"Course on Ocean-Atmosphere Interaction in the Tropics"
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"Convective Boundary Layers"

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Please note: These notes are intended for internal distribution only
1) Definitions

2) Fluxes and Entrainment in a Mixed Layer

3) Convective boundary layers: structure and dynamics

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1) Definitions:

a. Friction velocity \( u_f \):

\[ u_f = \frac{171}{e} \]

So that \( u_f \) defines some sort of characteristic velocity characteristic of the stress in a medium of density \( \rho \).

For stress at the air-sea interface

\[ \gamma = (\rho u_f^2)_{\text{air}} = (\rho u_f^2)_{\text{water}} \]

\[ \therefore \frac{(u_f^2)_{\text{air}}}{(u_f^2)_{\text{water}}} = \frac{\rho_{\text{water}}}{\rho_{\text{air}}} \approx 1000 \]

\[ \therefore (u_f)^{\text{air}} = 30 (u_f)^{\text{water}} \]

Typically for \( \gamma = 1 \text{ dyn/cm}^2 \)

\[ (u_f)^{\text{water}} = 1 \text{ cm/sec} \]

\[ (u_f)^{\text{air}} = 30 \text{ cm/sec} \]

b. Monin - Obukhov Length

\[ L = \frac{u_f}{\kappa g B \omega} \]
\[ \frac{1}{2}\frac{\partial w'}{\partial z} \] is the work done by buoyancy (against gravity).

\[ \frac{1}{2} \alpha \cdot u^2 \] is the work done by mechanical stirring.

The Monin-Obukhov length is a length over which the mechanical and buoyancy effects are comparable. If we have a mixed layer of depth \( h \)

If \( h > L \), the layer is convectively driven. If \( h \approx L \), the layer is mechanically driven.

**EXERCISE:**

If there are 4 mm/day of evaporation and the Bowen ratio is 1, what is the Monin-Obukhov length in meters?

2) **Fluxes and Entrainment in the mixed layer.**

Let the heat flux at the surface be \( \langle \theta' w' \rangle \).

The mixed layer temperature satisfies

\[ \frac{\partial}{} \]

\[ \frac{\partial \theta_m}{\partial t} = - \frac{\partial^2 \theta'_m}{\partial z^2} \]

Since \( \frac{\partial \theta_m}{\partial z} = 0 \)

\[ 0 = - \frac{\partial^2 \theta'_m}{\partial z^2} \]

and \( \langle \theta' w' \rangle = ax + b \) in the interior of the fluid mixed layer.

At \( z = 0 \), \( b = \langle \theta' w' \rangle_s \)
At \( z = h \), one might think that there is no heat flux, but \( z = h \) marks the interface between turbulent fluid for \( z < h \) and laminar fluid for \( z > h \). The turbulent fluid will necessarily
entrain laminar fluid.

Definition: The entrainment velocity is the
volume per unit area crossing the interface per unit time.

\[ \frac{dh}{dt} \] is the entrainment velocity.

There is an inversion at the interface and the
heat balance at the inversion is

\[ \int_{\theta_m}^{\theta} \left( \frac{dh}{dt} \right) d\theta = -\langle \theta' w' \rangle; \]

\[ \Rightarrow \frac{dh}{dt} = \theta_m \]

At \( z = h \), \( a h + b = \langle \theta' w' \rangle; \)

and

\[ \frac{d\theta_m}{dt} = a = \frac{\langle \theta' w' \rangle}{h} \]

To know how convective boundary layer rises,
we need additional equations:

\[ \frac{d\theta}{dt} = \frac{d\theta_0 - d\theta_m}{dt} = \langle \theta' w' \rangle \]

\[ = \left( \frac{\langle \theta' w' \rangle}{h} \right) \]

If, in the presence of \( w \neq 0 \), we look
for an equilibrium solution \( \frac{d\theta}{dt} = \frac{dh}{dt} = 0 \)
\[ h = \frac{v - v'}{\omega} \]

and \( \omega \theta = -\langle v \omega \rangle \).

and we still need a relation for \( \langle v \omega \rangle \).

Tennekes (1971) recognized that it takes work
to bring heat down from above the mixed layer into
the mixed layer since warm air is less
than cooler air.

The rising plumes gain
buoyancy and energy from the surface buoyancy
and lose some to dissipation in the interior
of the mixed layer. For most situations,
\( \langle v \omega \rangle \) seems to be the right choice.

\[ h = \frac{1.2 \langle v \omega \rangle}{\omega} \]

\[ \Delta \theta = \frac{2 \langle v \omega \rangle}{\omega} \]

so if we knew the surface forcing, the \( \theta \) temperature
gradient into which the mixed layer is rising,
and the vertical velocity in the environment, then
we can find \( h \) and \( \Delta \theta \) in the same mixed
layer.

Monin-Obukhov Similarity Theory:

Near the ground (within one Monin-Obukhov
length at the ground), the temperature is not well
mixed.

\[ \frac{d \Theta}{dt} = \frac{\Theta}{\kappa t} f \left( \frac{z}{L} \right) \quad \text{where} \quad \Theta^* = \frac{\omega^* \theta^*}{v^*} \]
This scaling seems to match the observations with

\[ f(\frac{\xi}{z}) = g\left(1 - \frac{\xi}{z}\right)^{-\frac{1}{4}} \]

Similarly,

\[ \frac{\partial u}{\partial z} = -\frac{u_s}{z} g\left(\frac{\xi}{z}\right) \]

where \( g\left(\frac{\xi}{z}\right) = \left(1 - \frac{\xi}{z}\right)^{-\frac{1}{4}} \)

Note that at \( z = 0 \), \( g\left(\frac{\xi}{z}\right)|_{z=0} = 1 \)

\[ \frac{\partial u}{\partial z} = \frac{u_s}{z^2} \]

\[ u = \frac{u_s}{k} \ln \frac{z}{z_0} \]

where \( z_0 \) is the "roughness length" and \( u \) 0.2 cm for the ocean

Note that under neutral conditions, i.e., \( z_0 \rightarrow \infty \), \( k \rightarrow 0 \)
and \( \frac{\partial u}{\partial z} = \frac{u_s}{z^2} \) also

Define the drag coefficient (Colburn as

\[ \frac{T}{T_s} = C_D u_s^2 = u_s^3 \]

where \( u_s = u(z = 10 \text{ meters}) \)

Thus \( u_s = \frac{u_s}{k} \ln \frac{1}{z_0} \)

\[ u_s^3 = u_s \left[ \frac{1}{k} \ln \frac{1}{z_0} \right] \]

so \( (C_D)n = \frac{1}{k^2 \ln^2 17z_0} \)
and mixing ratio are just in excess of moist adiabatic. A significant feature of the cloudy region cloud layer is that the moisture lapse rate in the upper two-thirds averages only 28% that of the lower third, which may thus be regarded as an extended (about 400m vertical extent) transition zone between unsaturated and saturated convective regimes. The trade inversion is defined by an abrupt increase in moisture lapse rate by a factor of about five and a concomitant stabilization in temperature lapse rate.

In clear areas (Figure 7B) the mixed layer is 75 m shallower than in cloudy areas and has a 15% steeper moisture lapse rate and 6-7% steeper temperature and virtual temperature lapse rates. The main distinction between clear and cloudy soundings lies, however, in the presence of a pronounced shallow transition layer between the mixed and cloud layers. This zone was studied by Bunker et al. in (4) who called it the "stable layer". Re-examination of the data in the light of present knowledge has suggested some revisions in nomenclature and interpretation. The "transition layer" will be defined as a narrow stratum just below or above the height of cumulus base in which the moisture lapse rate is not less than