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SMR.1498 - 9

SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

31 March - 8 April 2003

STRING INTERACTIONS IN PLANE WAVES

Lecture 2

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[Very Preliminary Draft — M. Spradlin]

2 The Hamiltonian of String Theory

In this lecture we will canonically second-quantize string theory in light-cone gauge and write down its Hamiltonian, which will be no more complicated (qualitatively) than

$$H = a^\dagger a + g_s(a^\dagger a a + a^\dagger a^\dagger a). \tag{1}$$

Students of string theory these days are not typically taught (certainly I was not!) that it is possible to write down an explicit formula for the Hamiltonian of string theory. The light-cone approach does suffer from a number of problems which will be discussed in detail in the next lecture.

However, light-cone string field theory is very well-suited to the study of string interactions in the plane wave background. For one thing, it is only in the light-cone gauge that we are able to determine the spectrum of the free string. Since other approaches cannot yet even give us the free spectrum, they can hardly tell us anything about string interactions. Although we hope this situation will improve, light-cone gauge is still very natural from the point of view of the BMN correspondence. The dual BMN gauge theory automatically provides us a light-cone quantized version of the string theory, and it is hoped that taking the continuum limit of the 'discretized strings' in the gauge theory might give us light-cone string field theory, although a large number of obstacles need to be overcome before the precise correspondence is better understood.

Many of the fundamental concepts which will be introduced in this and the following lecture apply equally well to all string field theories, and not just the light-cone version. It is therefore hoped that these lectures may be of benefit even to some who are not particularly interested in plane waves.

This portion of my lecture series is intended to be highly pedagogical. We will therefore start by studying the simplest possible case in great detail. Here is a partial list of simplifications which we will start with:

- During this lecture we will consider only bosons. Somewhat surprisingly, fermions complicate the story considerably, but we will postpone these important details until the next lecture. The reader should keep in mind that the plane wave background is not a solution of bosonic string theory, so strictly speaking all of the formulas presented in this lecture need to be supplemented by the appropriate fermions. (However, the flat space limit $\mu \to 0$ does make sense without fermions, although there is the issue of the tachyon...)

- Since the bosonic sector of string theory on the plane wave background is SO(8) invariant, we will completely ignore the transverse index $I = 1, \ldots, 8$ for most of the discussion. It is trivial to replace, for example, $x^2 \to \sum_{I=1}^{8}(x^I)^2$ in all of the formulas below.

- Finally, we will begin not even with a string in the plane wave background, but simply a particle in the plane wave background! A string can essentially be thought of as an infinite number of particles, one for each Fourier mode on the string worldsheet. A particle is equivalent to taking just the zero-mode on the worldsheet (the mode independent of σ). In flat space, there is a qualitative difference between this zero-mode and the 'stringy' modes: the former has a continuous spectrum (just the overall momentum of the string) while the stringy modes are like harmonic oscillators and have a discrete spectrum. However in the plane wave background, even the zero-mode lives in a harmonic oscillator potential, so it is not qualitatively different from the non-zero modes. Once we develop all of the formalism appropriate for a particle, it will be straightforward to take infinitely many copies of all of the formulas and apply them to a string.

Free Bosonic Particle in the Plane Wave

We consider a particle propagating in the plane wave metric

$$ds^2 = -2dx^+dx^- - \mu^2 x^2 (dx^+)^2 + dx^2. \qquad (2)$$

The action for a free 'massless' field is

$$S = -\frac{1}{2}\int \partial_\mu \Phi \partial^\mu \Phi = \int dx^+ dx^- dx\ \partial_+\Phi\partial_-\Phi - \int dx^+ H, \qquad (3)$$

where we have defined

$$H = \frac{1}{2}\int dx^- dx \left[(\partial_x\Phi)^2 + \mu^2 x^2 (\partial_-\Phi)^2\right]. \qquad (4)$$

Now let us canonically quantize this theory, so we promote Φ from a classical field to an operator on the multi-particle Hilbert space. The canonically conjugate field to Φ is $\partial_-\Phi$, so the commutation relation is

$$[\Phi(x^-, x), \partial_-\Phi(y^-, y)] = i\delta(x^- - y^-)\delta(x-y). \qquad (5)$$

Let us pass to a Fourier basis by introducing

$$\Phi(x^-, x) = \frac{1}{2\pi}\int dp_- dp\ \Phi(p_-, p) e^{i(p_- x^- + px)}. \qquad (6)$$

Note: from now on we **define**

$$p^+ = -p_-. \qquad (7)$$

In the supersymmetric string theory to be considered below, the supersymmetry algebra will guarantee that $p^+ \geq 0$ for all states. We will proceed with this assumption. The commutation relation is now

$$[\Phi(p^+, p), \Phi(q^+, q)] = \frac{1}{p^+}\delta(p^+ + q^+)\delta(p+q). \qquad (8)$$

Since Φ is real (as a classical scalar field), the corresponding operator Φ is Hermitian, which means that

$$\Phi(p^+, p)^\dagger = \Phi(-p^+, -p). \tag{9}$$

The Hamiltonian can now be written as

$$H_2 = \frac{1}{2}\int dp_- dp\ \Phi^\dagger(p^2 + (\mu p^+ x)^2)\Phi = \int dp^+ dp\ p^+ \Phi^\dagger h \Phi, \tag{10}$$

(normal ordering will always be understood) where h is the single-particle Hamiltonian

$$h = \frac{1}{2p^+}(p^2 + \omega^2 x^2), \qquad \omega = \mu|p^+|. \tag{11}$$

(We will always take $\mu \geq 0$.) The subscript "2" in (10) denotes that this is the quadratic (i.e., free) part of the Hamiltonian. Later we may add higher order interaction terms. The single-particle Hamiltonian may be diagonalized in the standard way:

$$a = \frac{1}{\sqrt{2\omega}}(p - i\omega x), \qquad p = \sqrt{\frac{\omega}{2}}(a + a^\dagger), \qquad x = \frac{i}{\sqrt{2\omega}}(a - a^\dagger), \tag{12}$$

so that

$$h = e(p^+)\mu(a^\dagger a + \tfrac{1}{2}), \tag{13}$$

where $e(x) = \text{sign}(x)$.

It is important to distinguish two different Hilbert spaces. The single-particle Hilbert space $\mathcal{F}$ is spanned by the vectors

$$|N; p^+\rangle \equiv (a^\dagger)^N |0; p^+\rangle, \qquad N = 0, 1, \ldots. \tag{14}$$

The operators a, $a^\dagger$ and h act on $\mathcal{F}$. The second Hilbert space is the multi-particle Hilbert space $\mathcal{H}$. Let us introduce particle creation/creation operators $A_N(p^+)$ which act on $\mathcal{H}$ and satisfy $(A_N(p^+))^\dagger = A_N(-p^+)$ and

$$[A_M(p^+), A_N(q^+)] = e(p^+)\delta_{MN}\delta(p^+ + q^+). \tag{15}$$

For $p^+ > 0$, $A_N(p^+)$ annihilates a particle in the state $|N; p^+\rangle$, while for $p^+ < 0$, $A_N(p^+)$ creates a particle in the state $|N; -p^+\rangle$. The vacuum of $\mathcal{H}$, denoted by $|0\rangle\rangle$, is annihilated by all $A_N(p^+)$ which have $p^+ > 0$. Normally in scalar field theory we do not introduce this level of complexity because the 'internal' hamiltonian h is so trivial.

We may refer to $\mathcal{F}$ as the 'worldsheet' Hilbert space and $\mathcal{H}$ as the 'spacetime' Hilbert space, since this is of course how these should be thought of in the string theory.

We now write the usual expansion for Φ in terms of particle creation and annihilation operators:

$$\Phi(p^+) = \frac{1}{\sqrt{|p^+|}} \sum_{N=0}^{\infty} |N; p^+\rangle A_N(p^+). \tag{16}$$

It is easily checked that the commutation relation (8) follows from (15). Note that we have written Φ as simultaneously a state in $\mathcal{F}$ and an operator in $\mathcal{H}$. This is notationally more convenient than the position space representation,

$$\Phi(p^+, x) = \langle x|\Phi(p^+)\rangle \sim \frac{1}{\sqrt{|p^+|}} \sum_{N=0}^{\infty} e^{-x^2} H_N(x) A_N(p^+), \tag{17}$$

which would leave all of our formulas full of Hermite polynomials $H_N(x)$.

Writing the field operator Φ as a state in the single-particle Hilbert space has notational advantages other than just being able to do without Hermite polynomials. For example, the equation of motion

$$\partial_+\partial_-\Phi - \frac{1}{2}\partial_x^2\Phi - \frac{1}{2}\mu^2 x^2 \partial_-^2\Phi = 0 \tag{18}$$

is just the Schrödinger equation on Φ:

$$i\partial_+\Phi = h\Phi. \tag{19}$$

There is a simple formula which allows us to take any symmetry generator on the worldsheet (such as h above, and later rotation generators j^{IJ}, supercharges $q, \ldots$) and construct a free field realization of the corresponding space-time operator (H, J^{IJ}, $Q, \ldots$):

$$G_2 = \int dp^+ dp\; p^+ \Phi^\dagger g \Phi. \tag{20}$$

We already saw this formula applied to the Hamiltonian in (10). When we further make use of the expansion of Φ into creation operators, we find the expected formula

$$H_2 = \int_0^\infty dp^+ \sum_{N=0}^{\infty} E_N A_N(-p^+) A_N(p^+), \qquad E_N = \mu(N + \tfrac{1}{2}). \tag{21}$$

Another trivial application is the identity operator — it is easily checked that

$$\int dp^+\; p^+ \Phi^\dagger \Phi = -i \int dx^-\; \Phi \partial_- \Phi = \int_0^\infty dp^+ \sum_{N=0}^{\infty} A_N(-p^+) A_N(p^+) = I. \tag{22}$$

Again, the subscript '2' in (20) emphasizes that this gives a free field realization (i.e., quadratic in Φ). Dynamical symmetry generators (such as the Hamiltonian, in particular) will pick up additional interaction terms, but kinematical symmetry generators (such as J^{IJ}) remain quadratic.

Interactions

Now let us consider a cubic interaction,

$$H_3 = g_s \int dx^- dx\; V, \tag{23}$$

where, for example, we might choose

$$V = \Phi^3 + \Phi(\partial_-^{17}\Phi)(\partial_x^9\Phi). \tag{24}$$

If we insert the expansion of Φ in terms of modes and do the integrals, we end up with an expression of the form

$$H_3 = \int dp_1^+ dp_2^+ dp_3^+ \delta(p_1^+ + p_2^+ + p_3^+) \sum_{N,P,Q=0}^{\infty} c_{NPQ}(p_1^+, p_2^+, p_3^+) A_N(p_1^+) A_P(p_2^+) A_Q(p_3^+). \quad (25)$$

The functions $c_{NPQ}(p_1^+, p_2^+, p_3^+)$ are obtained from V without too much difficulty: they simply encode the Fourier transform of the interaction, except that we Fourier transform to the number basis of a harmonic oscillator, rather than to the more familiar momentum representation.

Exercise. Compute $c_{NPQ}(p_1^+, p_2^+, p_3^+)$ for $V = \Phi^3$.

We now adopt a convention which is important to keep in mind. Because of the p^+ conserving delta function, it will always be the case that two of the p^+'s are positive and one is negative, or vice versa. We will always choose the index '3' to label the p^+ whose sign is opposite that of the other two. This means that the particle labelled '3' will always be the initial state of a splitting transition $3 \to 1 + 2$ or the final state of a joining transition $1 + 2 \to 3$.

In the state-operator correspondence, it is convenient to identify the operator cubic H_3 with a state in the 3-particle Hilbert space $|V\rangle$ (where V stands for 'vertex') with the property that

$$\langle N; p_1^+ | \langle P; p_2^+ | \langle Q; -p_3^+ | H_3 \rangle = c_{NPQ}(p_1^+, p_2^+, p_3^+), \qquad \text{for } p_1^+, p_2^+ > 0, p_3^+ < 0. \quad (26)$$

This state can be constructed by taking

$$|V\rangle = \sum_{N,P,Q=0}^{\infty} c_{NPQ}(p_1^+, p_2^+, p_3^+) |N; p_1^+\rangle |P; p_2^+\rangle |Q; -p_3^+\rangle. \quad (27)$$

Free Bosonic String in the Plane Wave

It is essentially trivial to promote all of the formulas from the preceding section to the case of a string. The field $\Phi(x)$ is promoted from a function of x, the position of the particle in space, to a functional $\Phi[x(\sigma)]$ of the embedding $x(\sigma)$ of the string worldsheet in space. In all of the above formulas, integrals over dx are replaced by functional integrals $Dx(\sigma)$, and delta functions in x are replaced by delta-functionals $\Delta[x(\sigma)]$. These are defined as a product of delta functions over all of the Fourier modes x_n of $x(\sigma)$.

The interacting quantum field theory of strings is described by the action

$$S = \int dx^+ dx^- Dx(\sigma)\ \partial_+ \Phi \partial_- \Phi - \int dx^+\ H, \quad (28)$$

where $H = H_2 + H_3 + \cdots$. The formula (20) is replaced by

$$G_2 = \int dp^+ Dp(\sigma)\ p^+ \Phi^\dagger g \Phi. \quad (29)$$

The worldsheet hamiltonian is now

$$h = \frac{e(p^+)}{2} \int_0^{2\pi|p^+|} d\sigma \left[4\pi p^2 + \frac{1}{4\pi}((\partial_\sigma x)^2 + \mu^2 x^2)\right] = \frac{1}{p^+} \sum_{n=-\infty}^{\infty} \omega_n a_n^\dagger a_n, \tag{30}$$

where

$$\omega_n = \sqrt{n^2 + (\mu\alpha' p^+)^2} \tag{31}$$

is the energy of the n-th mode and we have introduced a suitable basis of raising and lowering operators in order to diagonalize h. Note that a_0 is identified with the operator a corresponding to a particle, while for $n > 0$, $a_n = \alpha_n$ are the left-movers and $a_{-n} = \tilde{\alpha}_n$ are the right-movers.

The full Hilbert space $\mathcal{F}$ of a single string is obtained by acting on $|0; p^+\rangle$ with the raising operators $a_n^\dagger$ (for all n! Note that we do not use any convention like $a_n^\dagger = a_{-n}$). We therefore label a state by $|\vec{N}\rangle$, where the component N_n of the vector $\vec{N}$ gives the occupation number of oscillator n. Note that we have to impose the $L_0 - \overline{L}_0 = 0$ physical state condition

$$\sum_{n=-\infty}^{\infty} n N_n = 0. \tag{32}$$

The second quantized Hilbert space $\mathcal{H}$ is introduced as before. It has the vacuum $|0\rangle\rangle$, which is acted on by the operators $A_{\vec{N}}(p^+)$, which for $p^+ < 0$ create a string in the state $|\vec{N}; p^+\rangle$. The representation of the Hamiltonian at the level of free fields is just

$$H_2 = \int_0^\infty dp^+ \sum_{|\vec{N}\rangle \in \mathcal{F}} E_{\vec{N}} A_{\vec{N}}^\dagger(p^+) A_{|s\rangle}(p^+), \qquad E_{\vec{N}} = \frac{1}{p^+} \sum_{n=-\infty}^{\infty} \omega_n N_n \tag{33}$$

The Cubic String Vertex

Our goal now is to construct the state $|V\rangle$ in $\mathcal{F}^3$ which encodes the cubic string interactions, in the sense of formulas (25) and (27). What is the principle that determines the cubic interaction? It is quite simple: the embedding of the string worldsheet into spacetime should be continuous.

In a functional representation, the cubic interaction is therefore just

$$H_3 = g_2 \int \delta(p_1^+ + p_2^+ + p_3^+) f(p_1^+, p_2^+, p_3^+) \Delta[x_1(\sigma) + x_2(\sigma) - x_3(\sigma)] \prod_{r=1}^{3} \left(dp_r^+ Dx_r(\sigma)\ \Phi[p_r^+, x_r(\sigma)]\right). \tag{34}$$

There is one very important caveat: the principle of continuity requires the delta functional $\Delta[x_1(\sigma) + x_2(\sigma) - x_3(\sigma)]$, but it does not determine the interaction (34) uniquely because we have the freedom to choose the measure factor $f(p_1^+, p_2^+, p_3^+)$ arbitrarily. We will return to this point later.

Our convention about the selection of 'p_3^+' guarantees that string 3 is always the 'long string'. The interaction (34) mediates the string splitting $3 \to 1 + 2$, or its hermitian conjugate, the joining of $1 + 2 \to 3$. All we have to do now is Fourier transform this

Delta-functional into the harmonic oscillator number basis! Let us assemble the steps of this calculation.

Step 1. First we recall the definition of a Δ-functional as a product of delta-functions for each Fourier mode,

$$\Delta[x_1(\sigma)+x_2(\sigma)-x_3(\sigma)] = \prod_{m=-\infty}^{\infty} \delta\left(\int_0^{2\pi|p_3^+|} d\sigma\, e^{im\sigma/|p_3^+|}[x_1(\sigma)+x_2(\sigma)-x_3(\sigma)]\right). \quad (35)$$

Let us introduce matrices $X^{(r)}_{mn}$ which express the Fourier basis of string r in terms of the Fourier basis of string 3 (so that, clearly, $X^{(3)}=1$). Then we can write

$$\Delta[x_1(\sigma)+x_2(\sigma)-x_3(\sigma)] = \prod_{m=-\infty}^{\infty} \delta\left(\sum_{r=1}^{3} X^{(r)}_{mn} p_{n(r)}\right). \quad (36)$$

These matrices are obtained by simple Fourier transforms,

$$X^{(1)}_{mn} = \frac{1}{\pi}(-1)^{m+n+1}\frac{\sin(\pi m y)}{n-my}, \qquad X^{(2)}_{mn} = \frac{1}{\pi}(-1)^{n}\frac{\sin \pi m(1-y)}{n-m(1-y)}, \quad (37)$$

where $y = p_1^+/|p_3^+|$ is the ratio of the width of string 1 to the width of string 3.

Step 2. The expansion of the field Φ in position space is given by

$$\Phi[p^+, x(\sigma)] = \frac{1}{\sqrt{|p^+|}}\sum_{\vec{N}} A_{\vec{N}}(p^+)\prod_{n=-\infty}^{\infty}\psi_{N_n(x_n)}, \quad (38)$$

where $\psi_N(x) = \langle x|N\rangle$ is a harmonic oscillator wavefunction for the N-th excited level. When we plug (38) into the cubic action (34), we find that the coupling between the three strings labelled by $\vec{N}_{(1)}$, $\vec{N}_{(2)}$ and $\vec{N}_{(3)}$ is simply

$$c(\vec{N}_{(1)},\vec{N}_{(2)},\vec{N}_{(3)}) = \int \prod_{n=-\infty}^{\infty} \psi_{N_{(1)n}}(x_{n(1)})\psi_{N_{(2)n}}(x_{n(2)})\psi_{N_{(3)n}}(x_{n(3)})dM \quad (39)$$

where the measure is

$$dM = fDx_1(\sigma)Dx_2(\sigma)Dx_3(\sigma)\Delta[x_1(\sigma)+x_2(\sigma)-x_3(\sigma)]. \quad (40)$$

Step 3. The next step is to note that an x-eigenstate of an oscillator with frequency ω may be represented as

$$|x\rangle = (\text{constant})\exp\left[-\frac{\omega x^2}{4} + i\sqrt{2\omega}a^\dagger - \frac{1}{2}a^\dagger a^\dagger\right]|0\rangle. \quad (41)$$

It follows from this that

$$\sum_{N=0}^{\infty}|N\rangle\psi_N(x) = (\text{constant})\exp\left[-\frac{\omega x^2}{4} + i\sqrt{2\omega}a^\dagger - \frac{1}{2}a^\dagger a^\dagger\right]|0\rangle. \quad (42)$$

Step 4. Now let us assemble the couplings c into the state $|V\rangle$:

$$|V\rangle = \sum_{\vec{N}_1,\vec{N}_2,\vec{N}_3} = c(\vec{N}_{(1)},\vec{N}_{(2)},\vec{N}_{(3)})|\vec{N}_{(1)}\rangle|\vec{N}_{(2)}\rangle|\vec{N}_{(3)}\rangle. \tag{43}$$

Using (39) and (42), we arrive finally at

$$|V\rangle = \int dM \exp\left[\sum_{k=-\infty}^{\infty}\sum_{r=1}^{3}\left(-\frac{\omega_{k(r)}^2}{4}x_{k(r)}^2 - \frac{1}{2}(a_{k(r)}^\dagger)^2 + i\sqrt{2\omega_{k(r)}}x_{k(r)}a_{k(r)}^\dagger\right)\right]|0\rangle. \tag{44}$$

The functional measure is just a product over all Fourier modes:

$$dM = \prod_{m=-\infty}^{\infty}\left(\sum_{r=1}^{3}\sum_{n=-\infty}^{\infty}X_{mn}^{(r)}x_{n(r)}\right)\prod_{r=1}^{3}\prod_{k=-\infty}^{\infty}dx_{k(r)}. \tag{45}$$

The delta-functions allow us to replace all of the modes of string 3 in terms of the modes of strings 1 and 2. Then (44) is just a Gaussian integral in the infinitely many variables $x_{k(1)}$, $x_{k(2)}$, $k=-\infty,\ldots,+\infty$!

Step 5. The Gaussian integral is easily done, and one finds

$$|V\rangle = f(p_1^+,p_2^+,p_3^+)(\det\Gamma)^{-1/2}\exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty}a_{m(r)}^\dagger\overline{N}_{mn}^{(rs)}a_{n(s)}^\dagger\right]|0_{(1)}\rangle|0_{(2)}\rangle|0_{(3)}\rangle, \tag{46}$$

where we have absorbed a constant into f (it was undetermined anyway), and we have introduced the matrices

$$\Gamma_{mn} = \sum_{r=1}^{3}\sum_{p=-\infty}^{\infty}\omega_{p(r)}X_{mp}^{(r)}X_{np}^{(r)}, \tag{47}$$

and

$$\overline{N}_{mn}^{(rs)} = \delta^{rs}\delta_{mn} - 2\sqrt{\omega_{m(r)}\omega_{n(s)}}(X^{(r)\mathrm{T}}\Gamma^{-1}X^{(s)})_{mn}. \tag{48}$$

Alternate, Simpler Derivation

After Fourier transforming, the Delta-functional can be expressed as local conservation of momentum density on the worldsheet:

$$\Delta[p_1(\sigma)+p_2(\sigma)+p_3(\sigma)]. \tag{49}$$

We are trying to find a state $|V\rangle$ which is an oscillator representation of these delta-functionals. Now recall the elementary identity

$$x\delta(x) = 0. \tag{50}$$

The state $|V\rangle$ must therefore satisfy

$$(p_1(\sigma)+p_2(\sigma)+p_3(\sigma))\,|V\rangle = (x_1(\sigma)+x_2(\sigma)-x_3(\sigma))|V\rangle = 0. \tag{51}$$

Let us take the Fourier transform of these equations with respect to the m-th Fourier mode of string 3. Then we make use of the same matrices $X^{(r)}$ introduced above, and we find the following equations, which must vanish for each m:

$$\sum_{r=1}^{3}\sum_{n=-\infty}^{\infty} X^{(r)}_{mn} p_{n(r)}|V\rangle = 0, \qquad \sum_{r=1}^{3}\sum_{n=-\infty}^{\infty} e(p_r^+) X^{(r)}_{mn} x_{n(r)}|V\rangle = 0. \tag{52}$$

If we make an ansatz for $|V\rangle$ of the form

$$|V\rangle = f(p_1^+, p_2^+, p_3^+) \exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} a^\dagger_{m(r)} \overline{N}^{(rs)}_{mn} a^\dagger_{n(s)}\right] |0_{(1)}\rangle|0_{(2)}\rangle|0_{(3)}\rangle, \tag{53}$$

for some coefficients $\overline{N}$, and then expand the x's and p's appearing in (52) into creation and annihilation operators, then one obtains some matrix equations whose unique solution is

$$\overline{N}^{(rs)}_{mn} = \delta^{rs}\delta_{mn} - 2\sqrt{\omega_{m(r)}\omega_{n(s)}}(X^{(r)\mathrm{T}}\Gamma^{-1}X^{(s)})_{mn}. \tag{54}$$

It is clear that $f(p_1^+, p_2^+, p_3^+)$ remains undetermined by this method.

Summary

We have written the Hamiltonian of string theory in light cone gauge as a free term plus a cubic interaction. It turns out (at least for the bosonic string) that this is the whole story! One can use this simple (?) Hamiltonian to derive calculate the string S-matrix, to arbitrary order in string perturbation theory, with no conceptual difficulties. (The remaining measure factor f will be determined in the next lecture.) Since this is a light-cone gauge quantization, the procedure is especially simple. There are no 'vacuum' diagrams, so one just uses the simple Feynman diagrammatic expansion of the S-matrix: the only interaction vertex is a simple string splitting or joining.