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(SUMMARIES)

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MOMENTUM SPACE BEHAVIOUR OF INTEGRALS IN NON-POLYNOMIAL LAGRANGIAN THEORIES

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Most Lagrangians of physical interest - e.g., the chiral Lagrangians for strong interactions, the intermediate boson mediated weak Lagrangian and Einstein's Lagrangian for gravity - appear to be of the non-polynomial form in the field variables. By suitable field-transformations they can in general be expressed in the form of rational functions. This class of Lagrangian was examined in an earlier paper [1)] (referred to as I) following a method due to Efimov and Fradkin [2)], with particular reference to the ultraviolet infinities of physical amplitudes. The discussion was carried out in x-space with the amplitudes defined as Borel sums of divergent series like $\sum a_{nm\ldots} \Delta_F^n(x)\, \Delta_F^m(y) \ \ldots$. The singularity behaviour of these Borel sums was examined in the limits, $x^2 \to 0$, $y^2 \to 0$, ... With Efimov and Fradkin we concluded that if the Dyson index D of these rational Lagrangians was less than or equal to 4, all ultraviolet infinities associated with amplitudes in these theories could be compensated by a finite number of counter-terms (Dyson index D is defined by the limit, $\lim_{\phi \to \infty} L(\phi) = \phi^D$) and in this respect the theories behave like renormalizable theories.

For their physical use we need the renormalized amplitudes not in x-space but in p-space. What we did not examine in I were the momentum-space Fourier transforms, their analyticity properties and their asymptotic behaviour. This talk is devoted to a consideration of these problems, following a method first discussed in this context by Volkov [3)] and which in its essentials goes back to a discussion (in the appropriate region of x and n) of the Fourier transform of $[\Delta_F(x)]^n$ by Gel'fand and Shilov [4)]. A study of the same Fourier transforms has recently been made by Lee and Zumino [5)] using different methods; we reproduce their results for the examples they consider. In particular we show:

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a) The Fourier transforms, if properly defined, give correctly the singularity structure consistent with the unitarity requirements.

b) Our method gives immediately the asymptotic behaviour for large and space-like p^2.

c) The discussion of ultraviolet infinities, previously carried out in x-space (I), is closely paralleled for p-space and the same conclusions are reached.

d) The closed loop integrations in p-space have exactly the same form in polynomial and non-polynomial Lagrangian theories. The methods of analytic renormalization[6] studied recently for polynomial Lagrangians are particularly appropriate to the p-space method discussed in this talk.

REFERENCES

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4) I.M. Gel'fand and G.E. Shilov, "Generalized Functions", Vol. I (Academic Press, New York and London 1964).

5) B.W. Lee and B. Zumino, CERN preprint TH.1053 (1969).

6) E.R. Speer, J. Math. Phys. 9, 1404 (1968).

Further references will be found in ICTP, Trieste, preprint:
Abdus Salam and J. Strathdee, IC/69/120.