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REMARKS ON WAVE FUNCTIONS AND FIELDS ON HOMOGENEOUS SPACES OF THE POINCARE GROUP

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If we want to treat the spin of the particles in an analogous way to the mass and give a dynamical role to the spin it may be nice to realize the unitary irreducible representations of the physical Poincaré group as complex valued distributions on some homogeneous spaces of the Poincaré group. In the conventional Lagrangian field theory it is hoped that a few building blocks (quarks or stable particles, etc.) in the free Hamiltonian H_0 and a correct interaction H_I will provide the full world. It may be more realistic to approximate our understanding in a less fundamental way. Suppose that the free Hamiltonian H_0' already describes an infinite idealized set of zero width resonances. If much of the dynamics of H_0 and H_I has already been "diagonalized" in H_0' it may then be hoped that a well chosen H_I' could be used in a perturbative way. The high degeneracy of states obtained for homogeneous spaces of the Poincaré group which are larger than the Minkowski space may not be too bad in this respect. The continuous degeneracy of every idealized (fixed mass and spin) resonance may represent the continuous infinity of states (mass distribution) occurring in the resonances of the real world.

Defining the Poincaré manifold by points $p = (x, A)$ where x is a point of the Minkowski space and A an SL(2, C) matrix

$$A = \begin{pmatrix} \xi_1 & \eta_1 \\ \xi_2 & \eta_2 \end{pmatrix} \quad ; \det A = 1 \quad . \qquad (1)$$

The action of the physical Poincaré group $\mathcal{P}_{phys}$ on a function (or distribution) $f(p)$ is defined by $(U^L_{p_1} f)(p) = f(p_1^{-1} p)$. Analogously $(U^R_{p_1} f)(p) = f(pp_1)$. Right and left actions commute. Denote by D^L and D^R the infinitesimal generators (differential operators of first order) of these two commuting Poincaré groups $\mathcal{P}$. The left generators, for example, are built out of a collection of generators which generate $Q = \mathcal{P} \otimes SL^{(1)}(2,C) \otimes SL^{(2)}(2,C)$, $\mathcal{P}$ describes the translation and orbital part (action on x) while $SL^{(1)}(2,C)$ and $SL^{(2)}(2,C)$ act only on ξ and η respectively. The algebra of $\mathcal{P}_{phys}$ is the diagonal algebra of Q. A complete set of commuting observables in the enveloping algebra of D^L and D^R can be constructed as follows. The Casimir operators of D^R and D^L are identical: $P^2 = P_R^2 = P_L^2$, $W^2 = W_R^2 = W_L^2$. In general, four commuting independent operators can still be defined both for left operators (A_L) and the right operators (A_R). The eigenvalues are denoted, respectively, by $m^2(P^2)$, $-\ell^2(W^2)$, α_R (A_R) and α_L (A_L). For physical reasons the α_L are usually chosen to be the momentum $\vec{k}$ and the helicity λ. Then, as proved by Nghiem Xuan Hai, the harmonic functions f are given by

$$f^{m^2,\ell^2}_{\alpha_L,\alpha_R}(p) = \langle \alpha_L | U^{m^2,\ell^2}(p) | \alpha_R \rangle \qquad (2)$$

the matrix elements of a unitary representation U^{m^2,ℓ^2} of the Poincaré group.

A generalized equation of motion derived from a Lagrangian leads to a relation between operators belonging to the enveloping algebra of D^R. A picturesque equation is, for example, given formally by $(W^2 = -M^2\, S(S+1)$ formally)

$$\frac{1}{\Gamma(S - aM^2 - b)}\, \phi(p) = 0 \qquad (3)$$

and describes a set of parallel linear Regge trajectories with infinite degeneracy α_R.

Restriction to homogeneous spaces of dimension lower than ten (the dimension of the Poincaré manifold) is obtained by setting to zero the infinitesimal generators of a closed subgroup K of the SL(2, C) subgroup of the unphysical Poincaré group acting on the right. If K is this SL(2, C) itself the homogeneous space reduces to the usual Minkowski space.

The free fields can be quantized quite easily. By using the obvious composition relations derived from (2), nice expressions can be constructed for the commutator. Local commutativity is usually lost, however; roughly, the larger the dimension of the homogeneous space the smaller the domain where the fields commute.

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