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NEW METHODS FOR THE COULOMB PROBLEM

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This is a report on further developments of the algebraic approach to problems involving the Coulomb interaction. The main features of the method are:

1) A unitary, irreducible representation of SO(4, 2) exists in the space of functions $\varphi(\underset{\sim}{r})$ with invariant inner product $\int \varphi_1^*(\underset{\sim}{r})\varphi_2(\underset{\sim}{r})\, d^3r/r$. Apart from the factor $1/r$ in the metric, this is the space of Schrödinger wave functions. (The factor $1/r$ is, of course, not ignored.) The generators of the representation are:

$$\mathcal{S}_{AB} \quad , \quad A, B = 0, 1, 2, 3, 4 \text{ (Examples: } \mathcal{S}_{ij} = \text{ang. mom.},$$

$$\mathcal{S}_{i4} = \text{Runge-Lenz vector},$$

$$\mathcal{S}_{i0} - \mathcal{S}_{i4} = r_i \,.)$$

$$\mathcal{S}_{A5} \equiv \Gamma_A \quad , \quad (\Gamma_0 - \Gamma_4 = r \;\; , \;\; \Gamma_0 + \Gamma_4 = r\underset{\sim}{q}^2 \;\; , \;\; \Gamma_i = rq_i .)$$

2) Instead of the usual Schrödinger wave function $\varphi(\underset{\sim}{r})$ we introduce a function $\psi(p) \equiv e^{i\underset{\sim}{p}\cdot\underset{\sim}{r}}\varphi(r)$ that depends parametrically on $\underset{\sim}{p}$ and whose $\underset{\sim}{r}$-dependence is always suppressed. In this way all photon wave functions are made to disappear from the matrix elements - instead we have formal Galilei covariance and momentum conservation. The Schrödinger theory of hydrogen now takes the form of a Galilei invariant Majorana theory with the Lagrangian

$$\mathcal{L} = \int d^4p \, \psi^\dagger(p) \, (\Gamma^A P_A + \mu e^2) \, \psi(p)$$

where $p_\mu = \{E, \underset{\sim}{p}\}$ and P_A is a five vector whose components are linear in E and quadratic in $\underset{\sim}{p}$.

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3) The r-dependence of $\psi(p)$ is replaced by tensor indices on which the Γ-operators act as matrices. Tensors also replace the usual angular momentum system of basis states, which simplifies all calculations tremendously.

In this formulation the matrix element for Compton scattering from bound electrons is simply

$$\psi^{\dagger}_{n'}(p')(\mathcal{E}'\Gamma)\,[(Q\Gamma)+\mu e^2]^{-1}\,(\mathcal{E}\Gamma)\,\psi_n(p)$$

which is easily evaluated in closed form if the external states are low level bound states. The main reason for the simplification that is achieved in the calculation, besides the disappearance of retardation factors and angular momentum decompositions, is the use of a complete, discrete set of off-shell intermediary states (the eigenstates of $Q\Gamma$).

The new results consist mainly in the discovery that the method works even better for problems in which the external states are disassociated. The matrix elements for Coulomb scattering, photo effect and bremsstrahlung are, respectively,

$$\psi^{\dagger}_{\underset{\sim}{q}'}(p)\,[(P\Gamma)+\mu e^2]^{-1}\,\psi_{\underset{\sim}{q}}(p)$$

$$\psi^{\dagger}_{\underset{\sim}{q}'}(p')\,[(P'\Gamma)+\mu e^2]^{-1}\,(\mathcal{E}\Gamma)\,\psi_n(p)$$

$$\psi^{\dagger}_{\underset{\sim}{q}'}(p')\,[(P'\Gamma)+\mu e^2]^{-1}\,(\mathcal{E}\Gamma)\,[(P\Gamma)+\mu e^2]^{-1}\,\psi_{\underset{\sim}{q}}(p)$$

and all can be evaluated easily and quickly. We have also developed a similar formulation of capture processes and have found a simple formula for radiative capture. The matrix element for this process is

$$\psi^{\dagger}_{cap}\,[(Q\Gamma)+\mu e^2]^{-1}\,(\mathcal{E}\Gamma)\,\psi_n \quad .$$

The three wave functions ψ_1 (ground state), $\psi_{\underset{\sim}{q}}$ (scattering state) and ψ_{cap} (captured state) are all represented by tensors of the form $\sigma_{A_1}\cdots\sigma_{A_N}$, with a different five vector σ_A in each case. Incidentally, bremsstrahlung in a Coulomb field is an exact Regge-multiperipheral production amplitude.

REFERENCES

The following is a list of papers that, if read in sequence, give a coherent account of our method. References to related work of other authors are given in these papers.

1. "Infinite multiplets and the hydrogen atom", Phys. Rev. 156, 1665 (1967).

The next three papers are not concerned with hydrogen, but some of the computational techniques are developed there:

2. "On the supplementary series of representations of semisimple non-compact groups", ICTP, Trieste, Internal Report 15/1967.

3. "Feynman rules for Reggeons", Phys. Rev. 168, 1845 (1968).

4. "Relativistic Lagrangian field theory for composite systems", Phys. Rev. 171, 1811 (1968).

The remaining papers are applications. (The work on radiative capture is in preparation.)

5. "Compton scattering from bound electrons", Phys. Rev. to appear.

6. "Simplified calculation of the Lamb shift using algebraic techniques" by R.W. Huff, UCLA preprint 1969.

7. "Bremsstrahlung in Coulomb field - An exact Regge-multiperipheral production amplitude", by C. Fronsdal and L.E. Lundberg, ICTP, Trieste, preprint IC/69/33.