IC/69/54

INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

TOPICAL CONFERENCE

ON

DYNAMICAL GROUPS AND INFINITE MULTIPLETS

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

9-14 June 1969

1969

MIRAMARE - TRIESTE

THE RELATIVISTIC TWO-BODY PROBLEM

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Two apparently disparate approaches to the relativistic two-body problem are outlined.

A. We have studied an ad hoc equation of the Majorana type and found that it represents two interacting scalar particles. The model is a relativistic analogue of the hydrogen atom both in the sense that the same group and the same representation plays an important role and because it reduces to Schrödinger's theory of hydrogen in the non-relativistic limit. The equation is (compare treatment of H-atom in the appropriate abstract)

$$\left[(p^2 + m_1^2 - m_2^2)\,\Gamma_4 - 2m_1 p\Gamma + \frac{e^2}{2}\,(p^2 - m_-^2)\right]\psi \equiv L(p)\,\psi = 0 \;,$$

with $m_\pm = m_1 \pm m_2$. The identification of this system with two interacting particles is made by a study of the spectrum and the non-relativistic limit, but in more detail by an examination of the analytic structure of form factors and scattering amplitudes. Singularities are identifiable as normal and false thresholds, anomalous thresholds and Mandelstam spectral function boundaries.

B. We have found that the above equation can be derived from quantum field theory by a line of reasoning that is closely related to the quasi-potential approach of Logunov and Tavkhelidze. It is an alternative to the Bethe-Salpeter equation and possesses all the advantages of the quasi-potential approach. In addition there are certain formal advantages in that the equation can be derived from a Lagrangian. We start from the equations for two free particles,

$$(p_1^2 - m_1^2)\,\psi(p_1 p_2) = 0 \qquad (1)$$

$$(p_2^2 - m_2^2)\,\psi(p_1, p_2) = 0 \qquad (2)$$

and note that the Bethe-Salpeter equation possesses far too many solutions when the interaction is turned off. [$(p_1^2 - m_2^2)(p_1^2 - m_2^2)\,\psi = 0$ is obviously weaker than (1), (2).] In order to avoid the introduction of too many degrees of freedom we follow the quasipotential approach and retain one of the two free equations as a subsidiary condition even when interactions are present.

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If we take (1) as a subsidiary condition then $\psi(q, p)$, $p = p_1 + p_2$ and $q = p_1$, is, for every fixed p_μ, a wave function on the hyperboloid $q^2 = m_1^2$. On such functions we may construct the realization of SO(4, 2) mentioned above. If an interaction term of the form const. Γ_4^{-1} is introduced in (2), then the result is identical with the infinite component equation studied under the heading A. From this we can conclude that some infinite component c-number field theories are considerably better approximations to quantum field theory than the Bethe-Salpeter equation and that all objectionable features will become successively less important in higher approximations.

REFERENCES

A. "Relativistic Lagrangian field theory of composite systems", Phys. Rev. 171, 1811 (1968).

B. This work, which is a collaboration with L.E. Lundberg, is in preparation.