle with one $\gamma_s \gamma_\mu$ and two $\gamma_\mu$ vertices) is done at present. An analogous treatment of "axial gauge invariance" seems to be rather difficult [4].

The technique of analytic regularization can be applied, after some minor technical adaptions, to the Lee model, which can then be studied without any cut-off in the lowest sector [5]. The results which one obtains are exactly the same as those obtained with cut-off (in the limit in which the cut-off is removed): there are always ghosts present, the only interpretable solutions are Heisenberg's dipole solution [6] and the one discussed recently by T.D. Lee and G. Wick [7] in which the ghost is unstable. Thus the regularization procedure does no harm to the solution in this model.

## REFERENCES


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