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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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TACHYON DYNAMICS IN OPEN STRING THEORY

Lectures 1, 2 and 3

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1. Static properties of the tachyon effective action
2. Time dependent solutions
3. Effective field theory description.

classical open string (field) theory

4. Quantum effects

We shall begin by stating the results.

$$\hbar = c = \alpha' = 1 \quad \text{convention.}$$

D-p-brane spectrum:

IIB	IIA	Bosonic
BPS for p-odd $\mathcal{T}_p = \frac{1}{(2\pi)^p g_s}$	BPS for p-even $\mathcal{T}_p = \frac{1}{(2\pi)^p g_s}$	D-p-brane for every p $\mathcal{T}_p = \frac{1}{(2\pi)^p g_s}$ ↓ Has a tachy of $(\text{mass})^2 = -$ ↓ Described by
Non-BPS for p-even $\tilde{\mathcal{T}}_p = \frac{\sqrt{2}}{(2\pi)^p g_s}$ ↳ Has a tachyon of $(\text{mass})^2 = -\frac{1}{2}$	Non-BPS for p-odd $\tilde{\mathcal{T}}_p = \frac{\sqrt{2}}{(2\pi)^p g_s}$ ↳ Has a tachyon of $(\text{mass})^2 = -\frac{1}{2}$	

→ a real scalar T

Dp - D$\bar{p}$ system in IIA/IIB for p even/odd

→ Has two tachyons of $(\text{mass})^2 = -\frac{1}{2}$

→ described by a complex scalar field T.

Tree level tachyon effective action

$$S_{eff}(T, \cdots) = \int d^{p+1}x \, \mathcal{L}_{eff}(T, \cdots)$$

$\hookrightarrow$ massless fields

$\rightarrow$ obtained by integrating out all the heavy open string modes.

$\equiv$ eliminate the heavy fields by their equations of motion.

$V_{eff}(T)$: negative ~~of~~ $\mathcal{L}_{eff}(T)$ for

① T independent of $x^0, \cdots x^p$

② Massless fields set to zero.

- $V_{eff}(T)$ has a maximum at $T=0$ since T is tachyonic

- For non-BPS D-brane in IIA/IIB

$$S_{eff}(-T, \cdots) = S_{eff}(T, \cdots)$$

For BPS Dp-$D\bar{p}$ system in IIA/IIB

$$S_{eff}(e^{i\alpha} T, \cdots) = S_{eff}(T, \cdots)$$

Q. Does $V_{eff}(T)$ have a (local) minimum?

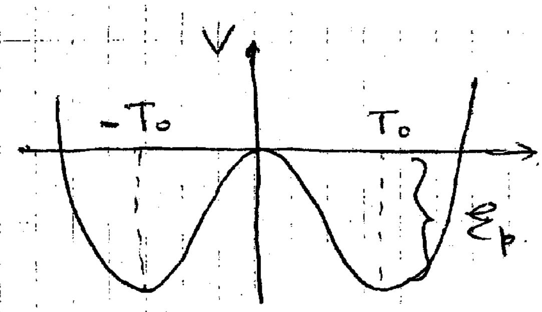
Results for IIA/IIB (Valid for non BPS Dp-brane as well as for Dp - $\bar{D}\bar{p}$ system)

1. $V(T)$ has a global ~~local~~ minimum at $|T| = T_0$, and

$$V(T_0) + \mathcal{E}_p = 0$$

↳ $\tilde{\mathcal{T}}_p$ for non-BPS Dp

$2\mathcal{T}_p$ for Dp - $\bar{D}\bar{p}$



⇒ Net energy density at $\pm T_0 = 0$

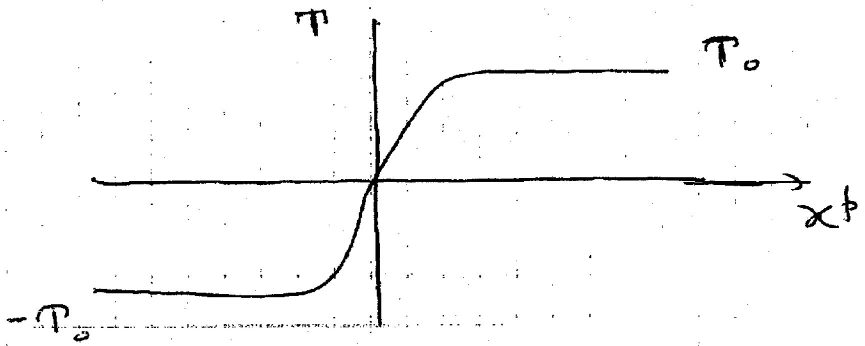
2. The configuration $|T| = T_0$ describes vacuum without any D-brane.

⇒ Perturbative quantization of the theory around $|T| = T_0$ should not give rise to any physical state representing open string

Counterintuitive for the point of view of a field theory as there are ∞ number of perturbative states around $T = 0$.

3. Various topological solitons in the tachyon effective field theory represent lower dimensional D-branes.

Example 1: Consider a kink on a non BPS D-p-brane.



$\rightsquigarrow$ describes a BPS D-$(p-1)$ brane

Example 2. Vortex on a Dp-$\overline{\text{D}p}$ pair.

$$T = f(\rho)\, e^{i\theta} \qquad \rho e^{i\theta} = x^{p-1} + i x^p$$

$$f(\infty) = T_0, \qquad f(0) = 0.$$

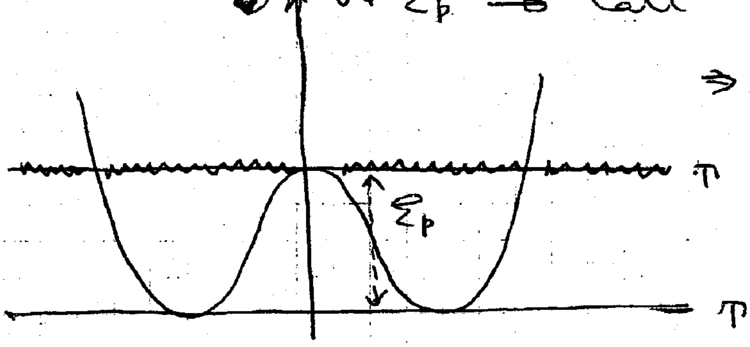
$\Rightarrow$ describes a BPS D-$(p-2)$ brane.

All D-branes in IIA/IIB can be described as a ~~topological~~ soliton on a certain number of non-BPS D9-brane ~~of~~ in IIA or D9-$\overline{\text{D9}}$ pairs in IIB

Time dependent solution:

$V + \mathcal{E}_p$ → call this $V(T)$.

⇒ $V(0) = \mathcal{E}_p$



What happens if we displace T a little from its maximum and let is roll down?

Take T to be spatially homogeneous for simplicity.

Ordinary scalar field motion

→ Characterized by 2-parameters

① initial position ② Initial velocity

Time translation : removes one parameter.

→ One parameter family of inequivalent solutions.

Can be taken to be the total energy density ρ

Case 1: $\rho < V(T=0)$

$\rightarrow$ Can choose $\partial_0 T = 0$, $T = \lambda$ at $x^0 = 0$

Case 2: $\rho > V(T=0)$

$\rightarrow$ Can choose $T=0$, $\partial_0 T = v$ at $x^0 = 0$.

General form of the energy momentum tensor:

$$T_{00} = \rho = \text{fixed} \qquad T_{i0} = 0$$

$$T_{ij} = p(x^0)\,\delta_{ij}$$

For ordinary scalar $p(x^0)$ oscillates about its average value 0 as the scalar oscillates about its minimum.

For tachyon on a non-BPS D-brane or Dp-D$\bar{p}$ brane pair we have a different behaviour of $p(x^0)$.

Case 1. $\rho < V(0) \equiv \mathcal{E}_p$

Parametrize $\rho = \frac{1}{2}\mathcal{E}_p\,(1+\cos 2\pi\tilde{\lambda})$

↳ parameter.

↳ $\sim T\,(x^0=0)$

$$p(x^0) = -\mathcal{E}_p\left[\frac{1}{1+\sin^2(\pi\tilde{\lambda})\,e^{\sqrt{2}x^0}} + \frac{1}{1+\sin^2(\pi\tilde{\lambda})\,e^{-\sqrt{2}x^0}} - 1\right]$$

$p(x^0) \to 0$ as $x^0 \to \infty$

$\tilde{\lambda} = \frac{1}{2} \Rightarrow \rho = 0,\ p(x^0) = 0$

i.e. $T_{\mu\nu} = 0$.

→ should correspond to placing the tachyon at the minimum of $V(T)$.

Case 2. $\rho > V(0) \equiv \mathcal{E}_p$.

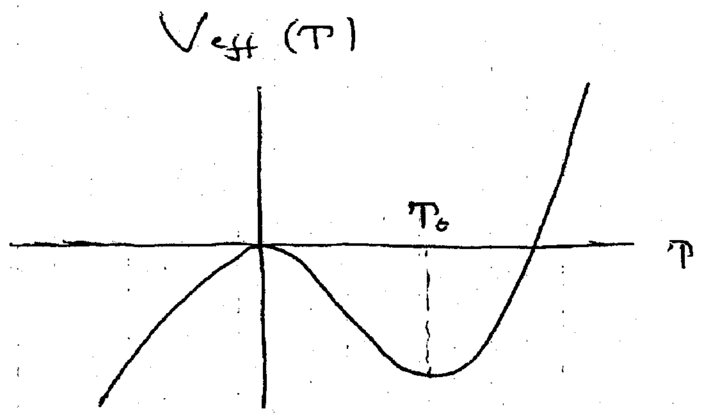
Parametrize $\rho = \frac{1}{2}\mathcal{E}_p\,(1+\cosh 2\pi\hat{\lambda})$

↳ $\sim \dot{T}\,(x^0=0)$

$$p(x^0) = -\mathcal{E}_p\left[\frac{1}{1+\sinh^2(\pi\hat{\lambda})\,e^{\sqrt{2}x^0}} + \frac{1}{1+\sinh^2(\pi\hat{\lambda})\,e^{-\sqrt{2}x^0}} - 1\right]$$

Again $p(x^0) \to 0$ as $x^0 \to \infty$.

Tachyon effective potential in bosonic string theory:



$V(T_0) + \mathcal{T}_p = 0$ Redefine $V(T) + \mathcal{T}_p$ as $V(T)$

$\Rightarrow$ Total energy density $= 0$ at $T = T_0$.

Note: the potential is unbounded from below for negative T.

$\Rightarrow$ $T = T_0$ is only a local minimum.

Q. What happens if we push the tachyon away from the maximum and let it roll.

As in the case of superstring, we can consider 2 cases

① Energy density $\rho < V(T=0) \equiv \mathcal{T}_p$ ② $\rho > \mathcal{T}_p$.

Result:

① $\rho < \mathcal{T}_p$.

Parametrize: $\rho = \frac{1}{2}\mathcal{T}_p\,(1+\cos(2\pi\tilde{\lambda}))$.

$$p(x^0) = -\mathcal{T}_p\left[\frac{1}{1+\sin(\pi\tilde{\lambda})\,e^{x^0}} + \frac{1}{1+\sin(\pi\tilde{\lambda})\,e^{-x^0}} - 1\right]$$

↳ pressure

Note: • For $\tilde{\lambda} > 0$, $p(x^0) \to 0$ for $x^0 \to \infty$

↳ Corresponds to displacing T towards its local minimum.

• For $\tilde{\lambda} < 0$, $p(x^0)$ blows up at

$$e^{x^0} = -\frac{1}{\sin(\pi\tilde{\lambda})}$$

→ Corresponds to pushing T to the side where the potential is unbounded from below.

• For $\tilde{\lambda} = \frac{1}{2}$, $\rho = 0$, $p(x^0) = 0$.

→ Corresponds to placing T at its minimum T_0.

(2) $\rho > \mathcal{T}_p$

Parametrize:

$$\rho = \frac{1}{2}\,\mathcal{T}_p\,(1+\cosh(2\pi\tilde{\lambda}))$$

$$p(x^0) = \mathcal{T}_p\left[\frac{1}{1+\sinh(\pi\tilde{\lambda})\,e^{x^0}} + \frac{1}{1-\sinh(\pi\tilde{\lambda})\,e^{-x^0}} - 1\right]$$

(Valid for $|\sinh(\pi\tilde{\lambda})| < 1$)

For $\tilde{\lambda} > 0$, $p(x^0) \to 0$ as $x^0 \to \infty$.

For $\tilde{\lambda} < 0$, $p(x^0)$ hits a singularity at $\exp(x^0) = -\frac{1}{\sinh(\pi\tilde{\lambda})}$

$\Rightarrow$ Corresponds to pushing the tachyon ~~towards~~ in the direction where the potential is unbounded from below.

Derivation of the results: Uses well-known correspondence between solutions of classical equations of motion in open string theory and boundary CFT (BCFT)

Strategy: Construct appropriate BCFT to describe various classical solutions

We shall illustrate this in the context of time dependent rolling tachyon solution in bosonic string theory.

$S_{eff}(T)$: Lorentz invariant

Suppose it has a classical solution

$$T = f(x)$$

$\hookrightarrow$ space-like coordinate

Then $T = f(ix^0)$ is also a solution

$\hookrightarrow$ time coordinate

Thus in order to construct a one parameter $(\tilde{\lambda})$ family of rolling tachyon solution, we need to construct a one parameter family of euclidean solution:

$$T = f(x; \tilde{\lambda})$$

Q. How do we choose $f(x; \tilde{\lambda})$?

Tachyon equation of motion near $T=0$

$$\partial_0^2 T = T + O(T^2) \qquad (\text{for } m^2 = -1).$$

$$\Rightarrow \quad T = A \cosh(x^0) + B \sinh(x^0).$$

Focus on case I: $\rho < V(T=0)$

Initial condition: $T=\lambda$, $\partial_0 T = 0$ at $x^0 = 0$

$$\Rightarrow T = \tilde{\lambda} \cosh(x^0) + O(\tilde{\lambda}^2) \quad ; \quad \lambda = \tilde{\lambda} + O(\tilde{\lambda}^2)$$

Obtained by $x \to i x^0$ replacement in

$$T = \tilde{\lambda} \cos(x) + O(\tilde{\lambda}^2)$$

Thus $f(x;\tilde{\lambda}) \to \tilde{\lambda} \cos x$ for small $\tilde{\lambda}$.

Corresponds to perturbing the world-sheet theory by

$$\tilde{\lambda} \int dt \, \underbrace{\cos(X(t))}_{\text{dimension 1 operator}}$$

t: Coordinate labelling world-sheet boundary

This deformation is known to be exactly marginal.

$\Rightarrow$ One parameter family of BCFT labelled by $\tilde{\lambda}$ $\leftrightarrow$ the solution $T = f(x;\tilde{\lambda})$

Rolling tachyon solution: $T = f(ix^0; \tilde{\lambda})$

→ corresponds to deformation of the world sheet CFT by

$$\tilde{\lambda} \int dt \, \cosh(X^0(t)).$$

Computation of $T_{\mu\nu}$

Given any BCFT describing a D-brane we have an associated boundary state
↓
A state of ghost number 3 in closed string state space containing information about what kind of source the D-brane produces for various closed string fields.

In particular in contains information about the source of the gravity field

$\Rightarrow T_{\mu\nu}$.

For $\tilde{\lambda} \int \cos X(t)$ perturbation:

$$T_{xx} = -\frac{1}{2} \mathcal{T}_p \left(1 + \cos(2\pi\tilde{\lambda})\right), \quad T_{xi} = 0 \quad i = 1, \cdots 25$$

$$T_{ij}(x) = \mathcal{T}_p \, F(x) \, \delta_{ij}$$

$$F(x) = 1 + \sum_{n=0}^{\infty} (-1)^n \sin^n(\tilde{\lambda}\pi)\,(e^{inx} + e^{-inx})$$

$$x \to i x^0$$

$$\Rightarrow \quad T_{00} = \frac{1}{2}\mathcal{T}_p\,(1 + \cos 2\pi\tilde{\lambda})$$

$$T_{0i} = 0$$

$$T_{ij}(x) = \mathcal{T}_p \quad F(ix^0) \quad \delta_{ij}$$

$$F(ix^0) = 1 + \sum_{n=0}^{\infty} (-1)^n \sin^n(\tilde{\lambda}\pi)\,(e^{nx^0} + e^{-nx^0})$$

$$= \frac{1}{1 + \sin(\tilde{\lambda}\pi)\,e^{x^0}} + \frac{1}{1 + \sin(\tilde{\lambda}\pi)\,e^{-x^0}} - 1$$

$\rightarrow$ Gives the result stated earlier.

Similar analysis holds for IIA/IIB string theory

References:

Time independent solutions: hep-th/9904207 (Review).

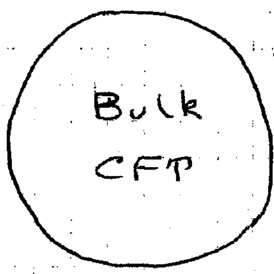
Time dependent solutions: hep-th/0203211, 0203265

Boundary state analysis (D-25-brane in bosonic string)

Consider a solution of the classical open string field equations on a D-brane

$\Rightarrow$ A boundary CFT

Unit disk D



$\to$ Conformally invariant (Boundary conditions + Boundary interactions)

V_c : A vertex operator of ghost no. 3 in the bulk CFT

$\leftrightarrow$ $|V_c\rangle = V_c(0)\,|0\rangle$ is a state of ghost no. 3 in closed string state space.

In general

$$\langle V_c(0)\rangle_D \neq 0$$

Boundary state $|B\rangle$: A ghost no. 3 state in closed string state space such that

$$\langle V_c|B\rangle = \langle V_c(0)\rangle_D$$

$\Rightarrow$ defines $|B\rangle$

A.2

Information contained in $|B\rangle$:

Physical closed string states:

$\rightarrow$ Ghost no. 2 states $|\psi_c\rangle$

Linearized equation of motion in absence of source terms:

$$(Q_B + \bar{Q}_B)\,|\psi_c\rangle = 0$$

We can expand $|\psi_c\rangle$ as:

$$|\psi_c\rangle = \int d^{26}k\,\Big[\,\tilde{\chi}(k)\,c_1\bar{c}_1\,|k\rangle$$

$$+ \tfrac{1}{2}\,\tilde{h}_{\mu\nu}(k)\,(\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1} + \bar{\alpha}^{\mu}_{-1}\alpha^{\nu}_{-1})\,c_1\bar{c}_1\,|k\rangle$$

$$+ \tfrac{1}{2}\,\tilde{b}_{\mu\nu}(k)\,(\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1} - \bar{\alpha}^{\mu}_{-1}\alpha^{\nu}_{-1})\,c_1\bar{c}_1\,|k\rangle$$

$$+ \tilde{\phi}(k)\,(c_1 c_{-1} - \bar{c}_1\bar{c}_{-1})\,|k\rangle + \cdots\Big]$$

$(Q_B + \bar{Q}_B)\,|\psi_c\rangle = 0 \Rightarrow$ linearized eq. of motion for $\tilde{\chi}, \tilde{h}_{\mu\nu}, \tilde{\phi}$ etc.

Fourier trs. of linearized graviton field

$\rightarrow$ A linear combination of $\tilde{h}_{\mu\nu}$ and $\tilde{\phi}$.

Presence of D-brane $\Rightarrow$ Source terms

$$(Q_B + \bar{Q}_B)\,|\psi_c\rangle = |B\rangle$$

Expand $|B\rangle$ as:

$$|B\rangle = \int d^{26}k\,[\,\tilde{S}(k)\,(c_0+\bar{c}_0)\,c_1\bar{c}_1\,|k\rangle$$

$$+\tfrac{1}{2}\tilde{A}_{\mu\nu}(k)\,(c_0+\bar{c}_0)\,c_1\bar{c}_1\,(\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}+\alpha^\nu_{-1}\tilde{\alpha}^\mu_{-1})\,|k\rangle$$

$$+\tilde{B}(k)\,(c_0+\bar{c}_0)\,(c_1c_{-1}-\bar{c}_1\bar{c}_{-1})\,|k\rangle + \cdots\,]$$

Expand both sides & compare

$\Rightarrow$ Source terms for $\tilde{\chi}$, $\tilde{h}_{\mu\nu}$, $\tilde{\Phi}$ etc.

$\Rightarrow$ Source terms for graviton.

$$\tilde{T}_{\mu\nu}(k) = (\tilde{A}_{\mu\nu} + \tilde{B}\,\eta_{\mu\nu}) \rightarrow \text{Fourier trs. of } T_{\mu\nu}(x)$$

For $\tilde{\lambda}\int dt\,\cos X(t)$ euclidean perturbation on D-25-brane (hep-th/9811237)

$$|B\rangle = \exp\Big(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\Big)\,(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle_{ghost}$$

$$\otimes\exp\Big(-\sum_{i,j=1}^{25}\sum_{n=1}^{\infty}\frac{1}{n}\,\delta_{ij}\,\alpha^i_{-n}\bar{\alpha}^j_{-n}\Big)\,|k\rangle_{c=25}$$

$$\otimes\Big[\sum_{m=-\infty}^{\infty}(-1)^m(\sin\tilde{\lambda}\pi)^m\,(1+\alpha^x_{-1}\bar{\alpha}^x_{-1})\,|k_x=m\rangle$$

$$-(1+\cos(2\tilde{\lambda}\pi))\,\alpha^x_{-1}\bar{\alpha}^x_{-1}|0\rangle + \text{terms involving } \alpha^x_{-n}, \text{ higher powers of } \alpha^x_{-1}\Big]$$

$$\Rightarrow \tilde{A}_{\mu\nu}(k),\ \tilde{B}(k) \Rightarrow T_{\mu\nu}(x)$$

Several properties of $S_{eff}(T)$ are counter intuitive from the point of view of a field theorist.

① Absence of perturbative states around $T=T_0$.

② $p(x^0) \to 0$ for large x^0 for rolling tachyon.

Is it possible to find a candidate for $S_{eff}(T)$ which captures the known properties, at least qualitatively?

hep-th/0003122, 0003221, 0204143, 0303057, 0303139

Choose:

$$S_{eff}(T) = -\int d^{p+1}x \; V_{eff}(T) \sqrt{-\det A}$$
$$\equiv V(T)$$

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \, \partial_\nu T \qquad \mu,\nu = 0, \dots p$$
$$\to -\det A = 1 + \eta^{\mu\nu} \partial_\mu T \, \partial_\nu T$$

[In presence of massless gauge fields A_μ and transverse scalars Y^I $(I = p+1, \dots 9)$

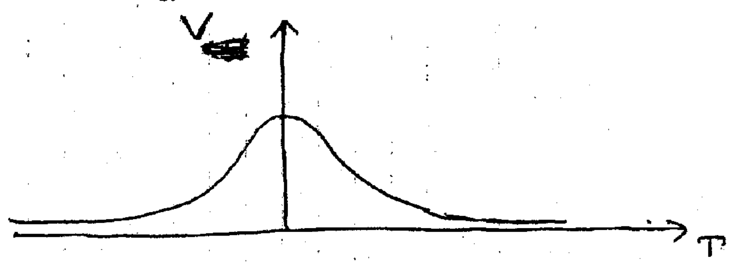
$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^I + F_{\mu\nu} + \partial_\mu T \, \partial_\nu T \;]$$

Energy momentum tensor

$$T_{\mu\nu} = -V(T) \, (A^{-1})^{\mu\nu}_S \sqrt{-\det A}$$

$\to$ Symmetric part

Choice of $V(T)$:



$T_0 = \infty$

Consider spatially homogeneous rolling tachyon

$$T = T(x^0).$$

$$\Rightarrow \quad T_{00} = V(T) / \sqrt{1-(\partial_0 T)^2}$$

$$T_{ij} = - \delta_{ij} \, V(T) \sqrt{1-(\partial_0 T)^2} \equiv p(x^0)\, \delta_{ij}$$

As $x^0 \to \infty$, $T \to \infty$, $V(T) \to 0$.

$\rho \equiv T_{00}$ = fixed by energy conservation

$\Rightarrow$ $\partial_0 T \to 1$ in this limit

$T_{ij} \to 0$ in this limit

Explains $p(x^0) \to 0$ result.

String calculation:

$$T_{ij}(x^0) \simeq - \delta_{ij} \, e^{-\sqrt{2}\, x^0} \quad \text{for large } x^0$$

(Superstring)

$$\Rightarrow \quad V(T) \simeq e^{-T/\sqrt{2}} \quad \text{for large } x^0$$

A convenient choice:

$$V(T) = \mathcal{E}_p / \cosh(T/\sqrt{2})$$

Q. Does this effective action satisfy the other expected properties of $S_{eff}(T)$?

① $V(T_0) = V(\infty) = 0$ by construction.

$\Rightarrow$ The energy density vanishes ~~at~~ at $T=\infty$

② Absence of perturbative states around $T=\infty$

$\Leftrightarrow$ Absence of plane-wave solutions.

Thus we need to analyze solutions of eqs. of motion around $T=\infty$.

Most convenient in Hamiltonian formalism

$$\Pi(\vec{x},t) \equiv \frac{\delta L_{eff}}{\delta(\partial_0 T(\vec{x},t))} = \frac{V(T)\,\partial_0 T(\vec{x},t)}{\sqrt{1+\eta^{\mu\nu}\partial_\mu T\,\partial_\nu T}}$$

$$\mathcal{H} = \int d^p x\,(\Pi(x)\,\partial_0 T(x)) - L_{eff}$$

$$= \int d^p x\,\mathcal{H}(x)$$

$$\mathcal{H}(x) = \sqrt{\Pi^2 + V(T)^2}\,\sqrt{1+(\vec{\nabla}T)^2}$$

For large T at fixed Π, $V(T) \ll \Pi$

$$\Rightarrow \quad \mathcal{H}(x) = |\Pi|\sqrt{1+(\vec{\nabla}T)^2}$$

We shall choose $\Pi > 0$.

(At late time $\partial_0 T > 0 \Rightarrow \Pi > 0$)

Equations of motion:

$$\partial_0 \Pi = -\frac{\delta H}{\delta T(\vec{x},t)} = \partial_j \left(\Pi \frac{\partial_j T}{\sqrt{1+(\vec{\nabla} T)^2}} \right)$$

$$\partial_0 T = \frac{\delta \mathcal{H}}{\delta \Pi(\vec{x},t)} = \sqrt{1+(\vec{\nabla} T)^2}$$

Energy momentum tensor:

$$T_{\mu\nu} = \frac{\Pi(x)}{\sqrt{1+(\vec{\nabla} T)^2}} \partial_\mu T \, \partial_\nu T$$

Define: $u_\mu = -\partial_\mu T$, $\quad \epsilon(x) = \dfrac{\Pi(x)}{\sqrt{1+(\vec{\nabla} T)^2}}$

$\Rightarrow$ Eqs of motion:

$$\eta^{\mu\nu} u_\mu u_\nu = -1, \qquad \partial_\mu (\epsilon(x) u^\mu) = 0$$

Energy momentum tensor:

$$T_{\mu\nu} = \epsilon(x) \, u_\mu u_\nu$$

$\Rightarrow$ Precisely agrees with the eqs. governing the motion of a fluid of non-interacting dust with curl-free velocity field.

u_μ: Local $(p+1)$ velocity $\qquad \epsilon(x)$: Local rest mass density

Such a system has no plane-wave solutions.

A compressed fluid remains compressed since pressure vanishes.

$\Rightarrow$ Absence of plane wave solutions.

$\rightarrow$ Absence of perturbative excitations.

3 Existence of a kink solution:

We look for a solution depending on one direction $x^p \equiv x$ such that

$$T(\pm\infty) = \pm T_0 = \pm\infty$$

Since $\partial_0 T = 0$, $\Pi = 0$.

$$\Rightarrow \text{Energy density } \mathcal{E} = \int \mathcal{H}\, dx = \int dx \; V(T(x)) \sqrt{1 + (\partial_x T)^2}$$

Energy / unit $(p-1)$ volume of the soliton

In order to construct a classical solution we need to minimize $\mathcal{E}$ subject to the b.c.

$$T(\pm\infty) = \pm\infty.$$

$$\mathcal{E} = \int_{-\infty}^{\infty} dx \; V(T(x)) \sqrt{1+(\partial_x T)^2} \geq \int_{-\infty}^{\infty} dx \; V(T(x))\, \partial_x T = \underbrace{\int_{-\infty}^{\infty} V(T)\, dT}_{\text{Fixed}}$$

Bound saturated for $\partial_x T \to \infty$

$$T(x) = a\, F(x) \;, \quad a \to \infty, \quad F(\pm\infty) = \pm F_0$$

Thus the soliton is infinitely thin, but still has finite energy density

$$\mathcal{E} = \int_{-\infty}^{\infty} dT \; V(T)$$

This is precisely how a D-(p-1)-brane should look.

For the choice $V(T) = \mathcal{T}_p / \cosh(T/\sqrt{2})$ one get:

$$\mathcal{E} = \mathcal{T}_p \int_{-\infty}^{\infty} \frac{dT}{\cosh(T/\sqrt{2})} = \mathcal{T}_p \cdot \frac{\sqrt{2}}{2\pi} = \tilde{\mathcal{T}}_{p-1}$$

→ precisely the tension of the D-(p-1)-brane

We can also study fluctuations around the kink

⇒ Reproduces DBI action.

$$(\xi^0, \cdots \xi^{p-1}) \equiv (x^0, \cdots x^{p-1})$$

→ World-volume coordinate on the kink.

$$T(x, \xi) = a\underbrace{F(x + y^p(\xi))}_{\text{The kink solution.}}$$

$$y^I(x, \xi) = y^I(\xi)$$

$$A_\alpha(x, \xi) = a_\alpha(\xi), \quad \alpha = 0, \cdots p-1$$

$$A_p(x, \xi) = 0$$

(a_α, y^I, y^p) → Gauge & scalar fields on D-(p-1)-br.