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FIELDS ON A HOMOGENEOUS SPACE DESCRIBING AN INFINITE NUMBER OF PARTICLES

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Different attempts have been made to widen the framework of field theory in order to be able to describe hadron physics. Generalizations which still carry the name field theory are infinite-component field theories [1] and field theories on spaces larger than the Minkowski space. We are here interested in the latter type. To be more specific, we shall study fields on a certain eight-dimensional homogeneous space $\mathcal{E}$ of the Poincaré group [2), 3)].

We can describe the space $\mathcal{E}$ in the following way. $\mathcal{E}$ is the homogeneous space $\bar{\mathcal{P}}/\mathcal{N}$ where $\mathcal{P}$ is the Poincaré group and $\mathcal{N}$ is the nilpotent subgroup of $\mathcal{P}$ generated by $L_{02} - L_{23}$ and $L_{01} + L_{31}$. A point in $\mathcal{E}$ can also be given as a pair (x, z) where x belongs to the Minkowski space and z is a complex two-spinor.

Wave functions $\psi_{m,j,n,\alpha+i\beta}(x, z)$ describing particles of mass m , spin j and structural "quantum" numbers $n, \alpha+i\beta$ can now be constructed starting from the reducible representation

$$T_{(a,\Lambda)}f(x, z) = f(\Lambda^{-1}(x-a), \Lambda^{-1}z)$$

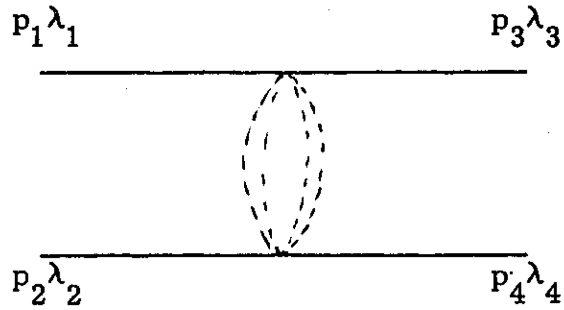
of $\bar{\mathcal{P}}$. Here $(a, \Lambda) \in \mathcal{P}$ and Λz is the action of the element $\Lambda \in SL(2, C)$ on a two-spinor. To reduce this representation one can use the two Casimir operators $P_\mu P^\mu$, $W_\mu W^\mu$ of $\mathcal{P}$ leading to m and j and two extra operators which introduce n and $\alpha + i\beta$. It must be stressed that different n , α and β define the same particle kinematically but when interactions are introduced they may behave differently.

Given the one-particle wave-functions, it is possible to define free fields using creation and annihilation operators on a Fock space. Such a field will then depend on (x, z) and is characterized by $(m, j, n, \alpha+i\beta)$.

It turns out that it violates microcausality in general. One way of restoring locality is to sum over j, n, β. But then the field can no longer be attributed to one particle only but to an infinite number of particles. Locality is essential if one wants to define the S-operator through the Dyson formula

$$S = T \exp\left(-ig \iint d^4x\, dz\; \mathcal{H}(x,z)\right)$$

where $\mathcal{H}(x,z)$ is a local product of fields $\overline{\Psi}(x,z)$. The time-ordering operator T will destroy relativistic invariance of S unless the fields commute at space-like separated points x. The big fields obtained by summing over j, n and β are still characterized by α. The assumption of local coupling is a restrictive one. Only two couplings are possible: tri-linear ones with two fermion fields $(\alpha = \frac{1}{2})$ and one boson field $(\alpha = 1)$ or quadri-linear ones with four fermion fields. In order to see what a scattering amplitude looks like let us consider the tri-linear coupling and fermion-fermion scattering corresponding to the graph



If the fermion has quantum numbers $(m\ j\ n\ \beta)$ then, to first order, the scattering amplitude becomes

$$A \sim \frac{1}{t-\mu^2} \iint \sin\theta\, d\theta\, d\varphi \frac{S^j_{\lambda_3 n}\, S^j_{\lambda_1 n}\, S^j_{\lambda_4 n}\, S^j_{\lambda_2 n}}{\left(\frac{p_3\mathcal{H}}{m}\right)^{\frac{1}{2}+i\beta}\left(\frac{p_1\mathcal{H}}{m}\right)^{\frac{1}{2}-i\beta}\left(\frac{p_4\mathcal{H}}{m}\right)^{\frac{1}{2}+i\beta}\left(\frac{p_2\mathcal{H}}{m}\right)^{\frac{1}{2}-i\beta}}$$

where μ is the boson mass, $\mathcal{H}$ is the four-component object $(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ and $S^j_{\lambda n}$ are spin functions depending on momenta and the angles θ, φ. Such an amplitude will show a forward peaking depending on β. It will obey a generalized crossing symmetry where the analytical continuation has to go via a branch point at infinity in the complex s, t plane.

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