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DYNAMICAL GROUPS, INFINITE MULTIPLETS AND MODEL OF HADRONS

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The methods of infinite multiplets and dynamical groups have been developed to understand some of the problems of strong interactions; it is therefore important to see how these ideas work quantitatively in explaining the properties of hadrons.

The dynamical group approach replaces the atomistic treatment based on "constituents + interactions between them" by a global treatment. A system of N bodies is replaced by a single system possessing internal degrees of freedom. In some cases this is only a reformulation (e.g., non-relativistic H-atom): in other cases the procedure transcends the atomistic formulation. In particular, the difficulties of relativistic treatment of N-body problem are completely circumvented, as the concepts of constituents and their interactions may no longer be convenient in the relativistic domain [1]. The fact that the global approach is indeed superior in the relativistic domain may be illustrated by the treatment of the H-atom by a relativistic infinite-component wave equation which gives the following binding energies B_n :

$$1 + \frac{1}{\mu} B_n + \frac{1}{2m_p m_e} B_n^2 = \sqrt{\left(1 - \frac{\alpha^2}{n^2}\right)} \quad (\mu = \text{reduced mass}) \ .$$

This formula also automatically contains, beyond the Dirac values, the corrections due to the motion of the nucleus:

$$-\frac{1}{8} \frac{m_e}{m_p} \left(\frac{\alpha}{n}\right)^4$$

(for the $|k| = n$ levels) [2]. The equation also gives a closed expression for the positronium levels (for $s = 0$, $\ell = n = 1$ levels):

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$$M_n^2 = 2m_e^2(1 + \sqrt{1-\alpha^2/n^2}) \quad \text{or} \quad E_n = -m_e \left(\frac{\alpha^2}{4n^2} + \frac{5}{64} \frac{\alpha^4}{n^4} + \ldots \right).$$

As to the structure of the proton, a particular model has emerged based on the form factors and mass-spectrum of hadrons according to which the effective constituents of the proton are held by long-range Kepler-type forces. In this model the analogue of the Bohr radius is 2.4 $(\mathrm{GeV})^{-1}$ (obtained from the slope of the form factor) and the wave length of the emitted "quanta" is 1 GeV^{-1} (obtained from the slope of the mass spectrum as a function of the principal quantum number n) so that the analogue of the fine structure constant is about $\alpha \sim 2.4$. And one can predict the slope of the diffraction peak of the pp-scattering which is about 10 $(\mathrm{GeV}/c)^{-1}$.[3]

Crucial stepsin the development: The starting point of the dynamical group approach was the symmetry-breaking problem in SU(3) and the successful algebraic calculation of mass differences within an SU(3) multiplet[4]. The need was felt then for a unified algebraic treatment of internal and external quantum numbers and for a dynamical group that would give the complete quantum numbers and the spectrum of the system[5]. Later the group SU(6) was proposed which solves this problem partly and approximately[6]. The relativistic version of SU(6) led to the groups SU(6, 6) [7] and SL(6, C) [8]. The complete mass spectrum was still not solved. At this point the mass spectra of well-known quantum mechanical systems were obtained algebraically[9]. To solve the mass problem in a general way one had to go from the rest frame states to states with momentum P_μ in order to calculate $P_\mu P^\mu$. Operationally such states can be excited by external interactions, so that the combination of the rest frame group and the Poincaré group is physically related to external interactions[10),1)]. On this basis the problem of mass-spectrum and transition probabilities of the H-atom was solved[11]. At this point the connection with the infinite-component wave equations was recognized. These general wave equations had been used by Majorana[12], Gel'fand and Yaglom[13]. They were used again by Nambu[14], Fronsdal, Budini et al.[15] and Barut and Kleinert[16].

The mass spectrum problem is much easier to solve by means of the wave equation than purely algebraically[17] but the two methods are equivalent. There are other related methods using internal coordinates or bilocal theory[18].

Applications to hadron physics: Any model based on the above ideas must choose A) a rest frame group, B) a particular representation of it, and C) a definite current operator (or wave equation). In the following I shall discuss a particular model. For baryon states, after studying first in detail the group SL(2, C), Kleinert and I proposed the group $O(4,2) \otimes SU(3)$ for the baryon rest frame states[19]. The reasons were: a) the existence of several $\frac{1}{2}^+$-baryon states requiring a new quantum number[20], b) the t-dependence of the form factors. With respect to the latter point, the group O(4, 2) is probably unique in giving a form factor behaving like $(1 - at)^{-2}$. As to the point B), the representation, it seems at the moment that the non-unitary representation $\mathcal{D}_{\text{most degenerate}} \otimes \mathcal{D}_{\text{Dirac}}$ of O(4, 2) is the appropriate choice for at least $I = \frac{1}{2}$ nucleon tower[20]. Here all states are parity doublets except the lowest levels $1/2^+$, $3/2^-$, $5/2^+$, $\cdots$ as in the relativistic H-atom. Finally, as to the point C) the current, Corrigan, Kleinert and I[21] have chosen $j_\mu = \alpha_1 \Gamma_\mu + (\alpha_2 + \alpha_3 S) P_\mu + i \alpha_4 L_{\mu\nu} q^\nu$ where α_i are constants, Γ_μ, S, $L_{\mu\nu}$ are O(4, 2) generators, and between two states, $P_\mu = (p + p')_\mu$, $q^\mu = (p' - p)_\mu$.

Results: With this model, we can fit mass spectrum, form factors and magnetic moments of proton and neutron[21] as well as some transition form factors[22]. Partial decay rates can be evaluated[23]. It is an interesting hypothesis that a universal O(4, 2) vector current also describes weak and strong interactions of hadrons[24]. In fact, on this basis the $K_{\ell 3}$-form factors have been calculated completely in agreement with experiment[25]. A contact vector interaction between two O(4, 2) further explains high s , high t behaviour of proton-proton interactions, in particular the remarkable lower limit of $d\sigma/dt$ for $s \to \infty$.[26]

The model has a number of parameters which, in a composite particle theory, would be related to the masses of constituents.

Problems: The problems of current interest are 1) the inclusion of antiparticles, 2) form factors in the time-like region, 3) the interpretation of space-like solutions of the infinite-component wave equations, and 4) treatment of exchange effects in scattering.

So far we have discussed transitions in a single composite system due to electromagnetic and weak interactions. In scattering problems and in annihilation problems, we have the interaction of two and more composite systems (e.g., two H-atoms). Consequently, in addition to the direct contact terms [26] there are exchange effects, i.e., exchange of a tower of mesons in the t-channel in the p-p-scattering. The vertex calculated by the form $\bar{\psi}(p')j_{\mu}\psi(p)$, where $\psi(p)$ are, for example, O(4, 2) wave functions, give the same anomalous threshold as a triangular diagram, but is more than a triangular diagram: it sums a lot of "radiative" corrections, cancelling the so-called normal threshold singularities in the form factor. For this reason, it is not clear that one can make another field theory with infinite-component wave functions and start summing up the corresponding Feynman diagrams again. Such a process destroys the fact that the infinite multiplet wave function already describes the final non-perturbative C-number solutions. Accepting this point of view, it is also possible to give a physical interpretation to the space-like solutions of the wave equation [27]. They are not asymptotic states (because they have negative norm) but intermediate states corresponding to singularities in the crossed channel; i.e., $P_{\mu}P^{\mu}$ is interpreted as s for the normal solutions but as t (or u) for the solutions with negative norm. The same interpretation might hold for the extra solutions of the Bethe-Salpeter equation which has too many solutions not all interpretable as asymptotic solutions.

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