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SMR.1498 - 11

SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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FLUX COMPACTIFICATIONS IN STRING THEORY

Lecture 2

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LECTURE II · AN EXAMPLE

MOSTLY BASED ON:
" Moduli Stabilization From Fluxes In a Simple IIB Orientifold "
Kachru, Schulz, Trivedi - 0201028.

Also: Tripathy & Trivedi - JHEP 03 028

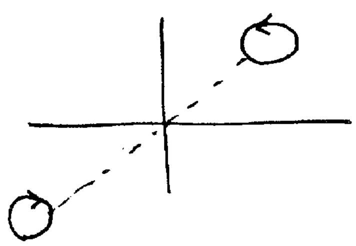
MOTIVATION:

- CAN CONSTRUCT AN EXPLICIT SUPERPOTENTIAL.
- ASK IF SUSY VACUA EXIST. HOW MANY?
  HOW WELL DO WE DO IN MODULI STABILISATION.
- EXAMPLE TRACTABLE, YET HAS MANY INTERESTING FEATURES

• MORE DETAILS. TAKE IIB ON T^6/Z_2 ORIENTIFOLD WITH FLUX.

① $\tilde{M}$: T^6/Z_2

Z_2: $\{1, \Omega(-1)^{F_L} R_{678945}\}$.



Ω IDENTIFIES OPPOSITE PTS ON T^6 ~~REFA~~ AFTER ORIENTATION REVERSAL OF STRING.

Orientifold T-Dual to TYPE I String after 6 T-dualities

- PRESERVES $N=4$ SUSY IN 4 DIM.
- Φ CONST.
- MASSLESS SPECTRUM.

$g_{\mu\nu}$: graviton

g_{ab} : 21 Scalars.

$(B_2)_{a\mu}, (C_2)_{a\mu}$: 12 gauge bosons

$(C_{(4)})_{abcd}$: 15 Scalars.

C_0, φ : 2 scalars.

$\boxed{\text{graviton, } 12 \text{ gauge bosons} + 38 \text{ Scalars}}$

(+ Fermionic partners).

In ADDITION: $2^6 = 64$ O_3^- planes with $-\frac{1}{4}$ units of D3-brane charge.

To cancell O_3^- charge: $16 = 64\left(\frac{1}{4}\right)$ D3 Branes are needed

Give Rise to:

$16 \times 6 = 96$ Scalars.

16 gauge bosons (at $U(1)^{16}$ pt.).

$\boxed{\text{TOTAL: } 38 + 96 = 134 \text{ Scalars}}$

FLUX :

$F_{(3)}$, $H_{(3)}$: CLOSED & HARMONIC $\in H^3(T^6)$

BASIS OF $H^3(T^3, \mathbb{Z})$

T^6: x^i, y^i $\quad 0 \leq x^i, y^i \leq 1$ Coordinates on T^6

$x^i \simeq x^i + 1$

$y^i \simeq y^i + 1$.

$$\alpha_0 = dx^1 \wedge dx^2 \wedge dx^3.$$

$$\alpha_{ij} = \frac{1}{2} \epsilon_{ilm} \, dx^l \wedge dx^m \wedge dy^j$$

$$\beta^{ij} = -\frac{1}{2} \epsilon_{jlm} \, dy^l \wedge dy^m \wedge dx^i$$

$$\beta_0 = dy^1 \wedge dy^2 \wedge dy^3.$$

Basis $\mathcal{B}$

$V \in H^3(T^6, \mathbb{Z})$ let C^i be any ~~___~~ 3-cycle in $H_3(T^6)$ then $\int_{C^i} V = \mathbb{Z}$

(for basis above this follows $\int_{T^6} \alpha_I \wedge \beta^J = \delta_I^J$)

DIRAC QUANTIZATION (DUE TO PRESENCE OF D/F STRINGS WHICH COUPLE TO $C_{(2)} / B_{(2)}$)

ON T^6:

$$\boxed{\begin{aligned} \frac{1}{(2\pi)^2 \alpha'} F_{(3)} &= a^0 \alpha_0 + a^{ij} \alpha_{ij} + b_{ij} \beta^{ij} + b_0 \beta^0. \\ \frac{1}{(2\pi)^2 \alpha'} H_{(3)} &= c^0 \alpha_0 + c^{ij} \alpha_{ij} + d_{ij} \beta^{ij} + d_0 \beta^0. \end{aligned}}$$

$\boxed{a^0, a^{ij}, b_{ij}, b_0, c^0, c^{ij}, d_{ij}, d_0 : \text{INTEGERS}.}$ "FLUX INTEGRS"

ON T^6/Z_2:

- FLUX MUST BE CONSISTENT WITH Z_2 ORIENTIFOLDING

$B_{(2)}$, $C_{(2)}$: ODD INTRINSIC PARITY

$[F_{(3)}]_{abc}$, $[H_{(3)}]_{abc}$: even under Z_2

THIS CONDITION IS MET.

$[\tilde{F}_5]_{abcde} \rightarrow -[\tilde{F}_5]_{abcde}$: Condition also met

- SECOND. QUANTIZATION CONDITION A LITTLE SUBTLE. BECAUSE OF HALF CYCLES LOCATED AT ORIENTIFOLD PTS. (FREY & POLCHINSKI)

e.g. $y^1 = y^2 = y^3 = 0$ $\quad 0 \leq x^i \leq 1$ with $x^i \simeq -x^i$

$$\int_C V = \frac{Z}{2}$$

where $V \in \mathcal{B}$

SIMPLEST WAY TO AVOID COMPLICATIONS:
TAKE ALL FL. INTEGERS TO BE EVEN.
(Else Need APPROPRIATELY PLACED EXOTIC ORIENTIFOLD PLANES).

• $\tilde{F}_5$ BIANCHI EQUATION:

TOTAL Q_3 CHARGE VANISHES.

$$\frac{1}{2\kappa_{10}^2 \mu_3} \int H_{(3)} \wedge F_{(3)} + Q_3^{\text{local}} = 0 .$$

$$\frac{1}{(2\pi)^4 \alpha'^2} \cdot \underset{\downarrow \; T^6/Z_2}{\left(\frac{1}{2}\right)} \int_{T^6} \underbrace{H_{(3)} \wedge F_{(3)}}_{\substack{\downarrow \\ \text{ALWAYS POSITIVE IF} \\ G_{(3)} : \text{ ISD}}} + N_{D3} = 16 .$$

SO $\boxed{N_{D3} < 16 .}$

• FINALLY REQUIRE $G_{(3)}$: (2,1) & PRIMITIVE.

IMPOSE (2,1) CONDITION BY CONSTRUCTING SUPERPOT.

$$W = \int G_{(3)} \wedge \Omega_{(3)} .$$

CHOOSE COMPLEX ST ON T^6:

$$\boxed{z^i = x^i + \tau^{ij} y^j} \qquad 0 \leq i, j, \leq 3$$

COMPLEX STR. COMPLETELY SPECIFIED BY τ^{ij} — PERIOD MATRIX.

$$\boxed{\begin{aligned} \Omega &= dz^1 \wedge dz^2 \wedge dz^3 . \\ &= \alpha_0 + \alpha_{ij}\tau^{ij} - \beta^{ij} (\mathrm{Cof}\,\tau)_{ij} + \beta^0 (\det \tau) \\ (\mathrm{Cof}\,\tau)_{ij} &= \frac{1}{2} \epsilon_{ikm} \epsilon_{jpq} \tau^{kp} \tau^{mq} . \end{aligned}}$$

THEN:

$$\boxed{\frac{1}{(2\pi)^2\alpha'} W = (a^0 - \phi c^0)\det\tau - (a^{ij} - \phi c^{ij})(\mathrm{cof}\,\tau)_{ij} - (b_{ij} - \phi d_{ij})\tau_{ij} - (b_0 - \phi d_0)}$$

NOW IMPOSE SUSY CONDITIONS.

$$\boxed{\begin{array}{ll} W=0. & \mathrm{I} \\ \dfrac{\partial W}{\partial \tau^{ij}} = 0. & \mathrm{III} \\ \dfrac{\partial W}{\partial \phi} = 0. & \mathrm{II} \end{array}}$$

THESE GIVE NON-LINEAR ALGEBRAIC EQS ~~CONDITION~~ THAT OVERDETERMINE τ^{ij}, Φ.

IN ADDITION PRIMITIVITY: $J \wedge G = 0$.

THIS IMOSES LINEAR CONDITIONS ON $g_{i\bar{j}}$ EASIER TO IMPOSE.

EXAMPLE.

$$\left(a^{ij}, b_{ij}, c^{ij}, d_{ij}\right) = (a, b, c, d)\, \delta_{ij}$$

$$\tau^{ij} = \tau\, \delta^{ij} \qquad \text{(Consistent with eqn's).}$$

$$\text{I-II}: \quad P_1(\tau) \equiv a^0\tau^3 - 3a\tau^2 - 3b\tau - b_0 = 0.$$

$$\text{II}: \quad P_2(\tau) \equiv c^0\tau^3 - 3c\tau^2 - 3d\tau - d_0 = 0.$$

$$\text{III}: \quad (a^0 - \phi c^0)\tau^2 - 2(a - \phi c)\tau - (b - \phi d) = 0.$$

$P_1(\tau)$ & $P_2(\tau)$ must share common quadratic root.

THIS IMPOSES CONDITIONS ON FLUX INTEGERS.

(example of fact that fluxes must be non-generic).

eg:

$$P_1(\tau) = \tau^3 - 1 = (\tau - 1)(\tau^2 + \tau + 1).$$

$$P_2(\tau) = (\tau + 2)(\tau^2 + \tau + 1).$$

$$(a^0, a, b, b_0) = (2, 0, 0, 2).$$

$$(c^0, c, d, d_0) = (2, -2, -2, -4). \quad : \ \underline{\text{even integers}}.$$

Soln.:

$$\tau = e^{2\pi i/3}.$$

$$\Phi = e^{2\pi i/3}.$$

T^6: $T^2 \times T^2 \times T^2$: ALL T^2's have equal mod. parameters

VERY SYMMETRICAL

$$\boxed{\begin{aligned} N_{FLUX} &= 12. \\ N_{D3} &= 16 - \frac{N_{FL}}{2} = 10. \end{aligned}}$$

$$G_{(3)} = dz^1 \wedge dz^2 \wedge d\bar{z}^3 + dz^2 \wedge dz^3 \wedge d\bar{z}^1 + dz^3 \wedge dz^1 \wedge d\bar{z}^2 .$$

(2,1) TYPE.

- PRIMITIVITY: $J = \sum_{a=1}^{3} r_a^2 \; dz^a \wedge d\bar{z}^{\bar{a}}$

THEN $J \wedge G_{(3)} = 0$

MORE GENERALLY 6 OF TOTAL 9 KAHLER MODULI NOT LIFTED.

- CAN SHOW EXAMPLE HAS $N=1$ SUSY.

<u>Moduli Stabilisation:</u>

- All complex structure moduli lifted + dilaton/axion lifted
- Some kahler moduli + partners lifted – 6 survive
- Some D3 branes replaced by flux Removes $6 \times 6 = 36$ moduli

<u>THESE CONCLUSIONS ARE GENERAL.</u>

Best one can do:

Replace all D3 branes by flux. leaves 3 Kahler moduli + partners unlifted : 6 moduli.

Lifted Moduli:

	134	Initial
	6	Final.
	128.	

ADDITIONAL COMMENTS:

- EXAMPLES WITH $N=2$, $N=3$ SUSY CAN BE FOUND.

- ALSO EXAMPLES ~~WH~~ WHERE MODULI SPACE OF COMPLEX STRUCTURES ONLY PARTIALLY LIFTED CAN BE FOUND. ONE SUCH EXAMPLE WILL BE RELEVANT FOR NEXT LECTURE.

- NOTICE $\varphi \sim \frac{\sqrt{3}}{2}$ IN EXAMPLE.

  MORE GENERALLY: $\varphi \sim O(1)$.

CORRECTIONS TO SUPERPOTENTIAL?

- α' EXPANSION CORRECTIONS MUST VANISH DUE TO A PQ SYMMETRY. & HOLOMORPHY.

- g_s EXPANSION: NON-PERT. CORRECTIONS POSSIBLE BUT MUST VANISH AT LARGE VOLUME (WITTEN: hep-th/9604030).

A LITTLE SUBTLE TO APPLY WITTEN'S DISCUSSION FOR THIS CASE DUE TO $T^6/\mathbb{Z}_2$ BEING SINGULAR IN F-THEORY LIFT.

- SIMILAR CONCLUSIONS ARISE FOR. $K3 \times T^2/\mathbb{Z}_2$. (ALTHOUGH METHOD OF ANALYSIS DIFF).

TRIPATHY & TRIVEDI.

IN THAT CASE ~~to~~ F-THEORY LIFT SMOOTH.
K3 x K3.

ONE CAN SHOW NO INSTANTON SUPERPOTENTIAL. HOWEVER A NON-PERT. SUPERPOTENTIAL DOES GET GENERATED, PROBABLY, DUE TO GAUGINO CONDENSATION. IN 10-DIM / ~~[illegible]~~ LANGUAGE THESE ARE FRACTIONAL EUCLIDEAN D3 BRANES WRAPPING K3. CORRECTION SMALL EXPONENTIALLY AT LARGE VOLUME.

- MORE NEEDS TO BE DONE IN UNDERSTANDING NON-PERTURBATIVE CORRECTIONS TO SUPERPOT.