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Comments on eddies, mixing and the large-scale ocean circulation

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Comments on eddies, mixing and the large-scale ocean circulation

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Abstract. Some recent laboratory and numerical studies of idealized ocean gyres and circumpolar currents are reviewed in the context of what they might tell us about the relative role of eddy transfer and mixing in setting the structure of the large-scale ocean circulation.

1. Introduction

We briefly review here some recent studies that address the relative role of geostrophic eddies and small-scale mixing in the large-scale ocean circulation and the maintenance of the thermocline.

The starting point for our discussion is the time mean buoyancy equation, written here in a familiar form:

$$\overline{\mathbf{v}} \cdot \nabla \overline{b} = \frac{\partial}{\partial z}\left(k_s \frac{\partial}{\partial z}\overline{b}\right) - \nabla \cdot \left(\overline{\mathbf{v}'b'}\right) \qquad (1)$$

where $\overline{\mathbf{v}}$ is the Eulerian mean velocity and, as usual, variables have been separated in to mean quantities (represented by an overbar) and perturbations from this mean (represented by a prime) due to transient eddies.

In Eq. (1) the effect of small-scale mixing processes has been represented by k_s acting on the vertical (in z) stratification. The effect of buoyancy transfer due to geostrophic eddies is the last term.

In ideal fluid thermocline theory — e.g. Luyten, Pedlosky and Stommel (1983) — both terms on the rhs of Eq. (1) are neglected and a surface buoyancy distribution is mapped adiabatically down in to the interior. In the diffusive thermocline — e.g. Samelson and Vallis (1997), Vallis 2003 (and references therein) — small scale mixing is retained and the eddy flux divergence is (conceptually at least) set to zero. Mixing plays a role at the base of the thermocline, 'joining' the ideal fluid above to the abyss below, as in the model of Salmon (1990).

It is commonly argued that the neglect of $\nabla \cdot \left(\overline{\mathbf{v}'b'}\right)$ in Eq.(1) is justified because:

1. geostrophic eddy fluxes are adiabatic anyway, aren't they? In this case $\nabla \cdot \left(\overline{\mathbf{v}'b'}\right)$ can be written entirely as an *advective* flux (as is made explicit by residual mean theory - see section 2) and so eddies cannot play any role in *diabatic* processes.

2. eddy fluxes are only large in special regions such as boundary currents and jets and are probably negligible elsewhere. So Eq.(1) with $\nabla \cdot \left(\overline{\mathbf{v}'b'}\right) \longrightarrow 0$ will suffice over most of the ocean most of the time.

Using evidence from idealized numerical experiments of eddying gyres and circumpolar currents, we will argue here that both of these statements are misleading: our conceptual models need to be modified to take in to account the role of eddies in diabatic processes and the global implications of local eddy effects. Point 1. is discussed in Section 2 and point 2 in section 3. In section 4 we summarize and conclude.

2. Geostrophic eddies and diabatic processes

The last term on the r.h.s. of (1) represents the eddy flux divergence. Some valuable lessons about its character can be learned from the case of small amplitude, non-breaking eddies on a zonal flow, for which it can be shown (*e.g.*, Plumb [1979], McDougall and McIntosh [1996]) that the i^{th} component of the flux can be written

$$\overline{v_i'b'} = -\left(K_{ij} + L_{ij}\right)\partial_j \overline{b}$$

where $\partial_j \equiv \partial/\partial x_j$ and where K and L are symmetric and antisymmetric tensors. The symmetric part is diffusive and can be related to processes (growth of parcel displacements and non-conservation of the tracer) known to lead to meaningful transport. The flux associated with L, however, is a "skew flux" which is equivalent to advection, rather than diffusion, and is frequently nonzero even when no real transport is occurring. The advective component can be subsumed in to a retransformation of the "mean" velocity using the 'residual mean' theory of Andrews and McIntyre (1976).

To illustrate how to proceed in a general framework in which the small amplitude assumption is not made, we specialize Eq.(1) to two dimensions thus[1]:

$$\overline{v}\frac{\partial \overline{b}}{\partial y} + \overline{w}\frac{\partial \overline{b}}{\partial z} = \frac{\partial}{\partial z}\left(k_s \frac{\partial}{\partial z}\overline{b}\right) - \left[\frac{\partial}{\partial y}\left(\overline{v'b'}\right) + \frac{\partial}{\partial z}\left(\overline{w'b'}\right)\right] \quad (2)$$

Our goal now is to express Eq.(1) in terms of the residual circulation, Ψ_{res}:

$$\Psi_{res} = \overline{\Psi} + \Psi^* \quad (3)$$

where $\overline{\Psi}$ is the streamfunction for the Eulerian mean flow and Ψ^* is the streamfunction for the advective component associated with eddies. The key step is to note that if the eddy flux $\overline{\mathbf{v}'b'}$ lies in the $\overline{b}$ surface, then $\nabla \cdot \left(\overline{\mathbf{v}'b'}\right)$ can be written entirely as an advective transport, $\mathbf{v}^* \cdot \nabla\overline{b}$, where, following Held and Schneider (1999), $\mathbf{v}^*$, the eddy-induced velocity, is defined in terms of a streamfunction Ψ^* given by:

$$\Psi^* = -\frac{\overline{w'b'}}{\overline{b}_y}. \quad (4)$$

Here $\overline{w'b'}$ is the vertical eddy buoyancy flux and $\overline{b}_y$ is the mean meridional buoyancy gradient.

More precisely, to express Eq.(1) in terms of Ψ_{res}, we eliminate $(\overline{v}, \overline{w})$ using (3) and (4), to obtain[2]:

$$\overline{v}_{res}\frac{\partial \overline{b}}{\partial y} + \overline{w}_{res}\frac{\partial \overline{b}}{\partial z} = \frac{\partial}{\partial z}\left(k_s \frac{\partial}{\partial z}\overline{b}\right) - \frac{\partial}{\partial y}\left[(1-\mu)\,\overline{v'b'}\right], \quad (5)$$

where μ is given by:

$$\mu = \left(\frac{\overline{w'b'}}{\overline{v'b'}}\right)\left(\frac{1}{s_\rho}\right). \quad (6)$$

and

$$s_\rho = -\frac{\overline{b}_y}{\overline{b}_z} \quad (7)$$

is the slope of mean buoyancy surfaces.

Note that we have made no approximations here — Eq.(5) is simply Eq.(2) rewritten. But note also that

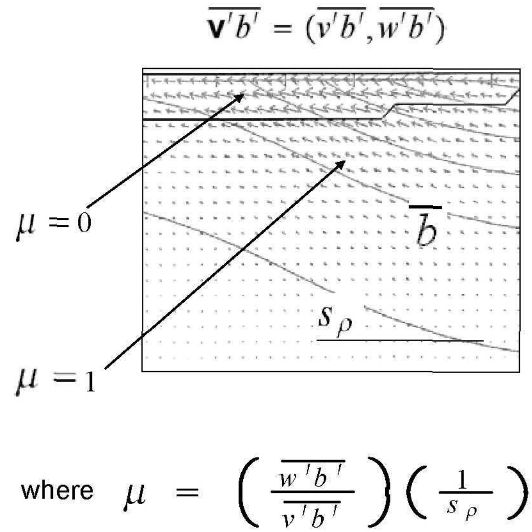

Figure 1. Eddy fluxes, $\overline{\mathbf{v}'b'} = \left(\overline{v'b'}, \overline{w'b'}\right)$, and (time and azimuthally-averaged) mean buoyancy surfaces, $\overline{b}$, with slope s_ρ, diagnosed from the numerical simulations of circumpolar flow described in Karsten et al (2002). The parameter μ measures the degree to which $\overline{\mathbf{v}'b'}$ crosses $\overline{b}$ surfaces. In the interior $\mu \longrightarrow 1$ — the eddies are almost entirely adiabatic. But as the surface is approached $\mu \longrightarrow 1$ — $\overline{\mathbf{v}'b'}$ becomes horizontal crossing the outcropping $\overline{b}$ surfaces with strong diapycnal component.

we have not been entirely successful in absorbing the eddy flux in to a retransformation of the "mean". The last term on the rhs of Eq.(5) remains if the eddies are 'adiabatic'. The parameter μ controls the magnitude of the diapycnal eddy flux: if $\mu = 1$, then the eddy flux is entirely along $\overline{b}$ surfaces, the horizontal diapycnal component vanishes and the advective transport captures the entire eddy flux; if $\mu = 0$, horizontal diapycnal eddy transport makes a contribution to the buoyancy budget.

If eddies were adiabatic, then $\mu = 1$ and Eq.(5) reduces to Eq.(2) with the eddy terms set to zero. Then we would simply have to reinterpret our coarse-grained velocity field as a residual flow. But the following simple kinematic argument tells us that eddies cannot be adiabatic everywhere.

In the interior of the ocean eddy fluxes are indeed likely to be aligned in mean buoyancy surfaces because mixing in the thermocline is observed to be weak — see Ledwell et al (1993). But, as the surface is approached, vertical motion is inhibited and eddy fluxes must become directed horizontally, crossing outcropping buoyancy surfaces, as observed in the numerical experiment shown in Fig.1. Of course there are myriad mixing processes at work in the upper boundary layer of the

[1] The full 3-d treatment can be followed in, for example, Ferrari, 2003, in this volume.

[2] To arrive at (5) from (1), decompose the eddy fluxes $(\overline{v'b'}, \overline{w'b'})$ into an along $\overline{b}$ component $(\overline{w'b'}/s_\rho, \overline{w'b'})$ and the remaining horizontal component $(\overline{v'b'} - \overline{w'b'}/s_\rho, 0)$ — see Marshall and Radko, 2003. The divergence of the along $\overline{b}$ component is then written as an advective transport

$$\nabla \cdot (\overline{w'b'}/s_\rho, \overline{w'b'}) = \overline{v}^*\overline{b}_y + \overline{w}^*\overline{b}_z$$

where Ψ^* is given by Eq. (4). This is combined with mean flow advection in (1) to yield the lhs of (5). The divergence of the diapycnal (horizontal) eddy flux, leads to the last term on the right hand side of (5).

ocean to provide the small-scale mixing processes necessary to allow irreversible lateral eddy transfer. But what are the rate controlling processes? If it is the rate at which eddies strain tracer contours driving gradients on the small scale, then the small scale may be slaved to the large-scale eddy field stirring the surface.

Diagnosis of the eddy resolving 'polar cap' calculations presented in Karsten et al (2002), show that the interior eddy flux is indeed closely adiabatic — see Fig.1. But, as the surface is approached, $\overline{w'b'}$ tends to zero, leaving a horizontal eddy flux directed across $\overline{b}$ surfaces.

Having argued in this section that eddy fluxes have an important diapycnal component, in the next section we will show that, although often confined to local regions, they can have global implications.

3. Maintenance of the thermocline in a turbulent ocean

What is the probable (possible) role of eddies in the integral buoyancy budgets of ocean gyres? Consider Fig.2, showing a hydrographic section running $N \leftrightarrow S$ through the Atlantic ocean. We clearly see the 'bowl' of the thermocline dipping down in the subtropics of each hemisphere. This is created by the pumping down of warm water from the surface in the Ekman layers — as in the sketch — and the sucking up of cold water around the poles and at the equator. But what happens to the warm water pumped down from the surface. How is equilibrium established?

As sketched in the schema of the warm water lens at the bottom of Fig.2, there are two likely possibilities. The first is that the heat is 'diffused away' in to the interior by small scale mixing processes, presumably associated with the breaking of internal waves. Indeed Samelson and Vallis (1997) describe such a scenario in their numerical solutions of the planetary geostrophic equations, obtaining an interior, diffusively controlled boundary layer — Salmon (1990) — that 'joins' an upper ideal regime to the abyss. But there is another possibility. Large-scale eddies associated with the instability of the gyre (localized to the western boundary currents) could break off blobs of warm water and flux them laterally, thereby equilibrating the system, as sketched in the schematic diagram at the bottom of Fig.2. In terms of eddy fluxes this appears as a diapycnal flux directed laterally across outcropping buoyancy surfaces parallel to the sea surface, as shown in Fig.1.

3.1. A laboratory analogue: equilibration of a warm, pumped lens on an '$f-$plane'

This possibility has been recently studied in both a laboratory setting and by numerical experiment. Fig.3

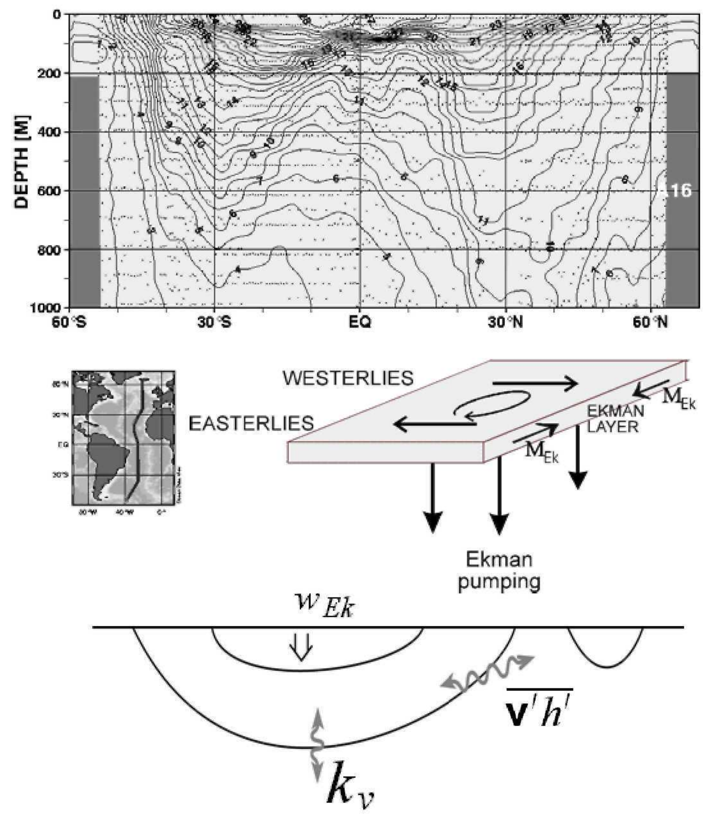

Figure 2. A hydrographic section (WOCE A16) through the Atlantic along the path marked, cutting across the subtropical gyres of both hemispheres. The schematic diagram shows the pattern of Ekman pumping which results in warm surface waters being pumped down in to the interior of the ocean. We discuss here the relative importance of small scale mixing acting in the interior, k_v, and lateral geostrophic eddy fluxes, $\overline{v'h'}$, where h is the depth of the warm lens, in allowing a steady state to be achieved.

shows a laboratory lens, formed by the pumping of warm water down from the base of a rotating heated disc. Experimental details can be found in Marshall et al (2002). As can be seen from the photograph looking down on the apparatus, geostrophic eddies form and sweep water away from under the disk allowing an equilibrium to be established in which the rate of creation of available potential energy by the action of pumping and differential heating, is exactly balanced by its rate of conversion in to eddy energy by baroclinic instability. This is just the balance hypothesized in Gill, Green and Simmons (1974). Simple theory discussed in Marshall et al (2002) predicts that, as supported by the graph reproduced in Fig.3, the depth of the lens, $h \sim \left(\frac{f}{B}\right)^{\frac{1}{2}} w_{Ek}r$, where f is the rotation rate of the tank, w_{Ek} is the Ekman pumping rate, r is the radius of the heating disc and $B = w_{Ek}g'$, is the imposed buoyancy flux associated with the pumping down of warm water of reduced gravity g'. In this experiment, then, the stratification of the lens at equilibrium is controlled by eddy transfer rather than small scale mixing. Diapycnal eddy fluxes play a key role in the integral buoyancy budget of the lens. But what about more realistic scenarios — with a β−effect and western boundary currents — that are closer analogues of ocean gyres?

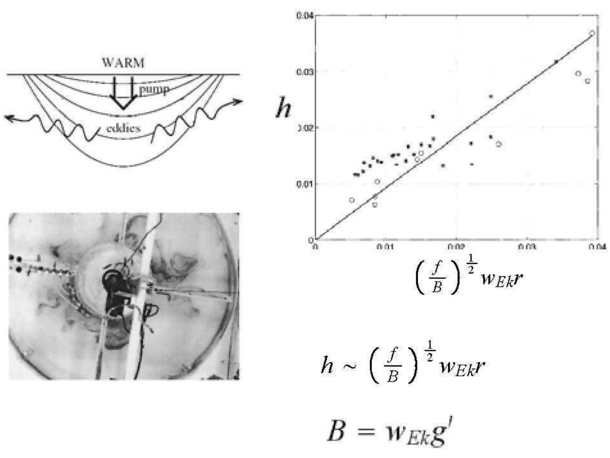

Figure 3. On the top left we show a schematic diagram of the warm, pumped lens studied in the laboratory by Marshall et al (2002). A photograph from the experiment itself is shown on the bottom left — we see eddies forming around the periphery of the warm lens which flux warm fluid away from under the rotating heated disc, arresting the deepening of the lens. The observed depth of penetration at equilibrium is plotted against the theoretical prediction on the top right. In this experiment, diapycnal eddy fluxes are central to achieving an equilibrium.

3.2. Equilibration of numerical ocean gyres on a 'β−plane'

In an attempt to address the role of β and the presence of boundaries, Radko and Marshall (2003) carried out high resolution numerical experiments of miniature ocean basins — both single gyre and double gyre — as can be seen in Fig.4. The depth of these gyres, in Sverdrup balance, scales as in classical theory $h \sim \left(\frac{w_{Ek}f^2}{\beta g'}r\right)^{\frac{1}{2}}$, rather than the Marshall et al (2002) f−plane result given in section 3.1. However, eddies play a crucial role in integral balances of these numerical gyres.

To diagnose the importance of eddies in the maintenance of the gyres, Radko and Marshall (2003) compared the diapycnal volume flux due to eddies with that due to small-scale mixing.

Both eddy and small scale mixing terms in Eq.(1) result in a finite cross isopycnal volume flux per unit area denoted below by w_{dia}[3]. In a statistically steady state, this flux can be conveniently expressed in terms of $\nabla \cdot \overline{(\mathbf{v}'b')}$ and $\frac{\partial}{\partial z}\left(k_s \frac{\partial}{\partial z}\overline{b}\right)$ by rewriting our equations using buoyancy (rather than z) as a vertical coordinate:

$$w_{dia} = w(x,y,\overline{b}) - \frac{D}{Dt}z(\overline{b}) = \overline{w} - \left[\overline{u}\frac{\partial}{\partial x}z(\overline{b}) + \overline{v}\frac{\partial}{\partial y}z(\overline{b})\right]. \tag{8}$$

where D/Dt is the total derivative. Eq.(8) is further simplified using the expression for the isopycnal slope in z-coordinates:

$$\begin{cases} \frac{\partial}{\partial x}z(\overline{b}) = -\frac{\overline{b}_x}{\overline{b}_z}, \\ \frac{\partial}{\partial y}z(\overline{b}) = -\frac{\overline{b}_y}{\overline{b}_z}. \end{cases} \tag{9}$$

When (9) is substituted in (8), the result is

$$\overline{\mathbf{v}} \cdot \nabla\overline{b} = w_{dia}\overline{b}_z,$$

Comparing this with the buoyancy equation (1), we arrive at the expression for the cross-isopycnal flux

$$w_{dia} = \frac{\frac{\partial}{\partial z}\left(k_s\frac{\partial}{\partial z}\overline{b}\right) - \nabla \cdot \overline{(\mathbf{v}'b')}}{\overline{b}_z}, \tag{10}$$

The eddy-driven component is thus readily isolated as:

$$w_{dia\_eddy} = \frac{-\nabla \cdot \overline{(\mathbf{v}'b')}}{\overline{b}_z}, \tag{11}$$

Clearly the relative importance of eddies and small-scale mixing in setting w_{dia} is an important measure

[3] Note that the diapycnal velocity w_{dia}, is often denoted by w^* in thermocline theory — see, for example, Pedlosky, 1996. Unfortunately the * notation is also commonly used in oceanography to represent eddy-induced velocities, a different quantity. Here, to avoid confusion we use w_{dia} to denote a diapycnal velocity.

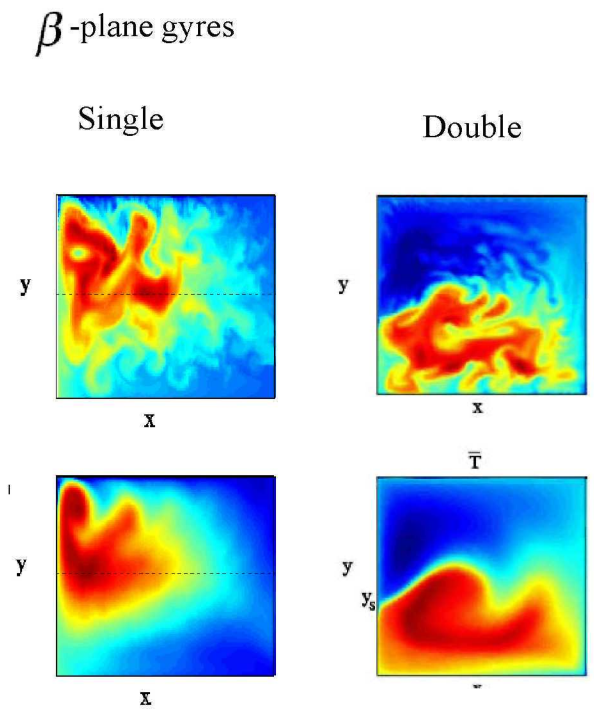

Figure 4. Near surface temperature fields from high resolution numerical simulations of ocean gyres on a β−plane — single gyre on the left, double gyre on the right. From Radko and Marshall (2003). Instantaneous fields are shown in the top panels, time-mean fields below. In each case the initial conditions were one of uniform temperature — interior stratification is set up through the collusion of patterns of surface warming/cooling, Ekman pumping, (resolved) eddy processes and small scale mixing.

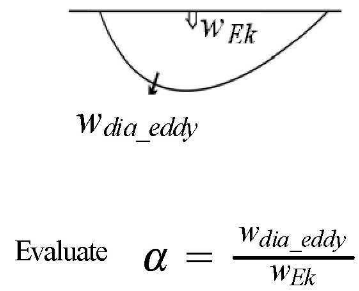

Figure 5. A schematic of a warm water lens. The continuous line is a temperature surface outcropping at the sea surface. The volume of fluid within the lens is increased by the pumping of warm water down from the surface, w_{Ek}; it can be reduced by the action of diabatic eddy fluxes, w_{dia_eddy}. Their ratio, α, is a measure of the importance of diapycnal eddy fluxes in the integral balance of the lens.

of the relative role of eddies and small scale mixing in the buoyancy budget. This, as sketched in Fig.5, can be ascertained by diagnosing the quantity

$$\alpha = \frac{\langle w_{dia\_eddy} \rangle}{\langle w_{Ek} \rangle}$$

from our simulation, where $\langle \rangle$ are appropriate area integrals.

Fig.6 (and legend) presents results of diagnostics of α from the double-gyre experiment shown on the r.h.s of Fig.4. We see that in the upper ocean (corresponding to the warmest fluid) α exceeds 0.85. In other words, of the volume of warm water pumped down from the surface Ekman layer, more than 85% of it is fluxed away by eddies. From the horizontal maps of w_{dia_eddy}, we see that this occurs almost entirely in the region of the model's separated Gulf Stream and is associated with diapycnal eddy fluxes. Deeper down eddies play a lesser role, but are more important than small scale mixing in the volume budget within all temperature surfaces.

If these numerical simulations are any guide, the role of small scale mixing in the thermocline of gyres may be supplanted by eddy transfer in global budgets of buoyancy and potential vorticity. Moreover eddies may have an increasingly dominant role to play as the surface is approached. Similar conclusions have recently been drawn by Henning and Vallis (2003), who find that small scale mixing remains a player at the base of the thermocline but is dominated by eddies on shallower layers. This is consistent with the results presented in Radko and Marshall (2003) — and summarized in Fig.6.

Motivated by the similarity in the pattern of w_{dia_eddy} and $\nabla^2 h$ (where h is layer depth) shown in Fig.6, Radko and Marshall (2003) modify LPS theory to take account

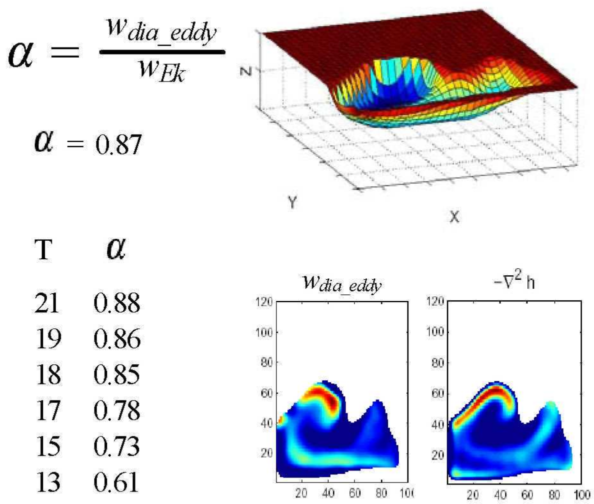

Figure 6. $\alpha(T)$ diagnostics from the double gyre experiment shown in Fig.4. The top right panel shows the geography of an outcropping temperature surface. The bottom panel shows the pattern of w_{dia_eddy} and $\nabla^2 h$, where h is the depth of the temperature surface shown in the top panel. Diabatic eddy fluxes are concentrated in the region of the model's separated Gulf Stream.

of w_{dia_eddy}. They are able to find solutions in closed basins in which eddy processes are responsible for the equilibration of the gyre. Moreover western boundary layers set the character of the interior potential vorticity distribution — the boundary layer is *active*.

3.3. The role of eddies in Circumpolar Currents

The laboratory experiment described in Marshall et al (2002) — see Fig.3 — and numerical studies of circumpolar flow by Karsten et al (2002), suggest that eddies may play a central role in setting the depth and stratification of the Antarctic Circumpolar Current. The ideas are further illustrated in a recent laboratory study by Cenedese et al (2003). As shown in Fig.7, the pumping down of light water in the subtropics, and the sucking up of dense fluid around Antarctica, is represented in the laboratory by (one needs to flip things over!) pumping of salty (and hence dense) fluid up from below over the periphery and extracting fresher (and hence lighter) fluid from the centre. In this way one creates a 'donut' of salty fluid which thickens through the action of pumping from below. But the thickening is arrested by baroclinic instability which sweeps blobs of dense eddying fluid toward the centre where it is sucked out of the system. The height of the equilibrated 'donut' scales like $h \sim \left(\frac{f}{B}\right)^{\frac{1}{2}} w_{Ek} r$, just as in the study of the lens of Marshall et al (2002) outlined in section 3.1. Again, these eddy fluxes are diapycnal in nature.

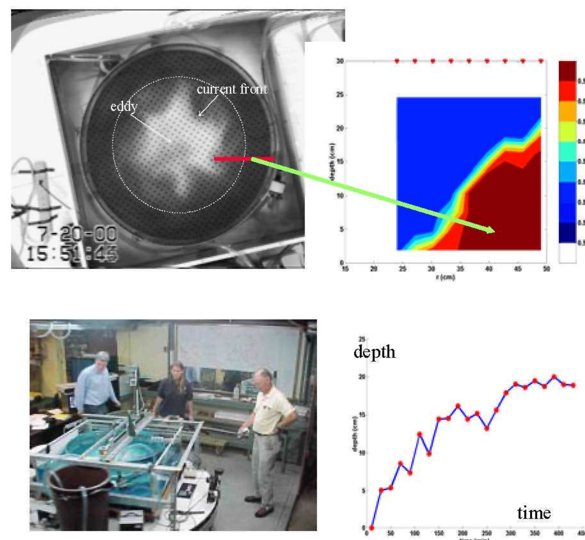

Figure 7. The laboratory experiment described in Cenedese et al (2003), enquiring in to the processes that control the depth and stratification of the Antarctic Circumpolar Current. On the top left we see the experiment from above. Salty (dense) fluid is pumped up through holes in the base of a circular, rotating tank. An equal mount of fluid is extracted over the centre of the tank. The deepening of the salty 'donut' of fluid (dark in the picture) – a vertical salinity section through which can be seen on the top right panel — is arrested by the baroclinic instability of the azimuthal current associated with it. Eddies sweep the salty fluid in toward the centre where it is sucked out of the tank. A time series of the depth of the salty donut measured close to the wall, is shown in the bottom right panel. We see the initial rise as fluid is pumped in from the base, and the equilibrium level as lateral fluxes begin to balance the vertical influx.

4. Conclusions

To conclude I want to make three points inspired by the foregoing examples and discussion:

1. Geostrophic eddy transfer is often erroneously neglected in favor of small scale mixing in discussions of large scale buoyancy balances.

2. Numerical and laboratory experiments suggest that eddy transfer, rather than small-scale mixing, may play a central role in maintaining the structure of the thermocline, particularly in the southern ocean, but also in subtropical ocean gyres.

3. Geostrophic eddies are likely to play a key role in diabatic processes, particularly near the upper surface, where eddy fluxes inevitably have a diapycnal component.

5. Acknowledgements

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