united nations
educational, scientific
and cultural
organization

international atomic
energy agency

the
abdus salam
international centre for theoretical physics

SMR.1498 - 10

SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

31 March - 8 April 2003

FLUX COMPACTIFICATIONS IN STRING THEORY

Lecture 1

S. TRIVEDI
Theoretical Physics Group
Tata Institute of Fundamental Research
Mumbai 400 005
INDIA

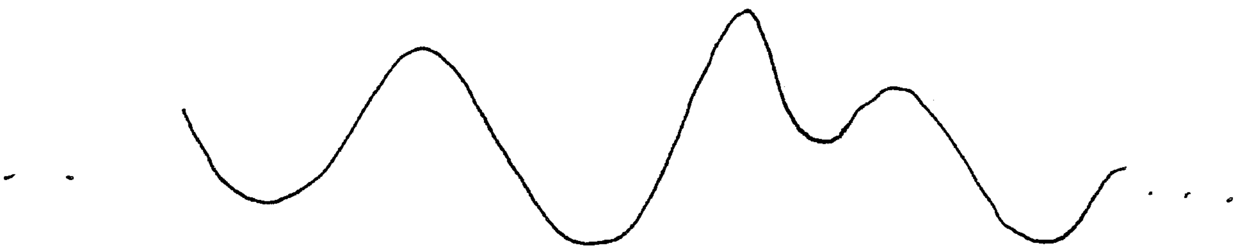
FLUX COMPACTIFICATIONS IN STRING THEORY.

- ESSENTIAL IDEA: Curl up extra directions while exciting various ~~fluxes~~ fluxes in internal directions.

- MOTIVATION:

1. IMPORTANT CLASS OF $N=1$ ($\& N>1$) SUSY STRING VACUA. PREVIOUS STUDY OF $N>1$ VACUA HAS BEEN VERY REWARDING. MOTIVATES STUDY OF THESE $N=1$ VACUA. (ALSO TIED TO $N=1$ SUSY GAUGE THEORIES & DUALS - ANOTHER THEME OF THIS WORKSHOP).

2. ALREADY FLUX VACUA HAVE HELPED CLARIFY ~~THE~~ SOME FEATURES ABOUT THE LANDSCAPE OF STRING VACUA IN GENERAL.
   E.g: WE EXPECT MANY, MANY, VACUA, WITH BROKEN SUPERSYMMETRY, ~~AND~~ NO MODULI, AND VARYING COSMOLOGICAL CONSTANTS.



VACUUM SELECTION PROBLEM.

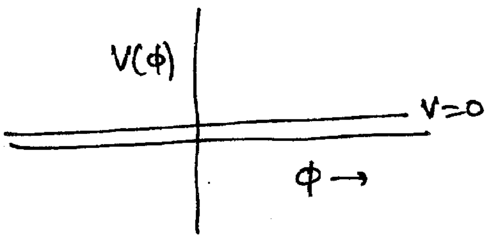
1. HOW DO WE FIND THE STANDARD MODEL VACUUM?
2. WHY DOES NATURE SELECT THE S.M. VACUUM?

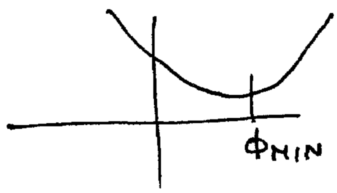
AS WE BEGIN TO FACE UP TO THIS KEY QUESTION IN STRING PHENOMENOLOGY, FLUX COMPACTIFICATIONS ARE LIKELY TO PLAY AN IMPORTANT ROLE. FLUXES, PROVIDE ADDITIONAL PARAMETERS, WHICH OFTEN RESULT IN BETTER CONTROL. THIS IS USEFUL AS WE BEGIN TO EXPLORE PROPERTIES ABOUT THE LANDSCAPE & BEGIN TO FACE UP TO QUESTIONS (1) & (2) ABOVE.

(3) ADDITIONAL MOTIVATION OF MORE SPECIFIC KIND:

a) MODULI PROBLEM: TYPICAL STRING VACCUA HAVE MANY FLAT DIRECTIONS. WE NEED TO UNDERSTAND

i) HOW THEY ARE LIFTED & WHERE MINIMA LIE.





WE WILL STUDY THIS IN LECTURE I & II MORE.

ii) MAKE MASS OF FLUCTUATION ABOUT MINIMUM HEAVY $m_{moduli} \geq 10$ TeV TO AVOID COSMOLOGICAL PROBLEMS.

b) WARPED COMPACTIFICATIONS: PROVIDE NEW (DUAL?) WAY TO SOLVE HIERARCHY PROBLEM OF THE STANDARD MODEL (RANDALL - SUNDRUM MODELS). NATURAL WAY TO IMPLEMENT THE~~[illegible]~~S IN STRING THEORY INVOLVES FLUX.

c) COSMOLOGY: CAN STRING THEORY GIVE RISE TO INTERESTING COSMOLOGIES?

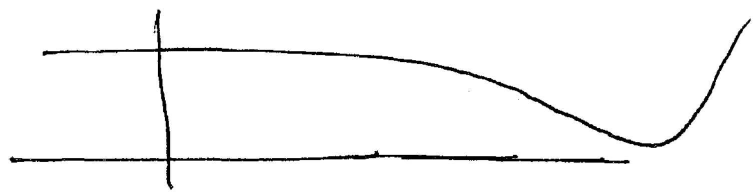
E.g:

1. de Sitter Space (with COSMOLOGICAL CONSTANT $\lambda^4 \sim M_{Pl}^2 H^2$, $H \sim$ HUBBLE SCALE)

   VARIOUS NO-GO THEOREMS. THESE CAN BE AVOIDED AS WE WILL SEE IN LAST LECTURE.

2. INFLATION? SO FAR NO MODEL TIED TO THE MODULI PROBLEM.

NEED POTENTIAL:



TO GET SLOW-ROLL INFLATION.

Partial, Roadmap to (huge) literature:

① Early Attempts: Polchinski & Strominger hep-th/9510227
Michelson: hep-th/9610151.

② Related Work on M-theory 4-folds: Becker & Becker: 9605053.
Gukov Vafa Witten: 9906070.

③ Early Work on IIB with Flux: Dasgupta, Rajesh, Sethi: 9908088
~~Holography & compactification~~

④ As WARPED COMPACTIFICATIONS: i) H. VERLINDE: 9906182
(ii) C. CHAN P. Paul & H. Verlinde: 0003236.
(iii) Greene, Schalm, Shiu: 0004103.
(iv)

⑤ ALTERNATE APPROACH AS GAUGED SUPERGRAVITY COMPACTIFICATIONS:
a) L. Andrianopoli, R. D'Auria, S. Ferrara & M-A. Lledo: 0203206
b) R.D. Auria et-al. 0206241.
c) Ferrara et-al. 0207135.
d) 0207255 0211027 0211116
e) 020624, 0212141 ... 0211027 0207135

(Also f) Dabholkar & Hervey: "Duality Twists Orbifolds Fluxes"
hep-th/0210209.)

g) Kaloper & Myers: hep-th/9901045

⑥ Recent Related Work:
i) J-Louis & A. Micu: "Type II Theories compactified on CY_3 ..."
hep-th/0202168

ii) G-Curio, A. Klemm, D Lust, S. Theisen: hep-th/0012213.

8. K. Becker & K. Dasgupta : hep-th/0209077.
Becker, Becker, Dasgupta ; Green :

Curio, Louis, Micu, Waldram : hep-th/0211102.

Cardoso, Curio, G. Dall'Agata, D. Lust et al. 0211118.

Frey, Polchinski : hep-th/0201029.

OUTLINE OF LECTURES:

SETTING: IIB STRING COMPACTIFICATIONS WITH FLUX
(MOSTLY CONST DILATON - AXION, BUT WILL MENTION MORE GENERAL CASE OF F-THEORY AS WE GO ALONG).

Lecture 1:

a) General Considerations imposed by EOM
b) Supersymmetry Conditions

Lecture 2:

A Simple Example: T^6/Z_2 with Flux

Lecture 3:

- Dual Descriptions of Simple Example
- Connection to other (earlier) Constructions

Lecture 4:

- dS vacua in String Theory & Related Discussion.

GENERAL CONSIDERATIONS.

Closely follow: Giddings, Kachru & Polchinski : 0105097

IIB Sugra: g_{MN} $B_{(2)}$, $C_{(2)}$ $C_{(4)}$, φ, $C_{(0)}$.

φ ↓ dilaton, $C_{(0)}$ ↓ axion

$$\Phi = C_0 + i e^{-\varphi}$$

$$G_{(3)} = F_{(3)} - \Phi H_{(3)}.$$

$$H_{(3)} = dB_{(2)}$$

$$F_{(3)} = dC_{(2)}.$$

$$\boxed{\tilde{F}_5 = dC_{(4)} - \frac{1}{2} C_{(2)} \wedge H_{(3)} + \frac{1}{2} B_{(2)} \wedge F_{(3)}.} \quad -(1)$$

(Gauge invariant 5 form).

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ R - \frac{\partial_\mu \Phi \partial_\mu \bar{\Phi}}{2(\operatorname{Im}\Phi)^2} - \frac{G_{(3)} \cdot \bar{G}_{(3)}}{12 \operatorname{Im}\Phi} - \frac{\tilde{F}_5^2}{4.5!} \right\}.$$

$$+ \frac{1}{8i\kappa_{10}^2} \int C_{(4)} \wedge \frac{G_{(3)} \wedge \bar{G}_{(3)}}{\operatorname{Im}\tau}$$

$$+ \quad S_{loc}.$$

↓ localised sources will say more soon.

(10 dim Einstein frame)

In addition impose:

$$\tilde{F}_5 = *_{10} \tilde{F}_5$$

$$\tilde{F}_5 = \underbrace{\epsilon_{M_1 \dots M_{10}}}_{\text{Tensor}} \tilde{F}_5^{M_6 \dots M_{10}}.$$

- Also from ①.

$$\boxed{d\tilde{F}_5 = -F_{(3)} \wedge H_{(3)} + \text{Source terms}} \quad (2)$$

- Compactifications which preserve 3+1 dim Poincaré Symmetry:

$$\boxed{ds_{10}^2 = e^{2A(y)} \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{-2A(y)} \underbrace{\sum_{m,n=1}^{6} \tilde{g}_{mn}\, dy^m dy^n}_{M_6}.} \quad (3)$$

x^μ : Spacetime
y^m : internal coordinates.
$M = \mu, m$: all coordinates.

- WARPED FORM due to $e^{2A(y)}$ prefactor.

. IN ADDITION : $G_{(3)}$ excited along y^m directions
: $\tilde{F}_5$ excited.

Also $\Phi(y)$: varying dilaton - axion allowed.
Mostly in this & next lecture we consider case of constant Φ, varying Φ related to F-theory vacua is easy to incorporate. We will make comments for this case also as we go along.

In particular $\Phi(y)$ need not be single valued. Monodromies Under $SL(2,\mathbb{Z})$:

$$\Phi \to \frac{a\Phi + b}{c\Phi + d}$$
$$G_{(3)} \to G_{(3)} / (c\Phi + d)$$

allowed.

- Sources we consider are D3/O3 branes/planes.
  D7/O7's wrapping 4-cycles.

(No 5-branes/planes here. Including them an interesting open problem).

- EINSTEIN EQUATIONS:

TAKE: $\tilde{F}_5 = (1+*_{10})\left[d\alpha \wedge dx^0 \wedge \cdots \wedge dx^3 \right]$.

(For now can replace $d\alpha$ by a one form if you like).

$$\boxed{R_{MN} = \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T^M{}_\mu \right)} \qquad (4)$$

$M, N = \mu, \nu$:

$$\boxed{R_{\mu\nu} = -g_{\mu\nu}\left(\frac{G_{mnp}\bar{G}^{mnp}}{48\, \mathrm{Im}\Phi} + \frac{e^{-8A}}{4} \partial_m \alpha\, \partial^m \alpha \right) + \kappa_{10}^2 \left(T_{\mu\nu}^{loc} - \frac{1}{8} g_{\mu\nu} T^{M\,loc}_M \right)}$$

$$T_{MN}^{loc} = \frac{-2}{\sqrt{-g}} \frac{\delta S_{loc.}}{\delta g^{MN}}. \qquad (5)$$

From (3)

$$R_{\mu\nu} = -\eta_{\mu\nu}\, e^{4A}\, \tilde{\nabla}^2 A \qquad \left(\tilde{\nabla}^2 A = \frac{1}{\sqrt{\tilde{g}}} \partial_m \left(\sqrt{\tilde{g}}\, \tilde{g}^{mn} \partial_n A \right) \right)$$

(5) Gives:

$$\boxed{\tilde{\nabla}^2 A = e^{-2A} \left(\frac{G_{mnp}\bar{G}^{mnp}}{48\, \mathrm{Im}\Phi} + \frac{e^{-8A}}{4} \partial_m \alpha\, \partial^m \alpha. \right) - \frac{\kappa_{10}^2}{8} e^{-2A} \left(T^\mu_\mu - T^m_m \right)} \qquad (6)$$

- Without localised sources, RHS must vanish pointwise due to its positivity, for a compactification.

This means $G_3 = 0$.

$$\partial_m \alpha = 0 \quad \Rightarrow \quad \hat{F}_5 = 0.$$

And so: $A = \text{const}.$

So no flux can be turned on & no warping allowed.

B. de Wit, D.J. Smit, N.D. Hari Das.

- ~~With~~ localised sources: Need negative tension sources.

$$S_{loc} = - \int_{R^4 \times \Sigma} d^{p+1}\xi \, \sqrt{-g} \, T_p \; + \; \mu_p \int_{R^4 \times \Sigma} C_{p+1}.$$

$$T^{loc}_{\mu\nu} = - T_p \, e^{2A} \, \eta_{\mu\nu} \, \delta(\Sigma).$$

$$T^{loc}_{mn} = - T_p \, \delta(\Sigma) \, \underbrace{\Pi^{\Sigma}_{mn}}_{\text{projector along } \Sigma}.$$

$$T^{\mu}_{\mu} - T^{m}_{m} = T_p \, \delta(\Sigma) \, [p-7].$$

(if tension positive need $p > 7$ Not possible in IIB theory).

$p = 7$ does not contribute.
So necessarily need O_3 planes.
Or 7 branes | planes wrapping 4-cycles with induced negative 3-brane tension: due to couplings:

$$\frac{\mu(7)}{96} (2\pi\alpha') \int C_4 \wedge \mathrm{Tr}(R_2 \wedge R_2)$$

$$\frac{\mu(7)}{96} (2\pi\alpha') \int \sqrt{g}\, d^4x\, \mathrm{Tr}(R_2 \wedge * R_2)$$

[In general in F-theory

$$Q_3^{D7} = -\frac{\chi(X_4)}{24} \longrightarrow \text{Euler Charac. of 4-fold}$$].

- One more comment. If we were interested in dS space vacua (6) would have an addition term ~~of~~ on RHS. of form:

$$\delta \mathrm{RHS} = \Lambda \eta_{\mu\nu} e^{-4A}.$$

So WITHOUT LOCALISED NEGATIVE TENSION SOURCES No dS COMPACTIFICATIONS Allowed.

(GIBBONS | MALDACENA - NUNEZ)

WE WILL REVISIT THIS THEOREM, ~~WITH~~ ALLOWING FOR O3 PLANES, SOON.

EQUATIONS FROM VARYING C_4:

$$d\tilde{F}_5 = H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 T_3 \rho_3^{loc.}$$

$$\boxed{\int H_3 \wedge F_{(3)} + 2\kappa_{10}^2 T_3 \int \rho_3^{loc} = 0 .} \qquad (7)$$

TOTAL D3-brane Charge must vanish: Gauss law in compact space.

$d\tilde{F}_5 = d * \tilde{F}_5$ Gives:

$$\boxed{\tilde{\nabla}^2 \alpha = \frac{i e^{2A} G_{mnp} *_6 \bar{G}^{mnp}}{12 \, \mathrm{Im}\,\Phi} + 2 e^{-6A} \partial_m \alpha \, \partial^m e^{4A} + 2\kappa_{10}^2 e^{2A} T_3 \rho_3^{loc.}} \qquad (7A)$$

(Subtleity in getting normalization of last term on RHS right).

$$\boxed{\begin{aligned} \tilde{\nabla}^2 (e^{4A} - \alpha) = {} & \frac{e^{2A}}{6\,\mathrm{Im}\,\tau} \left| i G_{(3)} - *_6 G_{(3)} \right|^2 + e^{-6A} \left| \partial(e^{4A} - e) \right|^2 + \\ & 2\kappa_{10}^2 e^{2A} \left[\tfrac{1}{4} \left(T^m_m - T^\mu_\mu \right) - T_3 \rho_3 \right]^{loc.} \end{aligned}} \qquad (8)$$

If sources are BPS equality is saturated for last term on RHS & it vanishes.

(True for D3 | O3 or D7/O7 wrapping susy 4-cycles).

Gives:

$$\boxed{*_6 G_{(3)} = i G_{(3)}} \quad (9)$$ IMAGINARY SELF DUAL (ISD) CONDITION.

$$\boxed{e^{4A} = \alpha} \quad (10)$$

[Question: what happens if BPS equality is not saturated & last term is positive?]

- OTHER EQUATIONS:

(1) $F_{(3)}$, $H_{(3)}$: Closed & Harmonic
$dF_{(3)} = d*F_{(3)} = 0$.

(2) $g_{mn}(y)$ $\Phi(y)$, solve EOM without flux in presence of 7-branes | planes, but with no 3-brane | plane sources.

THIS MEANS $\tilde{M}_6$, Φ : ~~Satisfy~~ F-THEORY COMPACTIFICATION

(e.g. with susy CY_4 elliptically fibred).

If Φ: Const $\tilde{M}_6$: Ricci flat. (upto orientifolding).
(e.g. CY_3 Orientifold).

SUMMARY OF REQUIRED CONDITIONS FOR FLUX COMPACTIFICATION.

1. Negative Tension O_3 planes are needed.
   (or 7-brane | planes with negative 3-brane induced charge).

2. $(\tilde{g}_{mn}, \Phi)$: Solve equations in absence of flux & 3-brane | plane sources.

3. $F_{(3)}$, $H_{(3)}$ Closed & Harmonic

4. TOTAL 3-BRANE CHARGE VANISHES.

THEN.

a) $G_{(3)}$: IMAGINARY SELF DUAL.

b) $e^{4A} = \alpha$.

AND DETERMINED BY 7A.

- COMMENT ON dS VACUA:
  RHS OF 8 GETS ADDITIONAL TERM.

$$\delta \text{RHS} = \Lambda e^{4A}$$

So EVEN WITH BPS O3 PLANES (& 7 brane | Planes, D3 branes) NO dS VACUA POSSIBLE.
(5-PLANES COULD CHANGE THIS, AT LEAST NOT DIRECTLY EXCLUDED)

BECKER & BECKER.
GRANA &
POLCHINSKI

SUPERSYMMETRY.

- IIB: 2 MAJORANA-WEYL SPINORS OF SAME CHIRALITY SPECIFY THE SUSY TRANSFORMATIONS
  (ϵ_L, ϵ_R).

  $$\epsilon \equiv \epsilon_L + i\,\epsilon_R .$$

- ϵ: SATISFIES B-TYPE CONDITION:

$$\Gamma^4 = i\,\gamma^0\gamma^1\gamma^2\gamma^3$$
$$\Gamma^6 = i\,\gamma^{\hat{y}_1}\cdots\gamma^{\hat{y}_6} \qquad (\hat{y}_i : \text{tangent space indices})$$

THEN: $\epsilon = \zeta \otimes \chi_1$

$$\Gamma^4 \zeta = \zeta .$$
$$\Gamma^6 \chi_1 = -\chi_1 .$$

AS A RESULT TERMS IN SPINOR VARIATIONS PROPORTIONAL TO ϵ & ϵ^* ARE INDEPENDENT. CONSIDERABLE SIMPLIFICATION.

TERMS PROPORTIONAL TO ϵ:

1. $\delta\psi_\mu \doteq 0$ GIVES $\boxed{e^{4A} = \alpha}$

2. ~~$\Omega = 0$~~ $\delta\psi_m = 0$

GIVES: ~~DILATON~~ ~~MUST~~ ~~VARY~~

$$\frac{1}{k}\left(D_m - \frac{i}{2} Q_m\right)\epsilon + \frac{i}{480}\gamma^{M_1\cdots M_5} F_{M_1\cdots M_5}\,\gamma_m\,\epsilon = 0 .$$

$\hat{M}$: KAHLER MANIFOLD. $\Rightarrow$ A COMPLEX STRUCTURE

IF $Q_m = 0$ SO DILATON IS CONSTANT

$\hat{M}$: SU(3) HOLONOMY: CY_3.

<u>Supersymmetry</u>:

$$\delta\lambda^* = -\frac{i}{\kappa}\gamma^M P_M^* \varepsilon + \frac{i}{4} G^* \varepsilon^* .$$

$$\delta\psi_M = \frac{1}{\kappa}\left(D_M - \frac{i}{2} Q_M\right)\epsilon + \frac{i}{480}\gamma^{M_1\cdots M_5}\left(\frac{1}{4\kappa}\right)\tilde{F}_{M_1\cdots M_5}\gamma_M \varepsilon - \frac{1}{16}\Gamma_M G\epsilon^* - \frac{1}{8} G\Gamma_M \epsilon$$

$$G = \frac{1}{6}\tilde{G}_{(3)MNP}\gamma^{MNP}.$$

$$B = \frac{1+i\Phi}{1-i\Phi} \qquad f^2 = 1 - BB^* .$$

$$P_M = f^2 \partial_M B \qquad Q_M = f^2 \,\mathrm{Im}(B\partial_\mu B^*) .$$

$$\tilde{G}_3 = \frac{1}{\kappa}\frac{i g e^{i\theta}}{\sqrt{\Phi_2}} \cdot \frac{F_{(3)} - \Phi H_{(3)}}{\sqrt{\Phi_2}} . \qquad e^{i\theta} = \left(\frac{1+i\Phi^*}{1-i\Phi}\right)^{1/2}$$

Spinors of type B: $\quad \varepsilon = \zeta \otimes \chi_1$

$$\Gamma^4 \zeta = \zeta \qquad \Gamma^4 = i\gamma^0\gamma^1\gamma^2\gamma^3 .$$

$$\Gamma^6 \chi_i = -\chi_i \qquad \Gamma^6 = i\gamma^{y_1}\cdots\gamma^{y_6} .$$

(defn. of spinors:

$$\delta\psi_\mu = \kappa^{-1}\partial_\mu \epsilon - \frac{1}{8}\gamma_\mu \gamma^m\left(\kappa^{-1}\partial_m \ln Z + \kappa^{-1}\Gamma^4 \partial_m h\right)\epsilon$$

$$h = -\frac{1}{Z}$$

$$ds^2 = Z^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{1/2} d\tilde{s}_6^2 .$$

3 $\delta\lambda = 0$ GIVES:

a) $\boxed{\text{DILATON VARIES HOLOMORPHICALLY}}$

(WITH RESPECT TO COMPLEX STRUCTURE OF $\tilde{M}$)

TERMS PROPORTIONAL TO ϵ^*:

$G_{(3)}$: OF TYPE $(2,1)$ & PRIMITIVE.

(2,1): $\boxed{G_{(3)\, ij\bar{k}} \neq 0}$ 9 ALL OTHER COMPONENTS VANISH.

$\boxed{G_{(3)} \wedge J = 0 .}$ 10 ~~ALSO~~: PRIMITIVE.

$J = i g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}}$ KAHLER TWO-FORM.

10 CAN ALSO BE EXPRESSED AS:

$$\boxed{G_{(3)\, ij\bar{k}}\, g^{j\bar{k}} = 0 .}$$

(FINALLY BIANCHI IDENTITY OF $\tilde{F}_5$ IMPLIES:

$$\nabla^2 \alpha = \underbrace{\frac{i}{12\, \mathrm{Im}\phi}\, G_{mnp} *_6 \bar{G}^{mnp}} + 2\kappa_{10}^2\, T_3\, \rho_3^{loc} .$$

POSITIVE IF $G_{(3)}$ OF TYPE $(2,1)$ & PRIMITIVE (or even ISD)

SO $Q_3^{loc} < 0$ NEEDED.)

- PRIMITIVITY AUTOMATIC IF $\tilde{M}_6$: CY_3

SINCE NO NON-TRIVIAL ONE-FORMS

SUMMARY OF SPINOR CONDITIONS:

1. $\tilde{M}$: KAHLER.
   (IF Φ CONST $\tilde{M}$: CALABI-YAU ORINTIFOLD)

   $(\tilde{M}, \Phi)$: F THEORY COMPACTIFICATIONS.

2. $G_{(3)} \equiv F_{(3)} - \Phi H_{(3)}$.

   a) TYPE $(2,1)$.
   b) PRIMITIVE.

ADDITION:

3. $F_{(3)}, H_{(3)}$ CLOSED, HARMONIC
   $\tilde{F}_5$ BIANCHI EQU. MET.

SUPERPOTENTIAL. (GUKOV, VAFA, WITTEN)

$$W = \int G_{(3)} \wedge \Omega_{(3)}.$$

$\Omega_{(3)}$: HOLOMORPHIC $(3,0)$ FORM ON $\tilde{M}$.

[FTHEORY: $W = \int G_{(4)} \wedge \Omega_{(4)}$].

Setting $W = 0$.

$\frac{\delta W}{\delta \tau^{ij}} = 0$, τ^{ij} : COMPLEX STR. MODULI.

$\frac{\delta W}{\delta \Phi} = 0$. Φ : DIL-AXION.

IMPLIES. $G_{(3)}$: $(2,1)$ TYPE.

$F_{(3)}, \quad H_{(3)} \quad \in \quad H^3(\tilde{M})$

So we consider them fixed in carrying out the above variation.

Next,

$W = 0 \quad \Rightarrow \quad \boxed{\text{No } (3,0) \text{ term}}$.

$\frac{\partial W}{\partial \tau^{ij}}$: $\frac{\partial \Omega}{\partial \tau^{ij}} = C_{ij}\,\Omega + \underbrace{\chi_{ij}}_{(2,1)\text{ form}}$

$\frac{\partial \Omega}{\partial \tau^{ij}}$ ↓ Most general

(2,1) form ↓ complete basis of (2,1) forms.

$\Rightarrow \frac{\partial W}{\partial \tau^{ij}} = \int G_{(3)} \wedge \chi_i = 0 \quad \Rightarrow \quad \boxed{\text{No } (1,2) \text{ term}}$

<u>Finally</u>: $\quad \frac{\partial W}{\partial \phi} = 0$

$$\Rightarrow \quad \frac{\partial}{\partial \phi} \int \left[F_{(3)} - \phi H_{(3)} \right] \wedge \Omega = 0$$

$$\Rightarrow \quad \int H_{(3)} \wedge \Omega = 0 .$$

$$\Rightarrow \quad \int \left(\frac{G - \bar{G}}{2\,\mathrm{Im}\,\Phi} \right) \wedge \Omega = 0 \quad \Rightarrow \quad \boxed{\text{No } (0,3) \text{ term}}$$

[<u>Aside</u>: ISD ALLOWS: (2,1) Primitive + (3,0)
+ (1,2) of type $J \wedge \omega \leftarrow (0,1)$].

- NOTE: IF $\hat{M}$ not CY_3 or (F-Theory Compac: CY_4) but say T^6 then PRIMITIVITY HAS TO BE IMPOSED IN ADDITION.

<u>IMPORTANT</u>:

Generically Susy Broken because we have one more condition than variables:

$$W = 0 \,.$$

$$\frac{\partial W}{\partial \phi} = 0 \,.$$

$$\frac{\partial W}{\partial \tau^{i}\text{'s}} = 0 \,.$$

MODULI - STABILISATION:

- ISD CONDITIONS:

$$*_6 G_{(3)} = i\, G_{(3)} .$$

$$\sqrt{g}\ [m_1 \dots m_6]\ g^{m_4 n_4} g^{m_5 n_5} g^{m_6 n_6} [G_{(3)}]_{n_4 n_5 n_6} = i\, [G_{(3)}]_{m_1 m_2 m_3} .$$

THIS INVOLVES METRIC (& Φ).
SO SOME COMPONENTS OF METRIC WILL BE FIXED.

$$g_{mn} \to \lambda\, g_{mn} .$$
$$\sqrt{g} \to \lambda^3 .$$
$$g^{mn} \to \lambda^{-1} g^{mn} .$$

So LHS does not change. Thus overall Volume not fixed.

- Susy Conditions:

$G_{(3)}$: (2,1) : ~~Primi~~ fixes All Complex Str. Moduli (one expects generically).
And Φ - dilaton/axion.

$G_{(3)}$: Primitive fixes some Kahler Moduli

$$J \wedge G = 0$$

$J \to \lambda J$: $\lambda J \wedge G = 0$.

SO NOT OVERALL VOLUME.

(Other moduli not coming from metric get eaten by gauge fields due to C-S - terms)

MASS OF LIFTED MODULI:

~~Potential~~

Moduli coming from metric have kinetic energy term:

$$\frac{1}{2k_{10}^2}\int\sqrt{g_{(4)}}\,d^4x\;v_6\,(\partial\phi)^2 .$$

Potential energy arises from terms in 10-dim theory of form

$$\frac{1}{2k_{10}^2}\int\frac{\sqrt{g_{(10)}}}{\mathrm{Im}\,\phi}\,G_{mnp}\,\bar{G}_{qrs}\,g^{mq}g^{nr}g^{ps}\,d^6y\,d^4x .$$

$$\frac{1}{2k_{10}^2}\int\frac{\sqrt{g_{(4)}}}{(\mathrm{Im}\,\phi)}\,\underbrace{\sqrt{g_{(6)}}\;G_{mnp}\,\bar{G}_{qrs}\,g^{mq}g^{nr}g^{ps}}_{\text{Independent of } V_6}\,d^6y\,d^4x .$$

So $\boxed{m^2 \sim \frac{\alpha'}{\sqrt{v_6}}}$

[STRICTLY SPEAKING WE HAVE TO INCORPORATE WARPING MORE CAREFULLY. HOWEVER IF WARPING SMALL (ie $e^{A}\sim$ ~~Const~~ 1 AT MOST PTS of $\tilde{M}_6$, THEN THE ABOVE A GOOD APPROX. ZERO MODES OF METRIC LOCALISED IN REGIONS WITH SMALL WARP].

SPECTRUM:

$m = \frac{1}{\sqrt{\alpha'}}$ ———— STRING MODES

$m = \frac{1}{R}$ ———— K.K MODES

$m = \frac{\alpha'}{R^3}$ ———— MODULI

$m \simeq 0$ ———— S. MODEL.

NOTE: SUGRA APPROX GOOD FOR LARGE V_6