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THE GROUP THEORETICAL STRUCTURE OF THE STRONG-COUPLING THEORY[1)]

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The group-theoretical structure of the symmetrical scalar fixed source meson theory is considered. The isospin part of the Hamiltonian of this system possesses SU(2) as an exact symmetry. For strong coupling an approximate additional invariance or a dynamical symmetry is suggested by the structure of the coupling term. This dynamical symmetry is described by a unitary representation U(G) of a non-compact so-called strong-coupling group $G = T_3 \otimes SU(2)$, isomorphic to the three-dimensional Euclidean group $\overline{E}_3$ with T_3 now interpreted as translation in the isospace. The reduction of $U(G) \downarrow SU(2)$ leads to an infinite number of SU(2) multiplets labelled by T. If U(G) is irreducible, different multiplets appear only once; for reducible U(G) any multiplet appears with a degeneracy, labelled by $\alpha(T)$.

We calculate the physical representation $U_{ph}(G)$ of G. We particularly want to decide whether $U_{ph}(G)$ is irreducible or not. For this purpose the Hamiltonian of the model has to be analysed very carefully. This is done in the present work. The method of splitting[2)] the Hamiltonian into a "bound" part, a "free" part and an interaction part, which was used in a series of papers by Wentzel[3)], Dothan and Ne'eman[4)], Dullemond and van der Linden[5)], Bednar and Tolar[6)] and Melsheimer[7)], leads to a bound state Hamiltonian containing rotational and vibrational degrees of freedom. To decide whether the strong-coupling limit exists in this case, one has to check that the abovementioned interaction between the "bound" part and the "free" part will vanish in the limit where the coupling constant g becomes infinite. It can be seen that this limit cannot be verified so long as the vibrational terms are present. However, we can show that by a proper transformation of the field operators the vibrational degrees of freedom can be completely removed. We thus arrive at another kind of splitting method which was developed already in Refs. 8 and 9. This method leads to a bound state part with rotational degrees of freedom only. The strong-coupling limit can be verified for this method and the correct spectrum is obtained.

This result has an immediate group-theoretical consequence for the physical representation $U_{ph}(G)$. We prove that the vibrational part leads necessarily to a reducible representation with vibrational levels with energy

$$E = \hbar\omega \left\{ \alpha(T) + \frac{1}{2} \right\} .$$

Hence, since the vibrational levels are not present, we conclude that the physical representation of the dynamical group for the bound state Hamiltonian in the strong-coupling case is irreducible. This was already assumed earlier, i.e., in the work of Fierz [10], Singh [11] and of Bednar and Tolar [6] without, however, deriving this from the Hamiltonian of the system. Furthermore, in our lecture some generalizations to the intermediate coupling case are discussed and, finally, the derivation of the dynamical group G from the strong-coupling Low equation (see Refs. 9, 12 and 13) is analysed.

For simplicity, only the case of the symmetrical scalar fixed source meson theory is considered in this lecture. But the basic lines of the method will be the same in the more complicated symmetrical pseudoscalar case as well as in the case with SU(3).

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