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QUANTUM THEORY OF THE GENERALIZED WAVE EQUATION AND THE PROBLEM OF SPIN STATISTICS *

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In recent years, the usefulness of the infinite-component field equations and their algebraic formulations has been amply demonstrated in explaining various observed phenomena of particle interactions[1)-4)]. Attempts have been made to obtain solutions for the algebra of local current densities[5)]. Unlike the finite-component field equation case, one is able to treat here infinitely many mass and spin states satisfying the same wave equation. Quantum systems described by such equations indeed possess "internal structure"[6)].

However, in recent pathological treatments, the breakdown of spin-statistics relations, invalidity of CPT theorem, the difficulty of interpreting the so-called redundant solutions, have been cited[7), 3)]. It is the purpose of this talk to throw some light on the basic assumptions underlying these properties in finite-component field theories and to develop a systematic quantization scheme for infinite-component fields compatible with locality, CPT invariance and right spin-statistics connection[8)].

Our discussion is mainly as follows:

I. 1. The wave equation

2. a) Properties of the Majorana representation

b) Properties of the Dirac representation

c) Properties of SL(2, C) (Majorana) ⊠ SL(2, C)(Dirac) representations

3. Mass spectra

4. Construction of the basis vectors, the normalization and the completeness of the solutions of the field equation

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5. Quantization of the generalized fields

II. The spin-statistics connection and CPT invariance in finite- and infinite-component field theories

1) Spin-statistics relation (postulation of S-principle)

2) Restoration of CPT invariance[10] (by master analytic representation method)

We study a type of relativistic wave-equation first proposed by Bhabha and Corben[11] in connection with establishing the correspondence between the quantum theories and classical relativistic theory of spinning particles. This equation has been re-examined by Abers-Grodsky-Norton[7] from the point of view of infinite-component field theories. The wave equation we have investigated is as follows:

$$(i\,\gamma_\mu \delta_\mu - m_0 - \frac{1}{2} m_1\, \sigma_{\mu\nu}\, \Gamma^{\mu\nu})\, \psi(x) = 0 \qquad (1)$$

where $\psi(x)$ is a double-indexed field: the two indices characterizing respectively the Majorana representations and the Dirac representation. $\sigma_{\mu\nu}$ and $\Gamma^{\mu\nu}$ are, respectively, the generators of the non-unitary and unitary representations of SL(2, C).

In contrast to the pure Majorana wave equation, the solution of eq. (1) provides us the following mass-spectra for the particles[8]:

$$m_J^2 = 2m_1^2(J+\tfrac{1}{2})^2 + [(m_0 - m_1)^2 - \tfrac{1}{4} m_1^2] \pm 2m_1^2 \ [m_1^2(J+\tfrac{1}{2})^2 + (m_0 - m_1)^2 - \tfrac{1}{4} m_1^2]^{\frac{1}{2}}$$

$$(\text{for } p^2 > 0 \text{ , time-like case}) \qquad (2)$$

and

$$m_\nu^2 = 2m_1^2\, \nu^2 - [(m_0 - m_1)^2 - \tfrac{1}{4} m_1^2]$$
$$\pm\, 2m_1^2\, \nu\, [m_1^2\, \nu^2 - \left\{(m_0 - m_1)^2 - \tfrac{1}{4} m_1^2\right\}]^{\frac{1}{2}} \quad , \qquad (3)$$

$$(\text{for } p^2 < 0 \text{ , space-like})$$

ν characterize here the SU(1, 1) subgroup of $SL(2, C)_\mu \otimes SL(2, C)_D$.

Quantization

The fields $\Psi(x)$ contain both +ve and -ve frequency solutions and satisfy the local commutativity, namely,

$$[\Psi_\alpha(t,\underset{\sim}{x}) \, , \, \psi^{+}_\beta(t,\underset{\sim}{x}')]_{\pm} = \delta^3 (\underset{\sim}{x}-\underset{\sim}{x}') \, \delta_{\alpha\beta} \quad . \tag{4}$$

We should like to remark here that, contrary to pure Dirac fields or the generalized fields of our present discussion, the pure Majorana fields do not possess any symmetry between the +ve and -ve frequency solutions. To redress this difficulty, one rather demands: i) $\Psi(x)$ and $\Psi^{+T}(x)$ to be treated on the same footing; ii) the action is invariant under the interchange of $\Psi(x)$ and $V\Psi^{+T}(x)$, where $V = e^{i\pi J_{12}}$. By constructing the fields in the above manner, one restores the usual spin-statistics relation[3].

CPT

We argue that the matrix element of $g(\sigma,\lambda) = e^{i\pi J_{12}} \, e^{i\lambda K_{03}}$ which is crucial for CPT invariance in finite-component field theories, does not exist in the case of infinite-component fields [e.g., in case of Majorana fields, $\langle \frac{1}{2}, j_3 \mid e^{i\lambda K_{03}} \mid \frac{1}{2}, j_3 \rangle = \dfrac{1}{\cosh^2 \frac{\lambda}{2}} \xrightarrow[\lambda = i\pi]{} \infty$; and this is precisely the point ($\sigma = \pi$, $\lambda = i\pi$) one needs to establish CPT invariance in finite-component field theories] . We suggest that this difficulty can be overcome if one deals with the master-analytic-representation method rather than the ones we have used so far[10].

Spin statistics

Scrutiny shows that the "quantization" schemes in pathological treatments do not possess symmetry between emission and absorption processes embodied in S-principle. By suitably amending the "action" for the infinite-component fields, one restores the usual connection between the spin of the fields and the type of statistics obeyed by the particles they describe. We just want to make a passing remark that there is a striking similarity between the Majorana fields and the non-relativistic Schrödinger fields.

Interpretation of space-like solutions

As emphasised by Fronsdal (vide these Proceedings), space-like solutions are necessary in order to obtain solutions for the algebra of local current densities. Two alternative physical interpretations of the space-like solutions have been furnished by Barut[12] and Sudarshan[13] respectively. Nevertheless, we rather insist on the existence of space-like solutions for our purpose of constructing a local quantum field theory within the framework of the infinite-component wave-equations.

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