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Lorentz-Covariant Lagrangians and Causality (*).

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Summary. — A class of Lorentz-covariant Lagrangians is described which seem to violate causality in the sense that the propagation velocity of wave fronts and particles can be larger than the velocity of light. As a simple model of a Lagrangian of this type we consider a point particle coupled to a massless rank-two tensor field. While it seems kinematically possible to accelerate a particle through the Minkowski light-cone, it turns out that dynamical reasons prevent this. The reaction force due to the radiation emitted by the particle diverges when the particle approaches the Minkowski light-cone. This simple model seems to indicate that Lorentz covariance is indeed sufficient to guarantee causality and no restrictions concerning the type of couplings which may be contained in the Lagrangian are necessary.

1. – Introduction.

A commonly accepted statement contained in most textbooks of special relativity (1) is that causality implies that all signals propagate with a velocity $v < c$. However, no general proof exists that the dynamical equations of an arbitrary system exclude possible acausalities due to $v > c$. Some recent papers (2,3) have therefore reconsidered this question in the framework of clas-

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(1) *E.g.*, J. L. ANDERSON: *Principles of Relativity Physics* (New York, 1968), p. 191;

sical Lorentz-covariant Lagrangian field theories. The conclusion of these authors is that certain couplings seem to imply superluminous behaviour ($v > c$) of the systems considered. This would mean that the usual formalism of Lorentz-covariant Lagrangians does not in itself guarantee the nonexistence of superlight velocities (*). VELO and ZWANZIGER conclude therefore that certain higher-spin interactions are inconsistent; in other words, causality would restrict the choice of possible interaction Lagrangians. This result would be of interest in elementary-particle field theory for instance. BLUDMAN and RUDERMAN use such superluminous models to arrive at equations of state for « superdense » matter.

It is the aim of this paper to investigate to what extent superluminous behaviour is possible in theories described by covariant Lagrangians. In Sect. **2** the usual argument that superlight velocities appear when the characteristics of the field equations are changed in the presence of interaction is discussed for a simple, purely field-theoretic model. However, since the solution of coupled field equations presents rather severe difficulties, we consider in Sect. **3** the case where the field that is expected to propagate acausally is approximated by a point particle. Mathematically the problem is then similar to the Čerenkov effect, where the source of the field has a velocity larger than the propagation velocity of the radiated field. In contrast to the Čerenkov effect, however, the main result of this Section is that the radiation reaction force diverges as the particle approaches the characteristics of the field. This reaction force is, of course, due to the fact that the particle loses energy when it radiates off the field to which it is coupled.

Finally, assuming that a similar result can also be obtained in a pure field theory, we discuss the possibility of noncausal effects in Lagrangian field theories.

2. – Characteristics of field equations and derivative couplings.

In this Section we briefly review the problem how noncausal effects may formally appear in a covariant Lagrangian field theory. It is well known that wave propagation is described by hyperbolic partial differential equations. The velocity of a wave front (or signal velocity) depends on the characteristic surfaces of the wave equation. These surfaces are determined by the coefficients

(*) We are not considering tachyons in this context, *i.e.* particles that are always faster than light [4]; for a discussion of the problems arising in tachyon theories see of the highest derivatives which are of second order for all known relativistic wave equations (*). The most obvious way to change the characteristics and thus the propagation velocity of the wave front is to add derivative couplings to the free Lagrangian. A simple model of a theory with derivative coupling consists of a scalar field $\varphi(x)$ coupled to a symmetric tensor field $\psi_{ik}(x)$, with a Lagrangian of the form (**)

$$L(x) = L(\psi) + \tfrac{1}{2}\left\{\varphi_{,i}(x)\varphi^{,i}(x) - m^2\varphi^2(x)\right\} + f\psi_{ik}(x)\varphi^{,i}(x)\varphi^{,k}(x)\,, \tag{1}$$

where we have not specified the free Lagrangian $L(\psi)$ for the ψ-field.

The field equation for the scalar field is

$$\varphi^{,ik}(x)\big(\eta_{ik} + 2f\psi_{ik}(x)\big) + 2f\psi_{ik,}{}^{i}(x)\varphi^{,k}(x) + m^2\varphi(x) = 0 \tag{2}$$

instead of

$$(\Box + m^2)\varphi(x) = 0 \tag{3}$$

in the case without interaction.

Thus the characteristics of eq. (2) are given by surfaces $u(x) = \text{const}$, which obey the following equation:

$$\big(\eta_{ik} + 2f\psi_{ik}(x)\big)\, u^{,i}(x)\, u^{,k}(x) = 0\,. \tag{4}$$

Since $u(x)$ depends on the tensor field $\psi_{ik}(x)$, it is possible that the bicharacteristics (or rays) belonging to the characteristic surface become spacelike. Of course, this would imply that the front velocity of a φ-wave could be larger than the velocity of light. However, it must be emphasized that only a detailed discussion of the complete system of equations—*i.e.* including the field equations for $\psi_{ik}(x)$—can give an answer to the question whether superlight velocities are actually present in the theory. At this point one might argue that one could always take $\psi_{ik}(x)$ as an external field in eq. (4), but it will become clear later why such a treatment is not satisfactory.

To obtain a realistic field-theoretic model, we thus would have to solve the coupled field equations, which is quite hopeless even if one chooses a simple free-field Lagrangian $L(\psi)$ for the tensor field. However, at least a partial solution to our problem can be given if one considers a point particle instead of the scalar field $\varphi(x)$ coupled to $\psi_{ik}(x)$. For one thing, the Euler equations will be much easier to handle in this case. Furthermore, one can ascribe a definite velocity to a point particle; this is not the case if one considers wave packets in a field theory. This fact allows one to formulate a precise condition which has to be fulfilled if the particle is accelerated through the light cone.

3. – Point-particle model.

In this Section we shall consider the problem of a mass point coupled to a symmetric massless tensor field ψ_{ik} in some detail. We start from a Lagrangian

$$L(x) = \tfrac{1}{2}\psi_{ik,l}(x)\psi^{ik,l}(x) - m\int \mathrm{d}\lambda\,\delta^4\big(x - z(\lambda)\big)\sqrt{\dot z^i(\lambda)\dot z^k(\lambda)\big(\eta_{ik} - 2f\psi_{ik}(x)\big)}\,. \tag{5}$$

The form of the free-field Lagrangian $L(\psi)$ is chosen to be as simple as possible. This choice implies, however, that ψ_{ik} us not a pure spin-two field, but contains additional spin-one and spin-zero parts. $z^i(\lambda)$ denotes the world-line of a particle as a function of the arbitrary parameter λ. Note that (5) is invariant against parameter transformations.

To first order in the coupling constant f this Lagrangian splits into a sum of the free-field Lagrangian, the particle part

$$L_p = -m\int \mathrm{d}\lambda\,\delta^4\big(x - z(\lambda)\big)\sqrt{\dot z^i(\lambda)\dot z_i(\lambda)} \tag{6}$$

and an interaction term of the form

$$L'(x) = f\psi_{ik}(x)\,T_p^{ik}(x)\,, \tag{7}$$

where $T_p^{ik}(x)$ is the energy-momentum tensor of the particle:

$$T_p^{ik}(x) = m\int \mathrm{d}\lambda\,\delta^4\big(x - z(\lambda)\big)\frac{\dot z^i(\lambda)\dot z^k(\lambda)}{\sqrt{\dot z^i(\lambda)\dot z^k(\lambda)\eta_{ik}}}\,. \tag{8}$$

From (5) one derives the following Euler equations:

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left[\frac{\dot z^k(\lambda)\big(\eta_{ik} - 2f\psi_{ik}(z)\big)}{\sqrt{\dot z^l(\lambda)z^k(\lambda)\big(\eta_{lk} - 2f\psi_{lk}(z)\big)}}\right] = \frac{-f\dot z^k(\lambda)\dot z^l(\lambda)\psi_{kl,i}(z)}{\sqrt{\dot z^l(\lambda)\dot z^k(\lambda)\big(\eta_{lk} - 2f\psi_{lk}(z)\big)}} \tag{9}$$

and

$$\Box\psi^{ik}(x) = fm\int \mathrm{d}\lambda\,\frac{\dot z^i(\lambda)\dot z^k(\lambda)\delta^4\big(x - z(\lambda)\big)}{\sqrt{\dot z^l(\lambda)\dot z^j(\lambda)\big(\eta_{lj} - 2f\psi_{lj}(x)\big)}}\,. \tag{10}$$

Equation (9) then reduces to

$$\ddot{z}^i(s) + \Gamma^i_{kl}\,\dot{z}^k(s)\,\dot{z}^l(s) = 0\,, \tag{12}$$

where Γ^i_{kl} are the Christoffel symbols for the « metric » $g_{ik} = \eta_{ik} - 2f\psi_{ik}$; eq. (11) is then a first integral of eq. (12), as it must be for consistency. It is well known that eq. (12) defines geodesics in a Riemannian space with metric g_{ik}. The condition (11) implies that the limiting velocity of the particle may exceed the velocity of light. This is most simply seen by assuming that g_{ik} is diagonal and that the particle moves in the x^1-direction (7); then for small deviations from η_{ik}, *i.e.* $|f\psi_{ik}| \ll 1$,

$$v^2 \equiv \left(\frac{\mathrm{d}z^1}{\mathrm{d}z^0}\right)^2 < \frac{g_{00}}{|g_{11}|}$$

and its limiting velocity is therefore given by

$$c(x) = \sqrt{\frac{1 - 2f\psi_{00}(x)}{1 + 2f\psi_{11}(x)}}\,,$$

which may be greater than one.

For arbitrary ψ_{ik} eq. (11) tells us that the four-velocity of the particle at a point $x^i = z^i(s)$ always lies inside the cone defined by

$$\big(\eta_{ik} - 2f\psi_{ik}(z)\big)\big(x^i - z^i(s)\big)\big(x^k - z^k(s)\big) = 0\,. \tag{13}$$

The field equation (10) implies that the ψ-field propagates along the ordinary light cone although the limiting velocity for the particle is changed.

Here we would to like emphasize the analogy to the Čerenkov effiect. In a dielectric the characteristics of the Maxwell equations depend on the dielectric constant ε. Waves travel with a phase velocity $u = 1/\sqrt{\varepsilon}$, while the velocity of the particle is only restricted to $v < 1$. Thus, ignoring indices, eq. (10) is similar to the situation in a dielectric where the source can move faster than the phase velocity of the field.

The main question is whether the coupled field and particle equations admit solutions where the particle breaks through the light cone as is the case for Čerenkov radiation. In other words, does the particle only see its « own light-cone » or is it also influenced by the light-cone of the ψ-field?

In order to give an exact answer to this question it would be necessary to solve eqs. (10) and (12), which is a complicated task. What we shall radiation reaction force diverges at the light-cone, thus keeping the particle from accelerating to velocities exceeding 1. This is in contrast to the Čerenkov radiation where the radiated energy does not diverge because of the frequency dependence of $\varepsilon(\omega)$. This introduces a cut-off for high frequencies (since $\varepsilon(\omega)\to 1$ for $\omega\to\infty$), thus allowing the particle to acquire velocities $v > u$.

The field equations (10) can be solved explicitly:

$$\psi^{ik}(x) = fm\int \mathrm{d}s\, \dot{z}^i(s)\, \dot{z}^k(s)\, D_{\mathrm{ret}}\big(x - z(s)\big)\,, \tag{14}$$

where we have omitted a possible incoming free-field solution. The Green's function is given by

$$D_{\mathrm{ret}}(x) = \frac{\delta(x^2)}{2\pi}\,\Theta(x^0)\,. \tag{15}$$

thus

$$\psi^{ik}(x) = \frac{fm}{4\pi}\,\frac{\dot{z}^i(s_0)\,\dot{z}^k(s_0)}{\big|\big(\dot{z}(s_0),\, x - z(s_0)\big)\big|}\,, \tag{16}$$

$$\big(x - z(s_0)\big)^2 = 0\,, \qquad x^0 - z^0(s_0) > 0\,.$$

We decompose the lightlike vector $x^i - z^i(s_0)$ into a timelike and an orthogonal spacelike vector

$$x^i - z^i(s_0) = \frac{\big(\dot{z}(s_0),\, x - z(s_0)\big)}{\dot{z}^2(s_0)}\,\dot{z}^i(s_0) + \gamma^i \tag{17}$$

with $\big(\dot{z}(s_0), \gamma\big) = 0$. From $\big(x - z(s_0)\big)^2 = 0$ one evaluates

$$\gamma^2 = -\frac{\big(\dot{z}(s_0),\, x - z(s_0)\big)^2}{\dot{z}^2(s_0)} = -r^2, \tag{18}$$

where r is the spatial distance between the space-time point x and the particle in the instantaneous rest frame of the particle; this rest frame is well defined as long as $\dot{z}^2(s_0) > 0$.

From (11)

$$\dot{z}^2(s_0) = 1 + 2f\psi_{ik}\big(z(s_0)\big)\,\dot{z}^i(s_0)\,\dot{z}^k(s_0)\,.$$

Because of

$$\dot{z}^2(s) = \left(\frac{\mathrm{d}z^0}{\mathrm{d}s}\right)^2 (1-\boldsymbol{v}^2)\,, \qquad \boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{z}}{\mathrm{d}z^0} \tag{19}$$

the velocity of the particle tends to 1 as $s \to s_0$.

Inserting $(\dot{z}, x-z)^2 = r^2\big(1 + 2f\psi_{ik}(z)\,\dot{z}^i\dot{z}^k\big)$ into (16) it turns out that $\psi^{ik}(x)$ diverges for fixed values of r as the velocity of the particle approaches 1. The same is true for all derivatives of the field; as an example we write down the expression for $\psi^{ik}{}_{,l}(x)$

$$\psi^{ik}{}_{,l} = \frac{fm}{4\pi|(\dot{z}, x-z)|^3}\cdot \\ \cdot\Big\{(\dot{z}, x-z)\big((x-z)_l[\ddot{z}^i\dot{z}^k + \dot{z}^i\ddot{z}^k] - \dot{z}^i\dot{z}^k\dot{z}_l\big) - \dot{z}^i\dot{z}^k(x-z)_l[(\ddot{z}, x-z) - \dot{z}^2]\Big\}\,. \tag{20}$$

Since we consider a coupled particle-field theory the divergence of the fields $\psi_{ik}(x)$ and all derivatives would, of course, heavily influence the trajectory of the particle via (9). In other words, the particle «feels» the Minkowski light-cone which is the characteristic cone of the ψ-field. This statement is only true if the forward light-cone lies inside the cone defined by (13) because only in this case $\dot{z}^2(s) = 0$ is compatible with (11).

Moreover, it is evident that an external-field approximation for $\psi_{ik}(x)$ is not justified in this case where the retarded fields, which are produced by the particle, diverge.

Let us consider in a little more detail the effect of the divergence of the radiated fields. From the Lagrangian (5) one derives by standard methods the symmetric conserved energy-momentum tensor [8]

$$T^{ik}(x) = \psi_{lj}{}^{,i}(x)\,\psi^{lj,k}(x) - \tfrac{1}{2}\eta^{ik}\psi_{lj,s}(x)\,\psi^{lj,s}(x) + 2\psi^i{}_{l,j}(x)\,\psi^{kl,j}(x) - \\ - \partial_l\big\{\psi^{jl,i}(x)\,\psi^k{}_j(x) + \psi^i{}_j(x)\,\psi^{kj,l}(x) - \psi^{ji,k}(x)\,\psi_j{}^l(x) + (i\leftrightarrow k)\big\} + \\ + m\int\mathrm{d}s\,\delta^4\big(x-z(s)\big)\,\dot{z}^i(s)\,\dot{z}^k(s) = T^{ik}_{\psi}(x) + T_{\text{part}}{}^{ik}(x)\,. \tag{21}$$

As the particle moves from a space-time point $z(s)$ to $z(s+\mathrm{d}s)$ it radiates off the ψ-field into a space-time region V, which is the region between the light-cones at $z(s+\mathrm{d}s)$ and $z(s)$, thereby inducing a change of the four-momentum of the ψ-field.

This change is given by

where σ is the boundary surface of V. In the rest frame of the particle the integration over this surface is essentially an integration over a sphere with radius r, where $r \to \infty$ eventually. Since the first and second derivatives of ψ decrease at least as $1/r$, the final expression is r-independent in the limit $r \to \infty$ and only depends on quantities at the space-time point $z(s)$ of the particle. In particular, each term in T^{ik}_{ψ} contributes a factor

$$\frac{1}{\left(1 + 2f\psi_{ik}(z(s))\dot{z}^i(s)\dot{z}^k(s)\right)^n},$$

where n is a natural number $\geqslant 1$. Thus $\mathrm{d}P^i_{\psi}/\mathrm{d}s$, which is the radiation reaction force on the particle, diverges as $|\boldsymbol{v}| \to 1$.

We shall not write down the explicit expression for $\mathrm{d}P^i_{\psi}/\mathrm{d}s$, mainly because of the following reason. Our choice of the free-field Lagrangian for the ψ-field implies that the energy of the field is not positive definite although the total energy is, of course, conserved ([7]). When negative energies appear in a model, one is, in general, rather hesitant to draw any physical conclusions from such results as the divergence of the radiation reaction force. Therefore we shall consider in the next Section the additional coupling to the electromagnetic field, the energy of which is positive definite. However, we want to emphasize that the main result of this and the next Section, namely the divergence of the radiation reaction force, is completely independent of the number of fields appearing in the theory. It is sufficient that there be one massless field in the theory, which propagates along the normal light cone and to which the particle is coupled. This immediately leads to the factor $|(\dot{z}, x - z)|$ in the denominator of the retarded field solution (stemming from the $\delta(x^2)$ in the Green's function), which causes the divergence for $\dot{z}^2(s) \to 0$.

4. – Reaction force due to electromagnetic radiation.

In this Section we shall show explicitly that the radiated energy (or more exactly the radiation reaction force) diverges when the particle approaches the velocity of light, as long as the radiated field propagates along the Minkowski light-cone.

Our Lagrangian is now that for a charged particle coupled to a tensor field ψ_{ik} and in addition to the electromagnetic field A_i:

$$(23) \qquad L(x) = -m\int \mathrm{d}\lambda\, \delta^4(x - z(\lambda))\cdot$$

We shall not specify the Lagrangian $L(\psi)$ of the tensor field. Therefore, the question of the characteristics of the tensor field is left open here. The dependence of our results upon the choice of these characteristics will be discussed in some detail in Sect. **5**. We only mention here that in (23) ψ_{ik} must not be identified with a gravitational field, since *e.g.* its interaction with the electromagnetic fields is missing in the Lagrangian. The equation of motion of the particle can be derived from (23) to be

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left[\frac{g_{ik}(z)\dot z^k(\lambda)}{\sqrt{\dot z^l(\lambda)\dot z^k(\lambda)g_{lk}(z)}}\right] = \frac{\dot z^l(\lambda)\dot z^k(\lambda)g_{lk,i}(z)}{2\sqrt{\dot z^l(\lambda)\dot z^k(\lambda)g_{lk}(z)}} + \frac{e}{m}F_{li}(z)\dot z^l , \tag{24}$$

where $g_{ik} = \eta_{ik} - 2f\psi_{ik}$. For the electromagnetic field we obtain the Maxwell equations

$$F^{ik}{}_{,k}(x) = j^i(x) = e\int \mathrm{d}\lambda\, \dot z^i(\lambda)\,\delta^4\big(x - z(\lambda)\big)\, . \tag{25}$$

Again eq. (24) permits a parameter transformation $\lambda = \lambda(s)$ with

$$\dot z^i(s)\,\dot z^k(s)\,g_{ik}(z) = 1\, ,$$

so that (24) becomes simply (from now on we write z instead of $z(s)$)

$$\ddot z^i + \Gamma^i_{kl}\dot z^k \dot z^l = \frac{e}{m}\dot z^l F_{lk}(z)\, g^{ki}(z)\, , \tag{26}$$

where g^{kl} is defined by $g_{ik}g^{kl} = \delta^l_i$ (note that $g^{kl} \neq \eta^{ki}\eta^{lj}g_{ij}$). As in the preceding Section it is possible to derive from (23) a symmetric conserved tensor:

$$T^{ik}(x) = m\int \mathrm{d}s\,\delta^4(x - z)\dot z^i \dot z^k + T_E{}^{ik}(x) + T_\psi{}^{ik}(x)\, , \tag{27}$$

where $T^{ik}_\psi(x)$ is the part of the ψ-field.

The energy-momentum tensor of the electromagnetic field

$$T^{ik}_E(x) = F^i{}_l(x)F^{lk}(x) + \tfrac{1}{4}\eta^{ik}F_{lj}(x)F^{lj}(x) \tag{28}$$

is the only part of T^{ik} which is relevant for us, since we are interested in the energy radiated in form of electromagnetic waves only. Solution of eq. (25) gives

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If this expression is inserted into (28), one finds for the radiation part, *i.e.* the part that only contains terms which do not decrease faster than r^{-2}, where r is the distance from the source,

$$(30)\qquad T^{ik}_{E,\mathrm{rad}}(x) = -\left(\frac{e}{4\pi}\right)^2 \frac{(x-z)^i(x-z)^k}{(\dot z,\, x-z)^4}\left\{\ddot z^l - \dot z^l\,\frac{(\ddot z,\, x-z)}{(\dot z,\, x-z)}\right\}^2\Bigg|_{s=s_0}.$$

In exactly the same way as in the last Section the radiated electromagnetic energy-momentum $\mathrm{d}P_E{}^i$ per line element $\mathrm{d}s$ is calculated (see also ref. [9,10] for details) and turns out to be

$$(31)\qquad \frac{\mathrm{d}P_E{}^i}{\mathrm{d}s} = -\frac{e^2}{4\pi}\,\frac{\dot z^i}{(1+2f\psi_{ik}(z)\dot z^i\dot z^k)^2}\left\{\frac{2}{3}\ddot z^2 - \frac{(\dot z,\ddot z)}{1+2f\psi_{ik}(z)\dot z^i\dot z^k}\right\}.$$

Of course, (31) reduces to the well-known expression

$$\frac{\mathrm{d}P_E{}^i}{\mathrm{d}s} = -\frac{e^2}{4\pi}\,\frac{2}{3}\,\ddot z^2(s)\,\dot z^i(s)$$

for $f=0$, taking into account $(\dot z, \ddot z)=0$ because of $\dot z^2(s)=1$.

From eqs. (31), (11) and (19) it follows again that $\mathrm{d}P_E{}^i/\mathrm{d}s$ diverges as the particle approaches the Minkowski light-cone.

Concerning the possible trajectories of the particle there are the following two cases which are distinguished by the position of the ray cone

$$F(x-z) \equiv \big(\eta_{ik} - 2f\psi_{ik}(z)\big)(x^i - z^i)(x^k - z^k) = 0\,.$$

a) $F(x-z)=0$ and $\eta_{ik}(x^i-z^i)(x^k-z^k)<0$, *i.e.* the generating rays are spacelike (Fig. 1 *a*)). In this case the radiation reaction force will prevent the particle from crossing the light-cone.

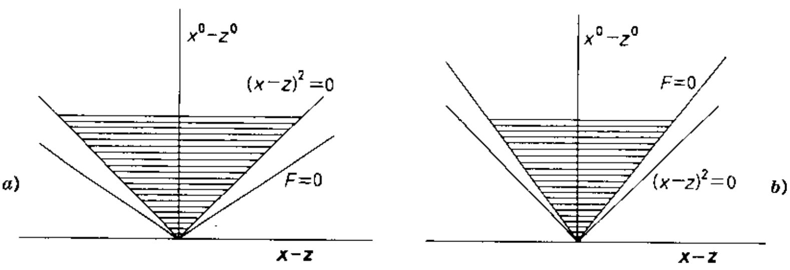

Fig. 1. – The regions to which the particle is limited; $F(x-z) \equiv (\eta_{ik} - 2f\psi_{ik}(z))\cdot$ $\cdot(x^i-z^i)(x^k-z^k)$. *a*) $F(x-z)=0$ and $2f\psi_{ik}(z)(x^i-z^i)(x^k-z^k)<0$; *b*) $F(x-z)=0$ and

b) $F(x-z)=0$ and $\eta_{ik}(x^i-z^i)(x^k-z^k)>0$, which means that the ray cone lies inside the light-cone (Fig. 1 *b*)). Thus the particle cannot even approach the light-cone.

In both cases *a*) and *b*) the limiting velocity of the particle is always determined by the innermost cone.

5. – General relativity.

Let us turn briefly to the field-theoretic treatment of gravitation [7,11]. In this approach to general relativity one starts out with a tensor theory of gravitation in a flat space-time (metric η_{ik}). The gravitational field ψ_{ik} is coupled to the energy-momentum tensor T_{ik} of all matter in a universal way, so that the interaction Lagrangian is given by

$$L_{\text{int}}(x)=f\psi_{ik}(x)\,T^{ik}(x) \tag{32}$$

($f=8\pi G$, $G=$ Newton's gravitational constant). It turns out that the metric is « renormalized » by this interaction and the observable metric g_{ik} is Riemannian

$$g_{ik}=\eta_{ik}-2f\psi_{ik}\,. \tag{33}$$

The original flat space-time metric η_{ik} loses its significance completely and becomes unobservable in principle. The Lagrangian (5) and the one used in the work of THIRRING are almost identical, except that the ψ-field part $L(\psi)$ is more involved in [7], due to the requirement of gauge invariance and positive definiteness of the energy. The equations of motion for a point particle in a gravitational field agree therefore with those derived in Sect. **3**, while the field equations for the gravitational field

$$\Box\psi^{ik}(x)=fT_p{}^{ik}(x) \tag{34}$$

take this simple form only in the Hilbert gauge (harmonic co-ordinates)

$$\psi_{ik,}{}^{k}(x)=0\,. \tag{35}$$

In (34) the energy-momentum tensor of the particle is given simply by

Thus the situation appears to be basically the same as in Sect. **3**. This seems to imply that the particle emits gravitational waves of infinite energy as its velocity approaches 1, *i.e.* the « unrenormalized » Minkowski light-cone. This seems to disagree with the interpretation given in ([7]) of $g_{ik} = \eta_{ik} - 2f\psi_{ik}$ as being the « physical » light-cone.

The solution of this apparent contradiction is of course that (34) is only consistent to first order in $f\psi$ due to gauge invariance. This follows *e.g.* from the fact that the gauge condition (35) would imply $T^{ik}_{p\ ,k} = 0$, which is certainly not true. The field equations used here imply that the energy-momentum tensor which is used as a source is conserved. One is therefore forced to use the complete energy-momentum tensor (*i.e.* including the one of the gravitational field) as the source terms in (34). Since T^{ik}_{grav} contains derivatives of the field, variation of the complete Lagrangian changes the characteristics of the field equations. It has been shown that gauge invariance to all orders of $f\psi$ leads to the complete equations of general relativity ([12]). Thus the field equations are changed exactly in such a way that all characteristics are determined by the « physical » metric g_{ik}. If both the geodesics of the particle and the propagation of the field are determined by g_{ik} instead of η_{ik}, then the flat metric loses its physical content. It is then not surprising that the energy of the particle or the radiation energy will no longer diverge at the normal light-cone.

6. – Conclusion.

The results of the preceding calculations have shown that a point particle cannot be accelerated to superlight velocities if at least one field which was assumed to be massless propagates along the normal (Minkowski) light-cone. The particle would radiate this field in the form of a Čerenkov-type radiation, when it moves with velocities $v > 1$. The reaction force due to the radiation diverges when one tries to accelerate the particle across the light-cone and therefore prevents this acceleration. It is at least plausible that these results also hold true in a more realistic field theory which does not contain point particles. Assuming this to be true we have to distinguish in general three cases:

1) There is only one field in the theory which is, however, self-coupled in such a way that the characteristics of the free-field equations are changed. This is the case *e.g.* in the field theoretic model for superdense matter which has been proposed by BLUDMAN and RUDERMAN. It is not surprising that

other field in the model which propagates normally. Therefore, the normal (Minkowski) light-cone exists only formally in the theory and has no physical significance whatsoever.

2) There are two interacting fields in the theory, one of which propagates normally, while the characteristics of the other field are altered by the interaction. This case is a field-theoretic analogue to the point-particle models discussed before. Let us assume that the conclusions derived there are also true in this case. This would mean that it would require an infinite amount of energy to accelerate a wave packet to velocities >1, which obviously would prevent the existence of signals faster than light. This would also invalidate the conclusions of VELO and ZWANZIGER, because an external-field approximation evidently loses its meaning due to the existence of infinite reaction forces.

3) Finally it may happen that all the fields in the Lagrangian have their characteristics changed in the same way. This is the case in the field-theoretical treatment of gravitation which we have briefly discussed in Sect. **5**. Thirring's work shows that the flat metric η_{ik} loses its significance in this case and the observable space-time becomes Riemannian with the metric g_{ik}. Since the characteristics of the electromagnetic field are determined by g_{ik}, as are the characteristics of all other fields, it is easy to see that the velocity of light is again the limiting velocity in this theory. Thus, no acausalities appear.

Our final conclusion, which we have proved for point-particle models only, is as follows. The requirement of covariant couplings in Lagrangian theories is not sufficient to exclude superlight velocities. If, however, at least one interacting field exists in the theory which propagates normally, so that the Minkowski light-cone does not lose its meaning completely, no signals can propagate with a velocity >1.

● RIASSUNTO (*)

Si descrive una classe di lagrangiane covarianti secondo Lorentz, che sembra violino la causalità nel senso che la velocità di propagazione dei fronti d'onda e delle particelle può essere più grande della velocità della luce. Come modello semplice di una lagrangiana di questo tipo si considera una particella puntiforme accoppiata ad un campo tensoriale di rango 2 privo di massa. Mentre, cinematicamente, sembra possibile accelerare una particella tramite il cono di luce di Minkowski, risulta che motivi dinamici lo impediscono. La forza di reazione dovuta alla radiazione emessa dalla particella diverge quando la particella si avvicina al cono di luce di Minkowski. Questo semplice modello sembra

Лорентц-ковариантные лагранжианы и причинность.

Резюме (*). — Описывается класс Лорентц-ковариантных лагранжианов, которые, по-видимому, нарушают причинность в том смысле, что скорость распространения волновых фронтов и частиц может превымать скорость света. В качестве простой модели лагранжиана этого типа, мы рассматриваем точечную частицу, взаимодействующую с полем, имеющим нулевую массу и описываемым тензором второго ранга. Хотя кинематически представляется возможным ускорить частицу через световой конус Минковского, оказывается, что динамические причины препятствуют этому. Сила реакции, обусловленная излучением частицы расходится, когда частица приближается к световому конусу Минковского. Эта простая модель, по-видимому, указывает, что Лорентц-ковариантность, в действительности, является достаточной, чтобы гарантировать причинность. Ограничения, касающиеся типа связей, которы могут содержаться в лагранжиане, не являются необходимыми.

(*) *Переведено редакцией.*