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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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FLUX COMPACTIFICATIONS IN STRING THEORY

Lecture 3

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LECTURE 3:

DUAL DESCRIPTIONS OF IIB WITH FLUX

Main Ref: "New Supersymmetric String Compactifications"
Kachru, Schulz, Tripathy, S.P.T. hep-th/0211182.

Motivation:

1. Duality has proved to be very useful.
2. Need to understand: all diff types of $N=1$ string vacua.
3. Mirror symmetry with flux, heterotic Duals.
4. Moduli Stabilisation: Both kahler & Complex structure?

- VERY LITTLE KNOWN TODATE. ONLY FEW PRELIMINARY RESULTS. RAPID PROGRESS EXPECTED IN COMING YEAR(S)

- WE WILL CONCENTRATE ON ONE $N=2$ susy IIB example AND CONSTRUCT VARIOUS DUALS. IN IIA/IIB AND M-THEORY. IF TIME PERMITS: HETEROTIC DUALS.

- EXAMPLE: $T^6/\mathbb{Z}_2$ IIB WITH FLUX (LAST LECTURE).

COMMENTS: a) NOT ALL COMPLEX STRUCT. MODULI LIFTED IN EXAMPLE, THIS WILL ENSURE THAT DUAL DESCRIPTION WEAKLY COUPLED IN SUITABLE REGIONS OF MODULI SPACE. THAT IS, CAN BE STUDIED RELIABLY IN SUGRA APPROX.

b) DUALS CONSTRUCTED USING BUSHER T-DUALITY RULES. THIS CAN BE VIEWED AS A SOLUTION GENERATING TECHNIQUE.

IT ENSURES THAT THE DUAL BACKGROUD SOLVES THE IIA|IIB| HETEROTIC EQUATIONS OF MOTION.

ESSENTIAL COMPLICATION IN UNDERSTANDING DUALS:

- THE MANIFOLD IS NO LONGER CALABI-YAU.
- NOT EVEN KAHLER.
- QUITE OFTEN, e.g. some IIA DUALS, NOT COMPLEX

(MORE PRECISELY, THE ALMOST COMPLEX STRUCTURE THAT SUPERSYMMETRY NATURALLY LEADS TO IS NOT INTEGRABLE, IN MANY DUAL CASES).

(CALABI-YAU COMPACTIFICATIONS ARE THE TIP OF A HUGE ICEBERG!)

ALL THESE FEATURES WILL BE ILLUSTRATED BY OUR EXAMPLE.

- BUT WE WILL NOT CHARACTERISE, IN ANY VERY USEFUL WAY, THE RESULTING NON KÄHLER MANIFOLDS.

- THE DUALS WILL BE RELATED TO (WARPED) SCHERK-SCHWARZ COMPACTIFICATIONS.

WARM UP TOY MODEL:

T^3 WITH $H_{(3)}$ FLUX (NOT SOLUT. TO EQM).

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad \begin{array}{c} 0 \le x \le 1 \\ 0 \le y \le 1 \\ 0 \le z \le 1 \end{array}$$

$$[H_{(3)}]_{xyz} = N.$$

$$B_{yz} = Nx.$$

T-duality along z:

$$\tilde{B}_{ab} = 0 .$$

$$\boxed{d\tilde{s}^2 = dx^2 + dy^2 + (dz + Nx\,dy)^2.}$$

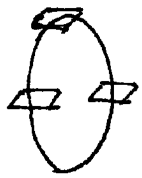
$$\left[\begin{array}{ll} \tilde{g}_{z\alpha} = B_{z\alpha} & \alpha \neq z \\ \tilde{g}_{\alpha\beta} = g_{\alpha\beta} - \frac{1}{g_{zz}}\left(g_{z\alpha}\, g_{z\beta} - B_{z\alpha} B_{z\beta} \right). & \\ \tilde{B}_{z\alpha} = -\frac{g_{z\alpha}}{g_{zz}}. & \\ \tilde{g}_s = \frac{g_s}{\sqrt{g_{zz}}} & \end{array} \right].$$

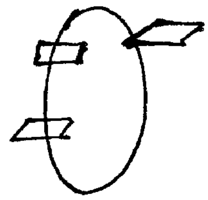
IDENTIFICATION: $(x, y, z) \simeq (x, y+1, z) \simeq (x, y, z+1)$.
$\simeq (x+1, y, z - Ny)$.

KEEPS METRIC SINGLE VALUED.

- MANIFOLD : <u>TWISTED T^3</u>

T^3: T^2 TRIVIALLY FIBERED OVER AN S^1.





TWISTED T^3: T^2 NONTRIVIALLY FIBERED. UNDERGOES. $SL(2,\mathbb{Z})$ TWIST. AROUND x CIRCLE.

$$x \to x+1 \qquad \begin{pmatrix} y \\ z \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ -N & 1 \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix}.$$

T^2 $\qquad\qquad\qquad \in SL(2,\mathbb{Z})$

- CAN BE VIEWED AS A <u>COSET</u>

$$g_N = \begin{pmatrix} 1 & y & -z/N \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix}$$

FORMS GROUP ISOMORPHIC TO HEISENBERG GROUP. $G_3^N(\mathbb{R})$.

$G_3^N(\mathbb{Z})$ subgroup where $y, z, x \in \mathbb{Z}$

Rt. Coset $G_3^N(\mathbb{R}) / G_3^N(\mathbb{Z})$. IS THE TWISTED TORUS.

$$\begin{pmatrix} 1 & y & -z/N \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b & -c/N \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & b+y & -\frac{c}{N} + ay - \frac{z}{N} \\ 0 & 1 & a+x \\ 0 & 0 & 1 \end{pmatrix}.$$

ALSO CALLED NIL-MANIFOLD.

EXAMPLE. $T^6/\mathbb{Z}_2$. WITH FLUX (Same Notation as Lectures I & II).

$$F_{(3)} = 2dx^1\wedge dx^2\wedge dy^3 + 2dy^1\wedge dy^2\wedge dy^3$$

$$H_{(3)} = 2dx^1\wedge dx^2\wedge dx^3 + 2dy^1\wedge dy^2\wedge dx^3.$$

COMPLEX STR. MODULI SPACE NOT LIFTED.
ALONG A LOCUS $T^6 : (T^2)^3 : \tau^1, \tau^2, \tau^3$: MOD. PARAM. OF T^2's.

$$\Phi\,\tau_3 = -1 \qquad (\Phi : \text{DIL-AXION}).$$
$$\tau_1\,\tau_2 = -1$$

DUAL METRIC T-DUALITY IN x^1 DIRECTION.

$$ds^2 = \underbrace{\frac{1}{R_{x^1}^2}\left(dx^1 + 2x^2dx^3\right)^2 + R_{x^2}^2(dx^2)^2 + R_{x^3}^2(dx^3)^2}_{\text{NIL MANIFOLD}} + \sum_{i=1}^{3} R_{y^i}^2 (dy^i)^2.$$

IN ADDITION IN IIA THEORY:

$F_{(2)}$, $F_{(4)}$, $H_{(3)}$ TURNED ON.

- $h^{1}(\hat{M}) = 5$
  ENOUGH TO SHOW: DUAL-MANIFOLD NON-KAHLER.

- SUPERPOTENTIAL IN DUAL DESCRIPTION CAN BE CONSTRUCTED. AN ALMOST COMPLEX STRUCTURE IS USED WHICH IS NOT INTEGRABLE.

- CAN BE VIEWED AS ~~TWISTED~~ A SCHERK-SCHWARZ COMPACTIFICATIONS:

- TORI: PARALLELISABLE MANIFOLDS. HAVE WELL-DEFINED NOWHERE VANISHING BASIS OF VIELBEIN FIELDS.
- TWISTED TORI: GENERALISATION. ALSO PARALLELISABLE. BASIS OF VIELBEIN FIELDS NOT CONSTANT, BUT STRUCTRE CONSTANT ARE CONSTANT.

$$\eta^a = U(x)^a{}_b \, dx^b .$$

Vielbein ↑ ↓ Coordinate ONE FORM.

$$d\eta^a = -\frac{1}{2} f^a_{bc} \, \eta^b \wedge \eta^c .$$

f^a_{bc} : STRUCTURE CONSTANTS ARE INDEED CONST.

(IN OUR CASE STR. CONST. ARE THOESE OF HEISENBERG ALGEBRA AND ARE CONST.)

- THESE FEATURES IN EXAMPLE WE CONSIDER WILL BE ~~BE~~ GENERALLY TRUE:

a) DUAL COMPACITFICATION IS TWISTED TORUS DUE TO $H_{(3)}$ FLUX TURNING INTO COMPONENTS OF METRIC.

b) IT CAN BE THOUGHT OF AS A COSET

c) ALSO AS A S-S. COMPACTIFICATION WITH STR. CONST. RELATED TO ~~&~~ GROUP IN b).

SUMMARY:

- DUAL DESCRIPTIONS INVOLVE NON-KÄHLER MANIFOLDS. NEED TO BE UNDERSTOOD BETTER.

- THE PARTICULAR EXAMPLE CONSIDERED, ACTUALLY DUAL TO IIA ON CY_3.

- MORE GENERALLY DON'T EXPECT THIS TO BE TRUE.

- HETEROTIC DUALS: SOME CAN BE EXPLICITLY CONSTRUCTED FOR IIB ON $K3 \times T^2/\mathbb{Z}_2$.
  NON KAHLER COMPLEX MANIFOLDS STUDIED FIRST BY STROMINGER.

  SUPERPOTENTIAL, ESP. GAUGE BUNDLE, NEEDS TO BE UNDERSTOOD BETTER.