"Course on Ocean-Atmosphere Interaction in the Tropics"
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"Linear Instability Analysis of the Coupled System"

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When an atmosphere, which itself has no instabilities, interacts with an ocean, which itself has no instabilities, instabilities of long time and space scales can arise. The surface of the eastern equatorial Pacific, for example, is normally colder than the western Pacific, and is under the influence of atmospheric mean divergence driven by the (convergent) region of persistent precipitation over the 'maritime continent' in the far west Pacific. If a warm sea surface temperature (sst) anomaly alters the atmospheric circulation in such a way that the induced anomalous surface winds enhance the anomalous sst, then the anomalies will grow in both the atmosphere and ocean.

In general, this will happen by inducing westerly surface wind anomalies to the west of the warm sst anomaly, which could then warm the surface of the ocean by a number of different processes. The westerly surface wind anomaly could force anomalous eastward ocean currents which, because of a pre-existing mean eastward sst gradient, would then warm by anomalous zonal advection. The westerly surface wind anomaly could weaken the normally poleward currents on both sides of the Equator which, because of a pre-existing mean equatorward sst gradient, would then warm by anomalous meridional advection. The westerly surface wind anomaly could reduce the magnitude of the upwelling which would then warm by anomalous vertical advection. The westerly surface wind anomaly could deepen the thermocline, thereby weakening the effective vertical temperature gradient, and the surface would then warm because of mean vertical advection acting on an anomalously weak vertical temperature gradient. These, and other permutations of terms in the ocean thermodynamic equation, could act singly, or in concert, to anomalously warm the surface of the ocean and thereby cause a growing coupled atmosphere-ocean instability. As it turns out, the nature of the induced instability depends sensitively on the mix of thermodynamic processes in the ocean that contribute to the anomalous sst changes.

The anomaly sst equation is, in general

\[ T_e + u (\bar{T} + T)_x + v (\bar{T} + T)_y + \bar{u} T_x + \bar{v} T_y + \left[ M(\omega + \bar{\omega}) - M(\bar{\omega}) \right] T_e + M(\omega + \bar{\omega}) T_e = -\alpha \bar{T} \]

(1)
arise intrinsically because of the coupling of the atmosphere and the ocean.

\[ \text{Warm SST} \rightarrow \text{Wetted windurons} \rightarrow \text{Advection or Upwelling} \]

Since both advection and upwelling involve dynamics of the ocean, we must consider 3 separate things:

**Atmosphere**: Takes SST, Gives Winds

**Ocean**: Takes winds, Gives thermocline depth, upwelling + currents

**Mixed Layer**: Takes winds, advection + upwelling, Gives SST

**Atmosphere Model**

\[
\begin{align*}
U_t - \beta y V + \Phi_x + AU &= 0 \\
V_t + \beta y U + \Phi_y + AV &= 0 \\
\Phi_t + CE_{z} (Ux+Vz) + \nabla \Phi &= -Q
\end{align*}
\]

**Ocean Model - Wave Dynamics**

\[
\begin{align*}
\eta_t - 
\beta y \phi + \eta h_x + au &= \frac{\tau_{(x)}}{H} \\
\phi_t + \nabla \psi + \eta h_x + au &= \frac{\tau_{(z)}}{H} \\
h_t + H(u_x + v_y) + b \phi &= 0
\end{align*}
\]
(I) \( T = x h \)

(II) \( T + u \bar{T}_x + \alpha T = 0 \) no entrainment

(III) \( T + u \bar{T}_x - K \bar{T} h + \alpha T = 0 \) full model

(IV) \( T - K \bar{T} h + \alpha T = 0 \) (entrainment dominated)

Coupling

\[ Q = K q T \]

\[ \frac{\bar{T}}{H} = -K_s U_i \]

Method: Express everything as \( C \)

The coupling reduces the freedom to extend variables in the system \((U, V, \phi, T, u, v, h)\) so the system can be formulated as an eigenvalue problem in \( \omega = \omega_r + i \omega_i \). Look for the modes with \( Im \omega \geq 0 \): these are the coupled unstable modes.

1. Always get instabilities if the coupling is large enough. Coupling measured by

2. Necessary condition for instabilities is

\[ J \cdot \mathbf{u} > 0 \quad \langle \phi \bar{\phi} \rangle > 0 \]
FIG. 1. Growth rates [Im(\sigma)] and frequencies [Re(\sigma)] of modes in Model 1 as functions of the product of coupling coefficients, \(K_0K_s\). Symbols refer to Kelvin (K), \(n = 1\) Rossby (R1), \(n = 2\) Rossby (R2), Yanai (Y) and \(n = 0\) inertia–gravity (IGEO) waves. \(\text{Im}(\sigma)\) is in \(10^{-2}\) nondimensional units, \(K_0K_s\) and \(\text{Re}(\sigma)\) are in nondimensional units. \(\text{Im}(\sigma) = 10^{-2} = (208\ \text{days})^{-1}\). Dashed line indicates representative \(K_0K_s\).

FIG. 2. Growth rates [Im(\sigma)] and frequencies [Re(\sigma)] of modes in Model 1 as functions of wavenumber (k). Symbols as in Fig. 1. Dashed lines indicate computed values in the absence of coupling. \(\text{Im}(\sigma)\) is in \(10^{-2}\) nondimensional units, \(\text{Re}(\sigma)\) and \(k\) are in nondimensional units. A wavelength of 16,000 km corresponds to \(k = 0.1\).
Fig. 6. Growth rates $[\text{Im}(\sigma)]$ and frequencies $[\text{Re}(\sigma)]$ of modes in Model II as functions of $K_0K_s$. Symbols "R2U" and "R2D" refer to unstable and damped $n = 2$ Rossby modes, respectively. Otherwise as for Fig. 1.

Fig. 7. As in Fig. 6 but as functions of wavenumber ($k$).
FIG. 4. Schematic illustration of equatorial motion associated with the Kelvin and $n = 1$ Rossby wave in Model I (A and B) and Model II (C and D). The solid horizontal line represents the ocean surface, the dashed curve the bottom of the mixed layer. Arrows indicate ocean current and atmospheric motion perturbations. Maximum positive and negative perturbations of SST are indicated by WARM and COLD, atmospheric heating by $Q+$ and $Q-$, of surface atmospheric pressure by $H$ and $L$, respectively.
FIG. 5. Eigenfunctions for (A) the Kelvin and (B) the $\pi = 1$ Rossby wave, in Model I. Solid, dashed and dotted lines indicate lower tropospheric pressure ($P$), mixed layer depth ($h$) and SST ($T$) perturbation contours; those for $P$ and $h$ are at 90%, 50% and 10% and for $T$ are at 30% of maximum values. The "+" and "−" indicate maximum positive and negative perturbations. Solid and dashed arrows indicate perturbation surface wind and ocean velocity. Coefficients and wavenumber have values as in Table 1, except that $K_0 = 2.5 \times 10^{-3} \text{ m}^2 \text{s}^{-1} \text{K}^{-1}$ for the Rossby wave. One unit of nondimensional distance corresponds to 250 km.
FIG. 9. As in Fig. 5 but for Model II when $K_0$ and $K_f$ have "representative" values.
Fig. 12. Growth rates $|\text{Im}(\sigma)|$ and frequencies $\text{Re}(\sigma)$ of modes in Model III as a function of $K_0 K_s$. The symbol U refers to the Model III unstable mode. Otherwise as for Fig. 1.
Fig. 15. Growth rates $[\text{Im}(\sigma)]$ and frequencies $[\text{Re}(\sigma)]$ of modes in Model IV as functions of wavenumber $k$. The symbol U refers to the Model IV unstable mode. Otherwise as for Fig. 2.
Fig. 14. Eigenfunction for the Model III unstable mode when $K_0$ and $K_p$ have "representative" values, otherwise as for Fig. 5.

Fig. 16. As in Fig. 14 but of Model IV.
FIG. 1. Schematic diagram illustrating sequence of studies reported herein: (a) unbounded ocean-atmosphere of period 15,000 km, (b) 15,000 km ocean bounded by thin barrier, (c) 15,000 km ocean bounded by wide continent. Solid line, shading and dashed line indicate land surface, ocean and top of atmosphere, respectively; $z$ is the vertical coordinate.
Fig. 10. \( x - t \) diagram showing the periodic time evolution of equatorial fields for the U1 mode in the wide continent case: (a) zonal current, (b) thermocline depth, (c) SST, (d) zonal wind and (e) pressure. Overall exponential growth is neglected. Contours are at 0, 30%, 40%, 60% and 80% of the maximum amplitudes attained on the \((x, y, t)\) domain. Dashed contours indicate negative values. Heavy vertical lines indicate positions of the ocean boundaries. One unit of time \((t)\) equals 2.1 days.
Fig. 3. Growth rate ($\sigma_i$) and frequency ($\sigma_s$) as functions of $K_0 K_s$ for the fundamental unstable mode in the unbounded ocean–atmosphere (solid), the thin oceanic barrier case (dashed) and the wide continent case (dot-dashed). The dashed vertical line indicates representative $K_0 K_s$. Units are nondimensional.

Fig. 5. Growth rate and frequency as functions of interval or basin width ($x_b$) for the fundamental unstable mode in the unbounded ocean–atmosphere (solid), the thin oceanic barrier case (dashed) and the wide continent case (dot-dashed). Dashed vertical line indicates $x_b$ of 15 000 km. Otherwise as in Fig. 3.
If we use the presence of a mixed layer, but without the meridional terms and put
\[ \frac{T_e}{H} = \frac{T - a(h)}{H} \]
the eq becomes
\[ T_e + u \frac{T_e}{H} + \frac{\bar{w}}{H} T = \frac{\bar{w} - a(h)}{H} \]
and we see that this is identical to model III

where
\[ K_T = -\frac{\bar{w} - a(h)}{H} \]

and the damping is
\[ \alpha \to \alpha + \frac{\bar{w}}{H} \]

so that the upwelling affects both \( K_T \) and the damping.

Weishaar & Saville (1949) showed that if instead of keeping \( K_T \) and \( \bar{w} \) constant with latitude, they include the latitude dependence of \( \bar{w}(y) \), then the unstable eastward propagating mode becomes stationary, see the walls, and satisfy the needed oscillating eq. The basic reason is that a meridionally confined \( \bar{w}(y) \) to \( \pm 100 \text{ km} \) no longer keeps the Rossby modes off them in turn see the W. boundaries.