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"ENSO Mechanisms: Simplified Models"

M. CANE
Lamont-Doherty Geological Observatory
Palisades, New York 10964
USA

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ENSO Mechanisms: Simplified Models

The Bjerknes-Wyrtki theory lacked an explanation for the perpetual turnabout from warm to cold states and back again. While the explanation is inherent in the physics hypothesized earlier, it did not emerge until after the development of the numerical models, and so is properly regarded as one of the fruits of numerical modeling. An early version (Cane and Zebiak, 1985; Cane et al. 1986) emphasized the recharging of the equatorial "reservoir" of warm water as a necessary precondition for the initiation of a warm event. On the basis of his analysis of sea level data, Wyrtki (1985) developed a very similar hypothesis. The aftermath of a warm event leaves the thermocline along the equator shallower than normal (i.e. equatorial heat content is low and SST is cold; this is the "La Nina" phase). Over the next few years the equatorial warm water reservoir is gradually refilled. Once there is enough warm water in the equatorial band, the rapid (for the ocean) equatorial Kelvin and Rossby waves allowed by linear equatorial ocean dynamics can move enough of the warm water to the eastern end of the equator to initiate the next event.

The theories of Suarez and Schopf (1988) and Battisti and Hirst (1989), which also have linear linear equatorial ocean dynamics at their core, provide a much clearer picture of how the ENSO cycle operates. More recent work along similar lines has expanded our understanding of this mechanism (Schopf and Suarez, 1990; Graham and White, 1988; Cane et al. 1990; Munnich et al. 1991: also see Cane and Zebiak, 1987), and we are now able to present a complete paradigm for the ENSO cycle.

As in nature, let the main wind changes be in the central equatorial ocean while the SST changes are concentrated in the east. Then the surface wind amplitude, which depends on the east-west temperature gradient, varies with this eastern temperature. Further simplify by assuming that this eastern SST is principally controlled by thermocline depth variations. These variations are driven by the changes in the surface wind stress according to the linear shallow water equations on an equatorial beta plane. If the eastern SST is warm (thermocline high) then the wind anomaly will be westerly, forcing a Kelvin wave packet in the ocean to further depress the thermocline in the east thus enhancing this state.

However, this excess of warm water must be compensated somewhere by a region of colder water (shallower than normal thermocline). Equatorial dynamics dictates that this be the in form of equatorial Rossby wave packets, which must propagate westward from the wind forcing region. When they reach the western boundary they are reflected as "cold" equatorial Kelvin waves, which propagate eastward across the ocean to reduce the SST there. Thus the original warm signal is invariably accompanied by a cold signal—but with a delay. This delayed oscillator mechanism accounts for the turnabout from warm to cold states. The wraparound Rovmuller diagram in the Figure from Schopf and Suarez (1988) illustrates this in their model.

To further appreciate the role of equatorial waves in sustaining the ENSO oscillation consider the state of affairs when the eastern thermocline and SST anomalies are near zero; for example, at the termination of a warm event. Then the wind anomaly must be near zero as well, so there is no direct driving to evolve the coupled system to its next phase. However, the previous warm event necessarily left a residue of cold Rossby waves in the
western ocean, which eventually reflect at the west into a Kelvin wave which will reduce the SST in the east. The wind then becomes easterly and the cycle continues.

This paradigm may be distilled into a very simple system such as a single ordinary differential equation with a delay (Suarez and Schopf, 1988; Schopf and Suarez, 1990; Battisti and Hirst, 1989) or a recurrence relation in a single variable (Cane et al., 1990; Münstich et al., 1991). Perhaps the simplest version is that of Battisti and Hirst (1989):

\[ \frac{\partial T}{\partial t} = -bT(t - \tau) + cT, \]

(1)

They derived this equation as well as values for the parameters \(b, c\), and \(\tau\), from Battisti's (1988) version of the ZC [Zebiak and Cane] numerical model. Here \(T\) is the SST anomaly in the eastern equatorial Pacific, and \(c\) is the sum of all the processes that induce local changes in \(T\), including horizontal advection, thermal damping, anomalous upwelling and changes in the local subsurface thermal structure (including local wave effects). The \(b\) term accounts for the effect of Kelvin waves generated at the western boundary as the reflection of Rossby waves; \(\tau\) is the delay associated with this reflection process. Growing, oscillating solutions to (1) - ENSO modes - exist when \(b\tau > \exp(c\tau - 1)\). A relation which holds for the parameters characteristic of the numerical model (see the Appendix to Battisti and Hirst, 1989).

Though other mechanisms can give rise to unstable oscillations in coupled tropical models (e.g. Hirst, 1986; 1998; Neelin, 1991), it is generally accepted that this paradigm accounts for the behavior of the numerical models discussed above, as well as that in the higher resolution coupled GCM which exhibits an ENSO-like oscillation. It is more difficult to establish conclusively that it operates in nature. It is consistent with the refill idea described above, which is supported by data (Wyrtki, 1985, and the additional time series available in the Climate Diagnostics Bulletin of NOAA). The role for western boundary reflection is further supported by the semi-empirical studies of Zebiak (1989) and Graham and White (1990). Finally, the ZC coupled model, in which this mechanism is clearly operative, has demonstrated the ability to predict warm events a year or more in advance.

While the restriction of these models to the tropical Pacific region serves to bolster Bjerknes' emphasis on this region, it also renders them incapable of simulating the global consequences of ENSO. What is perhaps more troubling is the inability of the paradigm to account for the changes in the western equatorial Pacific preceding the warming in the east (see the discussion in Cane et al., 1990). More generally, the SO is observed to exhibit some behavior distinct from El Niño, and this too is not reproduced. These tropical Pacific omissions suggest that connections essential to the ENSO cycle may have been overlooked.

The observed ENSO cycle is not regular, and some of the models share this feature. Nonetheless, the cause of the observed aperiodicity remains an unsettled issue. The results from Battisti's (1988) model and the experiments of Schopf and Suarez (1988) suggest that it is solely due to noise; that is, atmospheric or oceanic fluctuations distinct from the ENSO cycle. On the other hand, the low order ENSO model of Münstich et al. (1991) produces aperiodicity, doing so rather readily if a seasonal modulation is included.

Experiments and analysis with ENSO models have demonstrated very strong sensitivities to rather small changes in parameter values. (In addition to the references cited
above, also see Neelin, 1989.) In the anomaly models some of these changes are equivalent to changes in the mean background state. Since a greenhouse warming will alter this state, the implication of such sensitivity is that the characteristics of ENSO will be changed. There have been a few experiments to explore this possibility (Zebiak and Cane, 1989; Graham, private communication), but inferences must be highly tentative in deference to our limited confidence in the ENSO models and to the great uncertainties as to the nature of greenhouse induced changes. This area of research is likely to become quite active as climate modeling progresses.
APPENDIX: The Delayed Oscillator Equation

Battisti and Hirst (1989) derive a simple linear "delayed oscillator" model based on Battisti's (1988) version of the Zebiak and Cane (ZC; 1987) model. Note that this version is more regular than the original, (probably because it is more dissipative). They note that:

1. $\tau^e$, SST are in phase while thermocline depth $h$ leads slightly.
2. The signal in SST, $\tau^{(e)}$ is small west of dateline.
3. $\pm 5^\circ$ of the equator are all that matter.

They further simplify as follows:

4. Remove the seasonal cycle.
5. Linearize ocean thermodynamics and coupling. (Dynamics are already linear)
6. Use #2 as sketched here:

\[
\begin{array}{|c|c|}
\hline
\text{SST, } \zeta^{(e)} \text{ anomalies small:} & \text{SST, } \zeta^{(e)} \text{ vary:} \\
\text{only free waves in the ocean} & \text{forced motion} \\
\hline
\end{array}
\]

7. Use #3 to consider only $\tau^e < \langle SST \rangle$. $\langle \rangle =$ average here.

The linearized temperature equation contains many terms, which they fit from the output of the Battisti model. An important term is:

$$
\frac{\gamma w}{H_1} T_d(h) \approx \int \omega(h) h;
$$

They then take the dynamics as

$$
h = h_{RK} + h_L = -a_w < \tau^e(t - \tau) > + a_L < \tau^e >.
$$

The first terms is the reflected Kelvin wave, due to Rossby waves impinging on the western boundary. There is a delay $\tau$ from the time these waves were generated until the resulting Kelvin wave reaches the east. The second term is the locally generated, directly forced Kelvin wave.

All this leads to

$$
\frac{\partial T}{\partial t} = -b(T(t - \tau) + cT)
$$
which has solutions $T = T_0 \exp(\sigma t)$ if

$$\sigma = -be^{-\sigma \tau} + c.$$

BH estimate:

$$c = 2.2 \text{yrs}^{-1}; \; b = 3.9 \text{yrs}^{-1}; \; \tau = 180 \text{days}.$$

Oscillations if $b > \exp(c\tau - 1)/\tau$;
if $c\tau \approx 1$, then, approximately, if $b > c$.
Periods $> 2\tau$; and rapid periods for decaying (unless $b \gg 1$) modes.
Growth if, approximately,

$$b > \frac{\pi}{2\tau} - \left[\frac{\pi}{2} - 1\right]c.$$

For $c\tau \approx 1$ this is just $b > c$.

Nonlinearities in the model are:
(i) moisture convergence feedback
(ii) the quadratic wind stress law: $\tau = \rho CP U_2^2$
(iii) horizontal and vertical advection

They conclude (iii) is the important term - in particular, the nonlinearity in vertical thermal structure $T_d(h)$: This analysis yields the approximate equation:

$$\frac{\partial T}{\partial t} = -bT(t - \tau) + cT - e[T - \tau T(t - \tau)]$$

The simpler form with $\tau = 0$ is the model of Suarez and Schopf (1988), except that Suarez and Schopf (SS) take $b < c$ and $c > 0$. Oscillations are between the two states with

$$T = \pm \left[\frac{c - b}{e}\right]^{1/2}.$$
Figure 5. Upper layer volume of the tropical Pacific (10^4 m^3) from 1975 to 1983 relative to its mean value (70x10^4 m^3). The volume is estimated on the basis of tide gauge data for the region from 15°N to 15°S. (From Wyrtki 1985).

OBSERVED

HEAT CONTENT
P. CALCULATED FROM OBSERVED WINDS

OBSERVED NIÑO3 SST

YEAR

70 72 74 76 78 80 82 84 86 88

COUPLED MODEL

HEAT CONTENT

NIÑO3 SST

YEAR

0 2 4 6 8 10 12 14 16 18
Time-longitude behavior of the coupled model oscillator. (a) $\eta_0$ from 80°W (on left) to 120°E (on right). (b) $\eta_0$ from 120°E to 120°W. (c) Zonal surface wind on equator from 180°W to 123°W. (d) $\eta_0$ from 160°W (on left) to 120°E. (e) $\eta_0$ from 120°E to 80°W. In (a)-(c) positive anomalies are hachured. In (d) and (e) negative anomalies are hachured. From Schopf and Suarez, 1988.
FIG. 2. The complex solutions to the delayed oscillator equation (2.10) with a reference value of $r = 180$ days. The growth rate of the system (in yr$^{-1}$) vs the strength of the local instability term $c$ (in yr$^{-1}$) is plotted in (a), and the period of the oscillation (in yr) as a function of $c$ is plotted in (b). Each curve represents a different value for $b$ (in yr$^{-1}$). For $b = 1.3$ yr$^{-1}$, solutions are pure growth. The shading indicates the range of values for the coefficients $b$ and $c$ for the atmosphere-ocean system in the equatorial Pacific.

FIG. 3. The growth rate (yr$^{-1}$) (a) and period (yr) (b) of the coupled system vs time lag $r$, for $b = 3.9$ yr$^{-1}$ at $r = 180$ days. Each curve is for a difference reference value for $c$ (in yr$^{-1}$) (for the Pacific $c = 2.2$ yr$^{-1}$).

From Battisti & Hirst, 1989
Fig. 16. The solutions to the delayed oscillator equation (4.7) with a reference value of \( r = 180 \) days and \( \varepsilon = 0.07 \, \text{C}^{-1} \, \text{yr}^{-1} \). The final state amplitude, \( T_s \), of the system (in degrees C) vs the strength of the local instability term \( c \) is plotted in (a), and the period of the oscillation as a function of \( c \) is plotted in (b). Each curve represents a different value for \( b \). Variables \( b \) and \( c \) are in units of \( \text{yr}^{-1} \).

Fig. 17. The final state amplitude, \( T_s \), (in °C) (a) and period (yr) (b) of the coupled system vs time lag \( \tau \), for \( b = 3.9 \, \text{yr}^{-1} \), \( \varepsilon = 0.07 \, \text{C}^{-1} \, \text{yr}^{-1} \). Each curve is for a different reference value for \( c \) (in yr\(^{-1} \)).
ON INTERANNUAL VARIABILITY IN THE TROPICAL PACIFIC

A DISTILLATION OF THE BJERKNES-WYRTKI MODEL FOR ENSO

(drawing largely on work of Münich, Cane & Zebiak, J.A.S. 90a,b)

Every theory* of the course of events in nature is necessarily based on some process of simplification of the phenomena and is to some extent therefore a fairy tale.

Sir Napier Shaw
Manual of Meteorology

*B or model

BJERKNES (1969) HYPOTHESIS + WYRTKI (1975) THEORY:

WIND and SEA LEVEL: OCEAN DYNAMICS, NOT THERMODYNAMICS
REMOTE FORCING NOT LOCAL

THERMOCLINE DEPTH NOT W

IN FACT, LINEAR, SHALLOW WATER OCEAN DYNAMICS

NOT COASTAL: A BIG SIGNAL
\[ Z^{(s)} = A(t) \exp \left( -\frac{y^2}{L^2} \right) \]

\[ x_1, x_2 \rightarrow x_E / 2 \]

\[ L_a \sim 15^\circ; \quad \mu = 0.1; \quad r^{-1} = 2.5 \text{ years} \]

\[ \mu = 2 \left( \frac{L_e}{L_a} \right)^2; \quad Z \sim \exp \left( -\frac{L_e}{2} y^2 \right) \]

\[ h_e(t) = (1 + \mu)^{-1/2} e^{-r/2} A(t - 1) - \]

\[ \sum_{n=1}^{N} b_n a_n(\mu) e^{-(4n+1)r/2} A(t - 4n - 1) + \]

\[ \sum_{n=1}^{N} b_n e^{-4nr/2} h_e(t - 4n) \]

\[ b_1 = \frac{1}{2}, \quad b_j = \frac{1 \cdot 3 \cdot 5 \ldots (2j - 3)}{2 \cdot 4 \cdot 6 \ldots 2j}; \quad a_j(\mu) = \left( \frac{1 - \mu}{1 + \mu} \right)^{j-1} \frac{[(4j - 1)\mu + 1]}{(1 + \mu)^{3/2}} \]

\[ A = A(h_e; t) \]

**Linear:** \[ A = K h_e \]

**Nonlinear:** \[ K \approx \frac{\partial A}{\partial h_e} (h_E = 0). \]
Fig. 2. Forcing function \( A(h) \) given by Eq. (9).

\[
A(h) = \begin{cases} 
  b_+ - b_- \left\{ \tanh \left[ \frac{xa_-}{b_-} (h - h_-) \right] - 1 \right\}, & h_- < h \\
  xh, & h_- \leq h \leq h_-
\end{cases}
\]

We must have \( a_+ > 1 \) and

\[
h_- = \frac{b_-}{x a_-} (a_+ - 1); \quad h_+ = \frac{-b_-}{x a_-} (a_- - 1)
\]
Fig. 3. $h(t)$ for the symmetric forcing function $|b_3| = 1$ without an annual cycle and various $\kappa$; cf. Fig. 2 and Eq. (9). Ten Rossby waves included. $T$, the average period in years, is determined from the output. (a) $a_{\kappa} = 1$, (b) $a_{\kappa} = 2$.

Fig. 4. (a) $h(t)$ for the symmetric forcing function $a_{\kappa} = a_1 - a_3 = -b_3 = 1$, varying the curvature of the forcing ($a_{\kappa}$). $N = 10$ and $\kappa = 2.2$. $T$ gives the average period in years. (b) A 500-year sequence for the case $a_{\kappa} = 5$. 
FIG. 5. $h(t)$ for the symmetric forcing function varying the number of Rossby waves. (a) $\epsilon = 2.7, a_x = b_x = 1; (b) \epsilon = 2.2, a_x = 3.5, b_x = 1$. 
FIG. 6. \( h(t) \) for the symmetric version of function (9) with an annual cycle and 2 Rossby waves, \( N = 2, a_2 = 7, b_2 = 1, B = 0.25, \kappa = 1.9. \)

FIG. 7. \( h(t) \) for asymmetric forcing, \( a_* = 3a_-, b_* = 1.0, b_0 = 0.5, N = 10, \kappa = 1.9. \) Top: No annual cycle, \( a_* = 2. \) Middle: Weak annual cycle, \( B = 0.1; a_* = 2. \) Bottom: No annual cycle, \( a_* = 3. \)

FIG. 8. \( h(t) \) for asymmetric forcing and an annual cycle, \( \kappa = 2.1, a_* = 3, b_* = 1, b_0 = 0.6, B = 0.2, N = 4. \)