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RELATIVISTIC CORRECTIONS TO A VERTEX FOR COMPOSITE SYSTEMS

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The purpose of this talk is to discuss the connection between the covariant vertex which can be deduced from a Majorana-type equation representing a composite system, with non-relativistic internal motion, and the vertex which can be deduced for the same system with standard field-theoretical methods.

First, the covariant Bethe-Salpeter equation with a ladder kernel and no retardation is taken as the zeroth-order approximation for the covariant description*) of a system with slow (non-relativistic) relative motion[1),2)]:

$$(P^2 - H_1 - H_2)\varphi(p^T) = -\hat{P}^{(1)}\hat{P}^{(2)}\int G(p^T-q)\varphi(q)\delta(q\cdot P)d^4q \tag{1}$$

where

$$\hat{P}^{(1)} = \gamma_1\cdot P \qquad H_1 = m_1\hat{P}^{(1)} - iP\cdot\sigma^{(1)}\cdot p$$

$$p^T\cdot P = 0$$

$$P = \sqrt{P_\mu P^\mu} \qquad H_2 = m_2\hat{P}^{(2)} + iP\cdot\sigma^{(2)}\cdot p$$

then the equation for the Coulomb interaction and slow relative motion is algebrized and brought to the Majorana form (with no space-like solution):

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*) A similar formalism has been used by Brodsky and Primack to discuss relativistic correction to an electromagnetic vertex; see also Matveev et al. (Ref.3).

$$\left[P_\mu \Gamma^\mu \left(\frac{\partial^2}{2\mu} + M-P\right) + P\Gamma_4\left(\frac{\partial^2}{2\mu} + P-M\right) + e^2 a P\right]\psi_P = 0 \qquad (2)$$

and a vertex can be constructed from this point of view. (Eq. (2) can be brought to the relativistic form used by Fronsdal[4] and is deducible from a Lagrangian formalism by dropping terms of the order $(P-M)/M$.)

Then a scalar vertex in terms of $\varphi(p^T)$ is deduced starting from the covariant Mandelstam vertex[5]. It is shown that for not too high momentum transfer the integrations on the relative energy can be performed and the vertex takes a particularly simple form:

$$J(q) \sim \int \varphi^*_{P'}(p^{T'})\, \varphi_P(p^T)\, W(\beta)\, d^3p^T \quad . \qquad (3)$$

Here $W(\beta)$ is a known function connected with the contraction of the element of integration and $p^{T'}$ is a given function of p^T and q, which reduces to that of the Schrödinger theory:

$$p_T' = p_T - \frac{m_2}{M} q \quad \text{for} \quad q^2/P^2 \ll 1 \quad .$$

Expression (3) can be brought to the algebraic form[2),6)] provided one is able to define the SO(4,1) transformation among the Γ's properly, which brings p^T to $p^{T'}$ and express $W(\beta)$ in terms of functions of Γ's.

For high q values the retardation effects in the kernel become important and one has to use the integral equation with the correct four-dimensional kernel[7]. Even an approximate treatment of these effects, through a finite iteration of the integral equation, does not appear to come out straightforwardly from the algebraic vertex.

The conclusion is that, at low momentum transfer q, computation of relativistic corrections to the composite system vertex can relatively easily be taken into account by an appropriate dependence of the internal momentum on q. At high momentum transfer, dynamical corrections due to retardations in the binding forces become important and do not appear to be a straightforward generalization of the algebraic description.

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The detailed calculations concerning the present talk will be published elsewhere.