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Detection of quantum noise

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Noise in the Quantum Realm, What is Measured?

Excess Noise Detection

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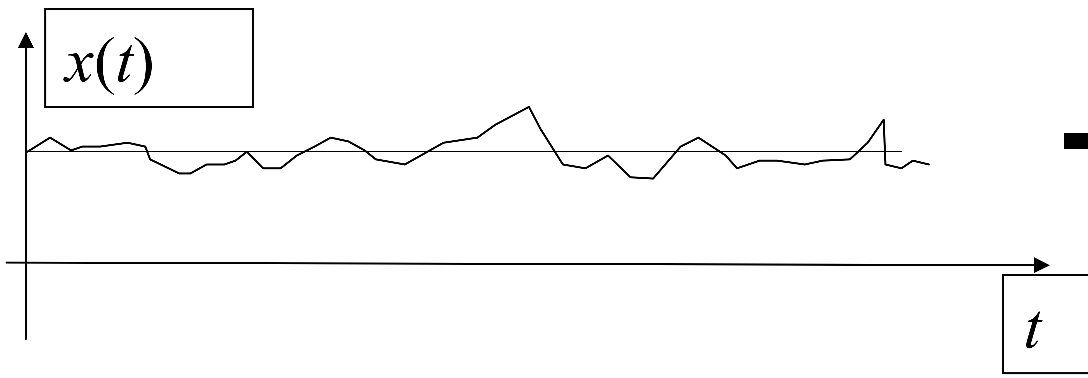
Thanks: E. Conforti, C. Glattli, M. Heiblum,
R. de Picciotto, M. Reznikov, U. Sivan

Outline:

- Quantum noise, Physics of Power Spectrum
- Fluctuation-Dissipation Theorem, in steady state
- Shot-Noise, Excess noise, dependence on full state of system
- What is detected in a quantum noise and in an excess noise measurement?

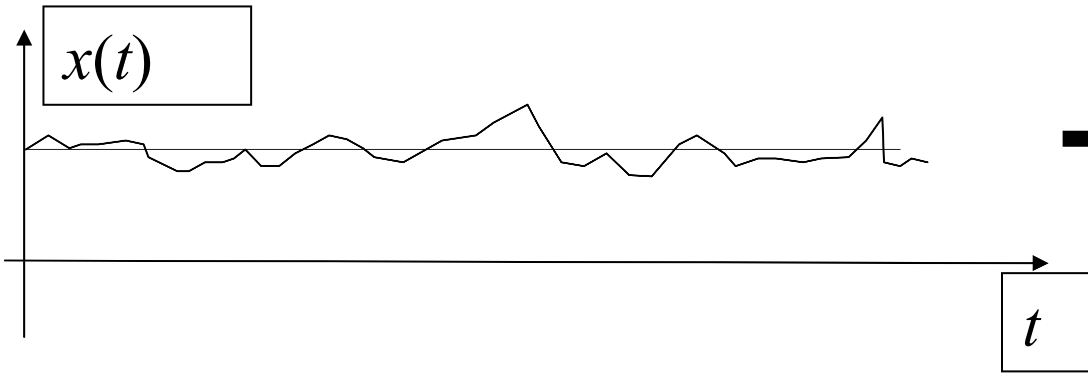
(nonuniversal vs. universal)

Classical measurement of time-dependent quantity, $x(t)$, in a stationary state.



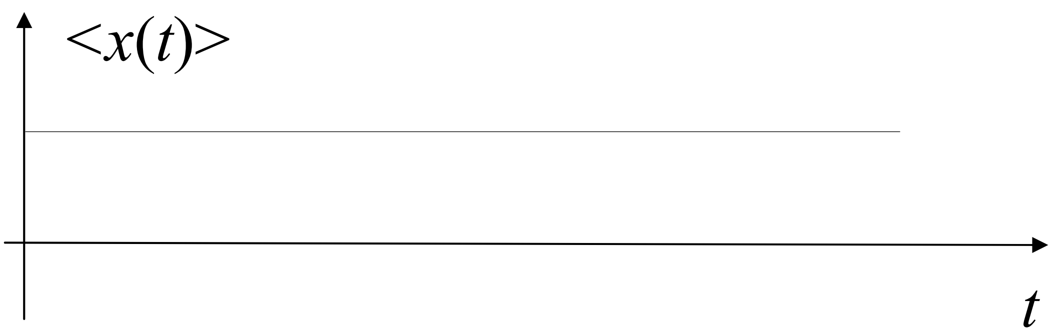
$$C(t)=<x(t)\ x(t')>$$

Classical measurement of a time-dependent quantity, $x(t)$, in a stationary state.



$C(t) = \langle x(t)\, x(t') \rangle$

Quantum measurement of the expectation value, $\langle x_{op}(t) \rangle$, in a stationary state.



$C(t) = ?$

STATISTICAL PHYSICS

by

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Volume 5 of *Course of Theoretical Physics*

Translated from the Russian by Peierls ?

This relation has to be regarded as a definition of the quantity which has been denoted here symbolically by $(x^2)_\omega$. Although the x_ω are complex, the quantity $(x^2)_\omega$ is evidently real. (It is sufficient to remark that the left-hand side of (118.4) differs from zero only when $\omega' = -\omega$, and the change to complex conjugate quantities means changing the sign of ω, i.e. the interchange of ω and ω').

Inserting (118.4) in $\phi(\tau)$ and carrying out the integration over $d\omega'$, we find

$$(x^2)_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi(\tau)e^{i\omega\tau}\,d\tau. \tag{118.7}$$

By treating the quantity x as a function of time, we have implicitly assumed it to behave classically. All the above formulae can, however, easily be re-written so as to apply to quantum-mechanical quantities. For this purpose one has to consider, instead of the quantity x, its quantum-mechanical operator $\hat{x}(t)$, and its Fourier transform

$$\hat{x}_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{x}(t)e^{i\omega t}\,dt. \tag{118.8}$$

The operators $\hat{x}(t)$ and $\hat{x}(t')$ for different instants of time do not, in general, commute, and the correlation function must now be defined as

$$\phi(t'-t) = \overline{\tfrac{1}{2}[\hat{x}(t)\hat{x}(t')+\hat{x}(t')\hat{x}(t)]}, \tag{118.9}$$

where the bar denotes averaging by means of the exact quantum-mechanical

DETECTION OF "QUANTUM NOISE"

Uri Gavish, Y. Levinson, YI .

ZPF exist and appear in well-known effects:

- Debye-Waller factor,
- Casimir force,
- Lamb shift, etc.

What does detector looking at ZPF see?

i.e. Which noise correlator measured (Lesovik-Loosen)?

Does ZPF dephase as $T \to 0$?

Shot-Noise , HBT , ...

- **NO PASSIVE ZPF DETECTION,**
- **NEED ENERGY: AMP/"DRIVER"...**
- **ZPF CORRELATOR FROM F.T. of**

  ABSORPTION SPECTRUM.

- **NO DEPHASING as $T \to 0$!**

(BUT: Nearly degenerate ground-state???)

Usual symmetrized correlator/power specrum –

NO GOOD, GIVES MISLEADING RESULTS!

What is the Physical Noise Correlator?

$C(t'-t) \equiv \langle j(t)j(t')\rangle = Fourier\ Transf \quad of \quad S(\omega)$

$S(\omega) \propto$ power exchanged with EM field.

Classically, both C and S real and symmetric.

Q. M.: $\hat{j}(t)$ operator, $[\hat{j}(t), \hat{j}(t')] \neq 0, \quad [t \neq t']$.

$$C(t) = C(-t)^* \neq C(-t), \qquad S(\omega) \neq S(-\omega).$$

$$S(\omega) = \hbar \sum_{if} P_i |\langle f|\hat{j}|i\rangle|^2 \delta(E_i - E_f - \hbar\omega),$$

$|i\rangle$ – eigenstates, energies E_i, populations P_i.

At equilibrium, temp $T, \Rightarrow$ **Detailed Balance**:

$$S(\omega) = S(-\omega)e^{-\hbar\omega/k_BT}.$$

$S(\omega) = S(-\omega)$ holds only for $\hbar|\omega| \ll T, \quad |t| \gg \hbar/T$.

$C(t)$ is not real and not symmetric,

$C(t)$ is not directly measurable,
except via $S(\omega)$, AT BOTH $\omega > 0, \omega < 0$.

Antenna coupled to EM field with N_ω photons.

$$Coupling \cong \frac{\vec{A}}{c} \bullet \int \vec{j} \; d^3r$$

$S(\omega)$ gives:

Emission x'section for $N_\omega = 0$, for $\omega > 0$.

Absorption x'section for $\omega < 0$, for $N_{|\omega|} = 1$.

Easily generalizable to finite $N_{|\omega|}$

$\Rightarrow$ The sign OF ω is RELEVANT

$\Rightarrow$ Symmetrized: $C_S(t'-t) \equiv (1/2)\langle \hat{j}(t)\hat{j}(t') + \hat{j}(t')\hat{j}(t)\rangle$,

is customary, but **BAD**, cf. Lesovik-Loosen.

NO NOISE DETECTED PASSIVELY at $T = 0$

NEED: Active Detector, Amplification, etc

$$\text{Emission} = S(\omega) \neq S(-\omega) = \text{Absorption},$$
(in general)

From field with N_ω photons, net absorption
(Lesovik-Loosen, Gavish et al):

$$N_\omega S(-\omega) - (N_\omega + 1) S(\omega)$$

For classical field ($N_\omega >>> 1$):

$$\textbf{CONDUCTANCE} \propto [S(-\omega) - S(\omega)] / \omega$$

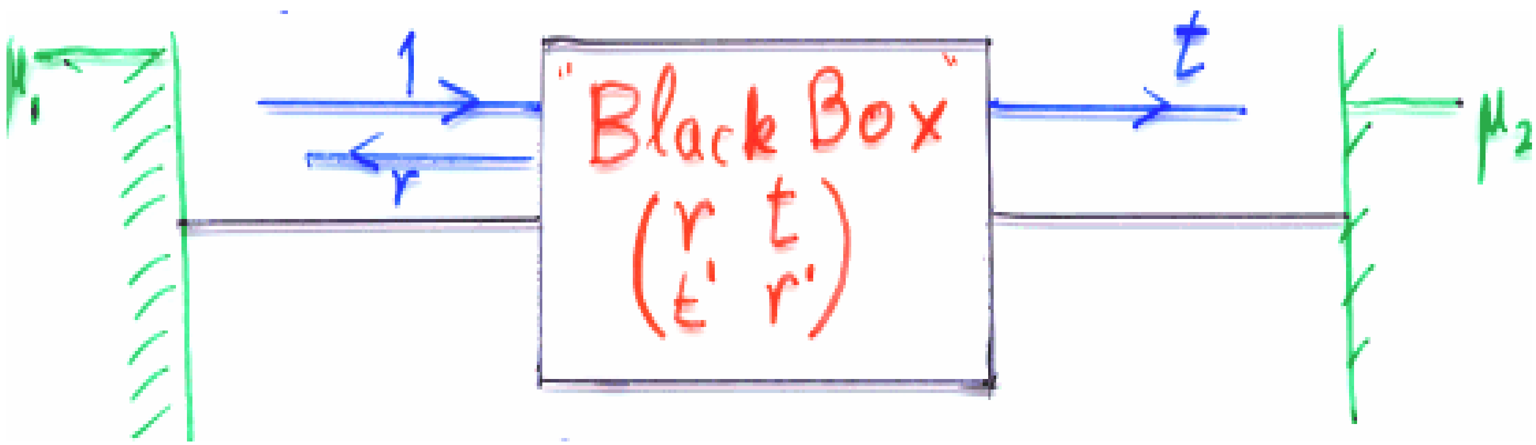
This is the Kubo formula!

Fluctuation-Dissipation Theorem (FDT)

Valid in a **nonequilibrium** steady state!!

Dynamical conductance-response to tickling ac field,
Given by $S(-\omega) - S(\omega)$ = F.T. of the commutator of
the temporal current correlator

Landauer: 2-terminal conductance = transmission



$G \equiv I/V = (e^2/\pi\hbar)\,|t|^2$, with spin.

$eV \equiv \mu_1 - \mu_2$

Equilibrium Noise in the Landauer Picture

$$|j_{ll}|^2 = |j_{ll}|^2 = (e\mathrm{v}T)^2\ ;\ |j_{lr}|^2 = |j_{rl}|^2 = (e\mathrm{v}\ \mathrm{T}(1\text{-}\mathrm{T}))^2$$

Since T(1-T) + T^2 = T, from van Hove-type expression for $S(\omega)$:

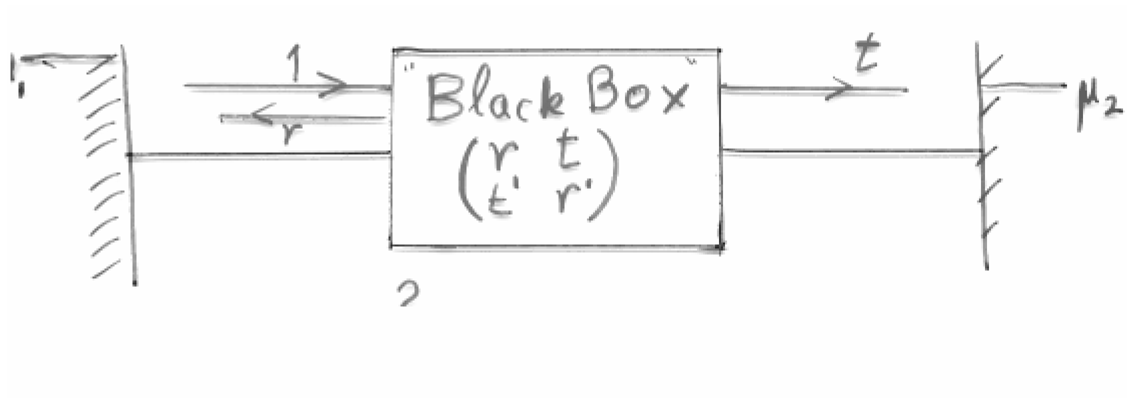
- Temp = 0: **$S(\omega) \propto$ G ω, (ω < 0 only)**
- Temp >> **ħω: $S(\omega) \propto$ G** ·Temp.

(Nyquist!)

Quantum Shot-Noise (Khlus, Lesovik)

For Fermi–Sea Conductors, different for BEAMS in Vacuum, for same current.

μ

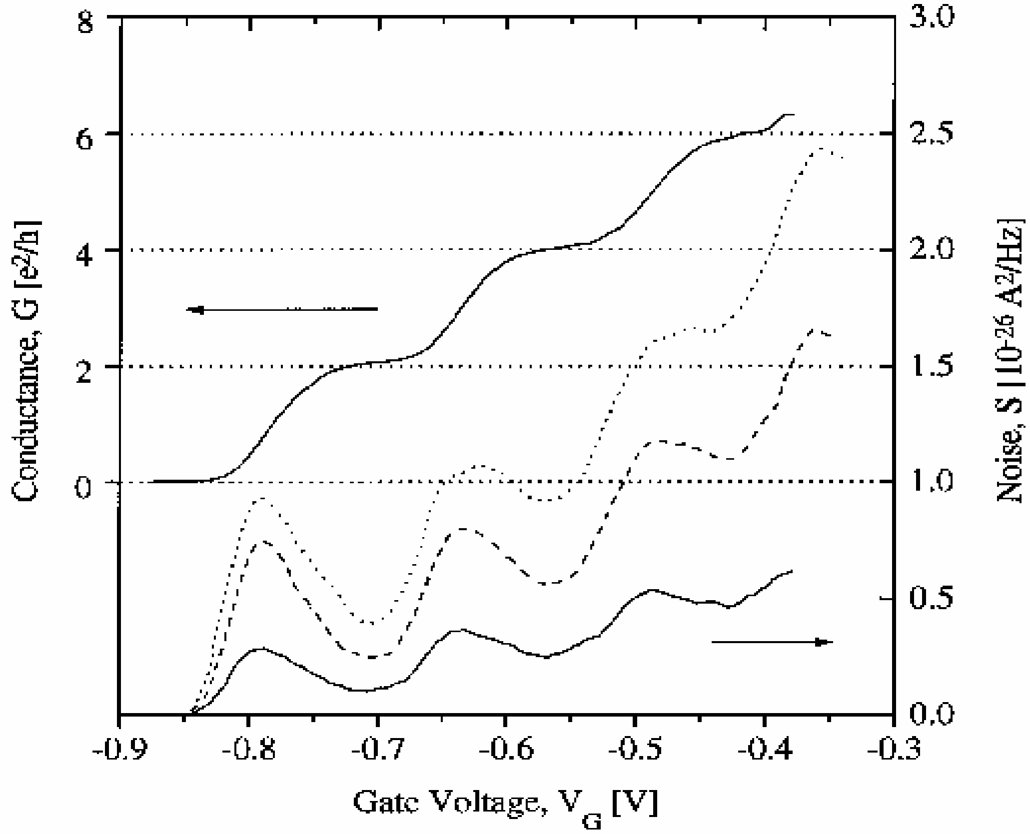


Left-coming Scattering state

$|\langle lk|\ j\ |rk'\rangle|^{2} = v_F^{2}\ TR$, for $(k- k' \ll 1/L)$

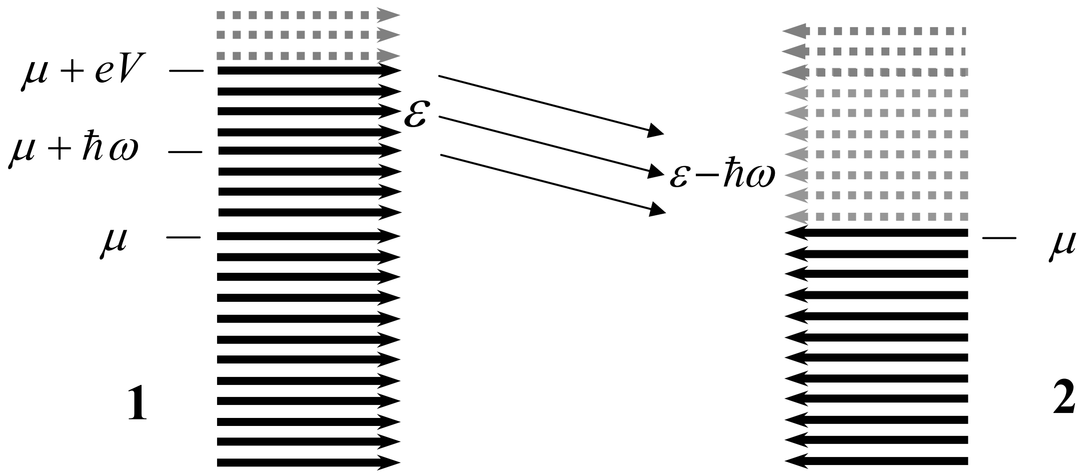
$\rightarrow S(\omega) = 2e(e^{2}V/\pi\hbar)\ T(1-T)$, **$\omega \ll V$**

$= 0$, $\omega > V$. This is **Excess Noise.**

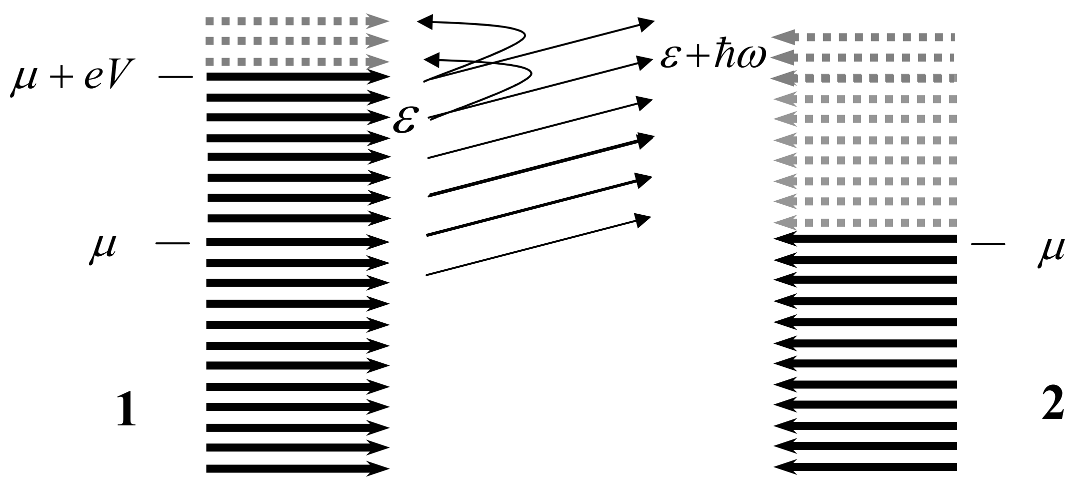


Exp confirmation, of T(1-T)

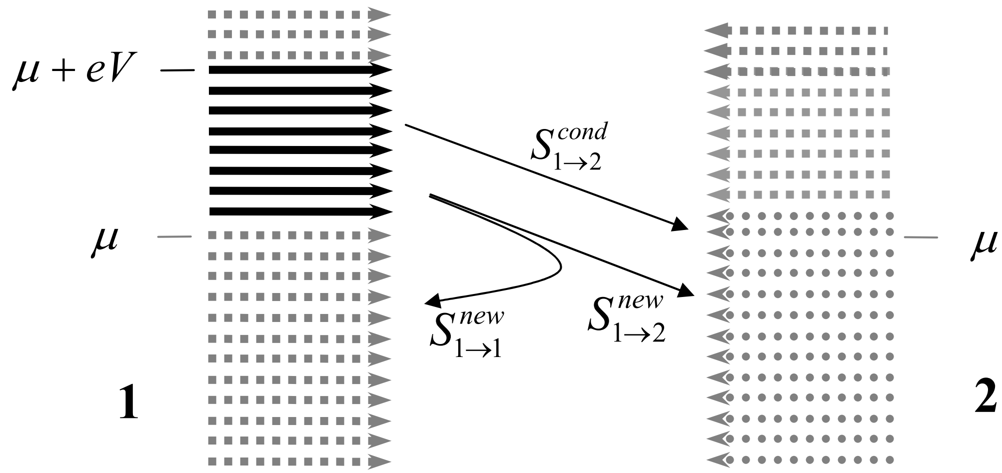
Reznikov et al, WIS, 1997



a. Emission

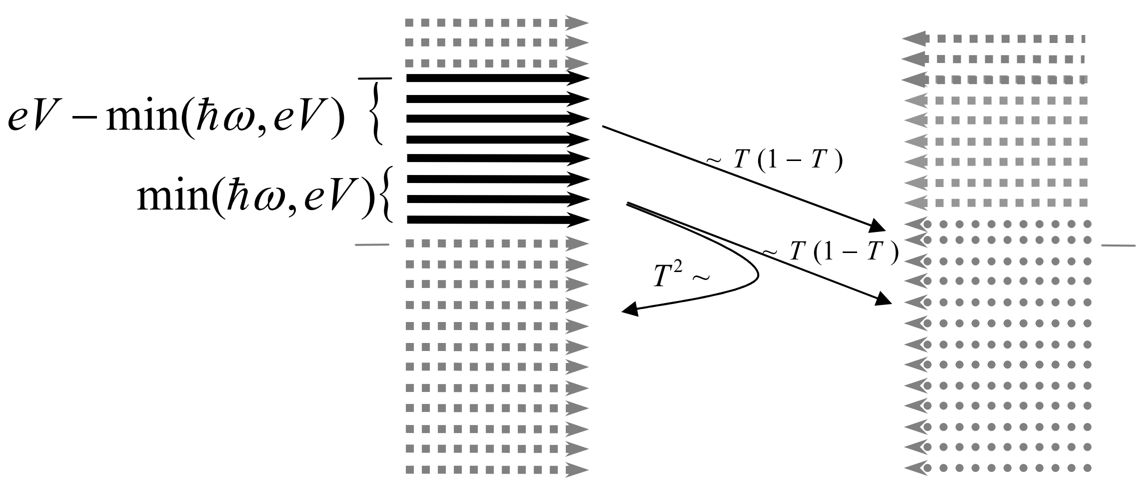


b. Absorption



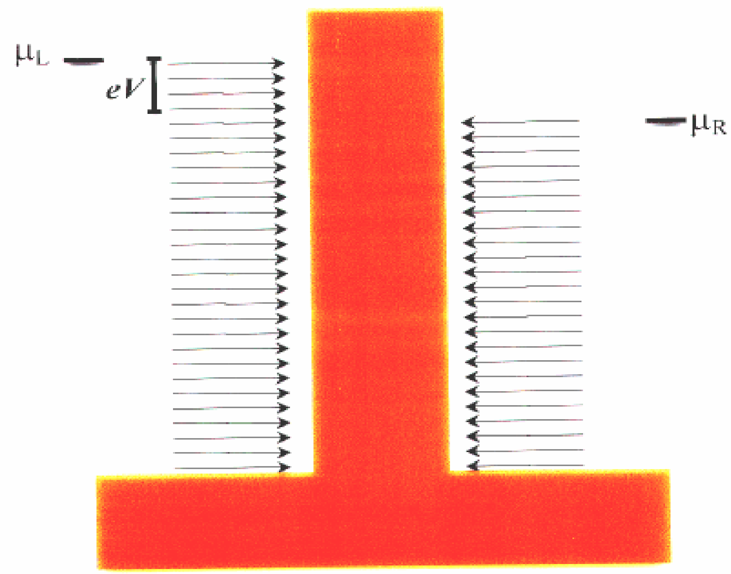
$\mu + eV$
$S_{1\to 2}^{cond}$
μ
$S_{1\to 1}^{new}$
$S_{1\to 2}^{new}$
μ
1
2

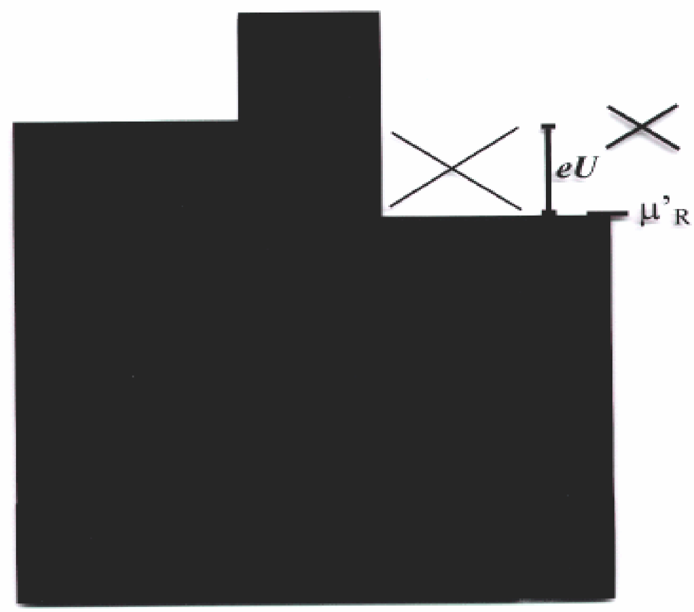
a.



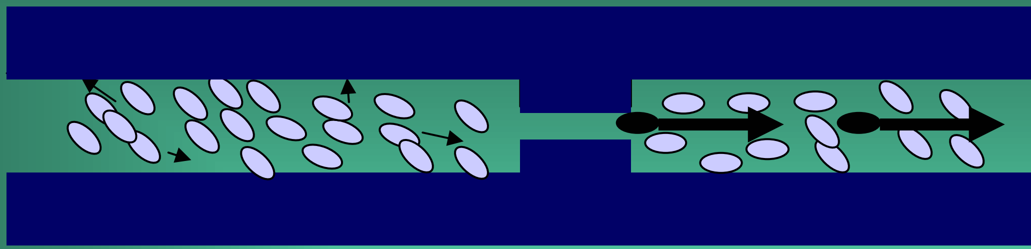
$eV - \min(\hbar\omega, eV)$
$\sim T(1-T)$
$\min(\hbar\omega, eV)$
$\sim T(1-T)$
$T^2 \sim$

b.


μ_L
eV
μ_R



CONCEPT OF NEW EXPERIMENT



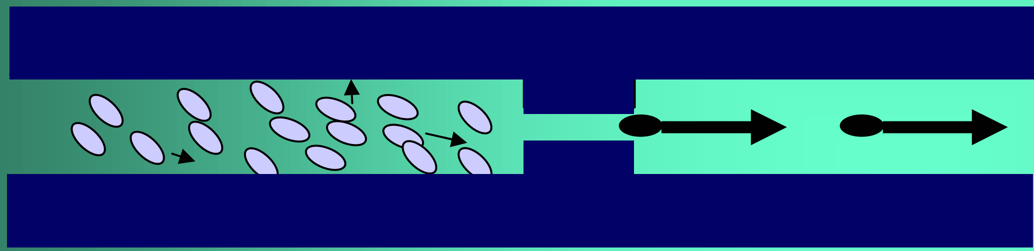
Current Noise Measurment in Quantum Point Contact

Shot noise measurment.

Pauli blocking effect between **all** particles.

Charge 1/3 detection (Weizmann – Saclay).

Interaction effects.



Current Noise Measurment in Beams in Vacuum.

Shot noise measurment.

Pauli blocking effect between the particles in the current **only**.

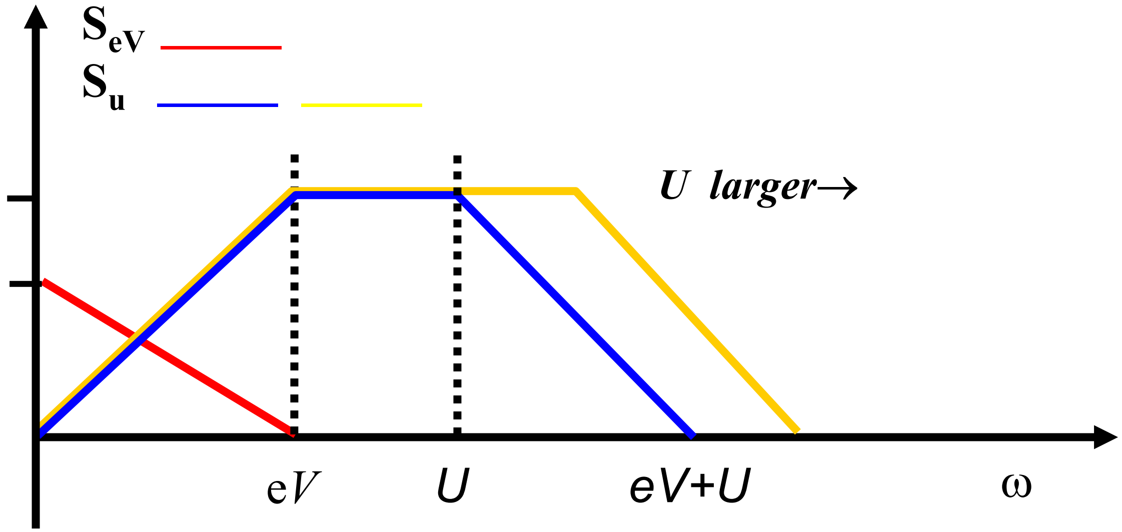
Charge 1/3 detection: impossible.

Interaction effects.

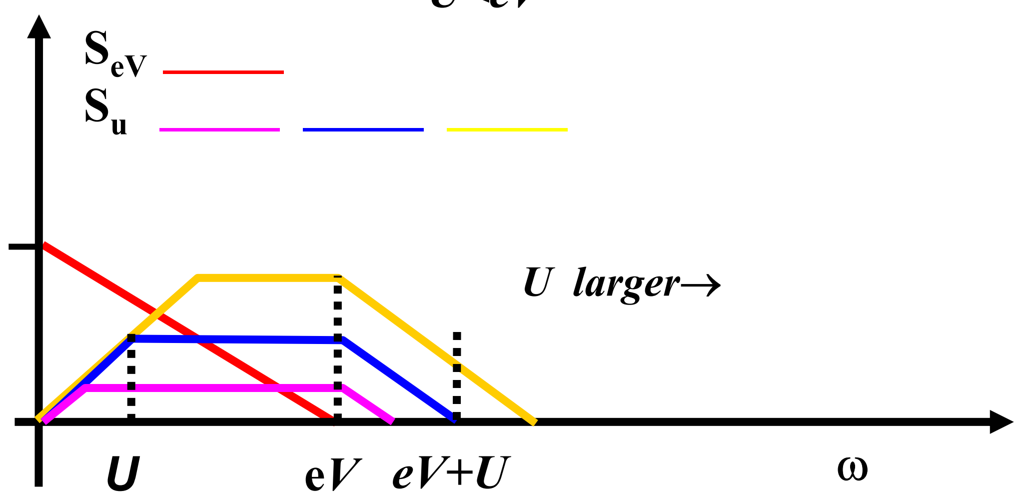
Is the current noise identical to a beam in vacuum?

Answer: NO. The Pauli principle blocks more transitions in the point-contact, so a different noise is emitted. By changing the occupancy at the sink (with a gate), this difference can be manipulated and the radiation spectrum can be controlled.

U>eV



U<eV



Hanbury-Brown & Twiss Correlations (1956)

$\langle \Delta n_k^2 \rangle = \langle n_k \rangle (\langle n_k \pm 1 \rangle)$ in a photon beam (+!)

$\langle I(t) I(t') \rangle$

Can use 1 counter, 2 counters, split beam
↓
"dead time" probem.

⇓ Purcell, Glauber ....

what about Fermions? — sgn.

- Narrow "beam" $(V \to 0) \Rightarrow C$ is played by v_F $[VG]$ $C_{II} \propto C_{nn}$
- Blocking by Fermi Sea - very important
  makes Lesovik's Theory valid.
- But, "Vaccum Beam" ≠ Fermi Sea (Transport)
- However, interesting in Beam-split case.

Present "HBT solid state" exps just confirm Lesovik

3-4 orders of mag missing for "Real HBT Physics"

"Fermi hole" ... ???

rimental input conductance at the plateau is $0.83G_Q$, rather than G_Q. With p used to characterize both the partial transmission through the input point contact and the back-reflection from the beam splitter, and using T to characterize the transmission at the beam splitter with $\bar{T} \equiv (1 - T)$, an analytic expression for the tion 6 is identical to the cross-covariance (Eq. 1) when $\langle V_2^2 \rangle = \langle V_3^2 \rangle$. For $p = 1$, this is true for any T. However, for $p < 1$, it is true only for $T = 1/2$; in our experiments, we maintained $T \approx 1/2$. The input currents throughout the experiment typically ranged from 20 to 40 nA, and the noise measurement sensitivity limit in each output branch was 5 nA.

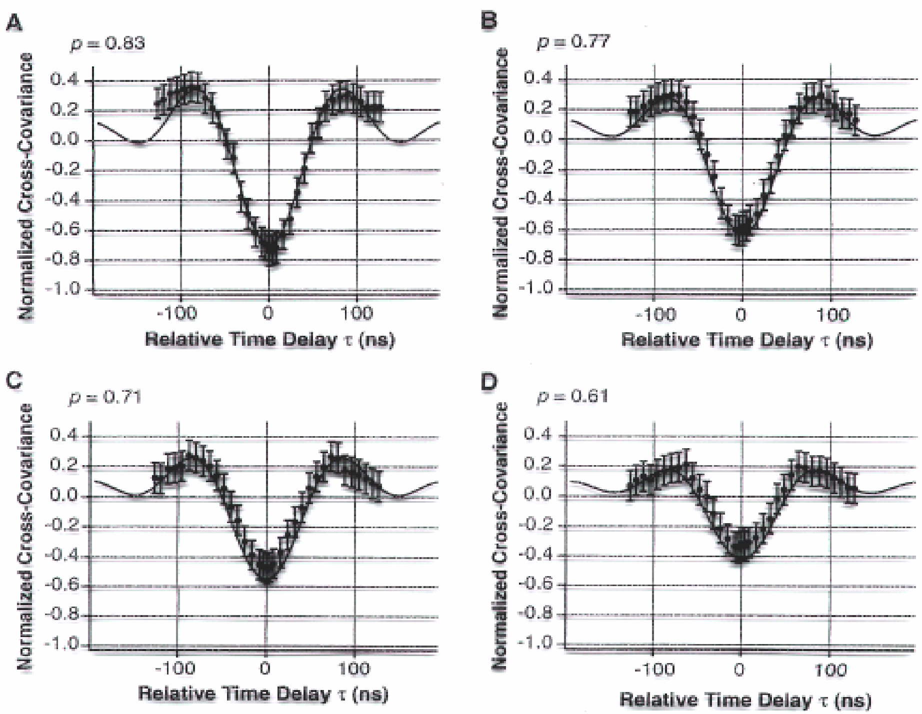


Fig. 3. The normalized cross-covariance is plotted as a function of τ at four values (**A** through **D**) of the input quantum point contact transmission probability: p = 0.83, 0.77, 0.71, and 0.61. In each case, the minimum cross-covariance occurs at delay time $\tau = 0$, corresponding to $\hat{\rho}(\tau = 0)$ shown in Fig. 2. The solid line represents the simulated cross-covariance for the actual measurement circuit.

$= 2 \cdot 10^{+11}/\text{sec} = \frac{30\,\text{nA}}{1.6 \cdot 10^{-19}\,\text{Coul}}$ $\boxed{\frac{e}{I} \approx \frac{1}{200}\ \text{nS}}$

W. D. Oliver, J. Kim, R.C. Liu and Y. Yamamoto, p. 299
also: Henny et al. p. 235; Büttiker, p. 275.

Partial Conclusions

- **The noise power is the ability of the system to emit/absorb (depending on sign of ω).**

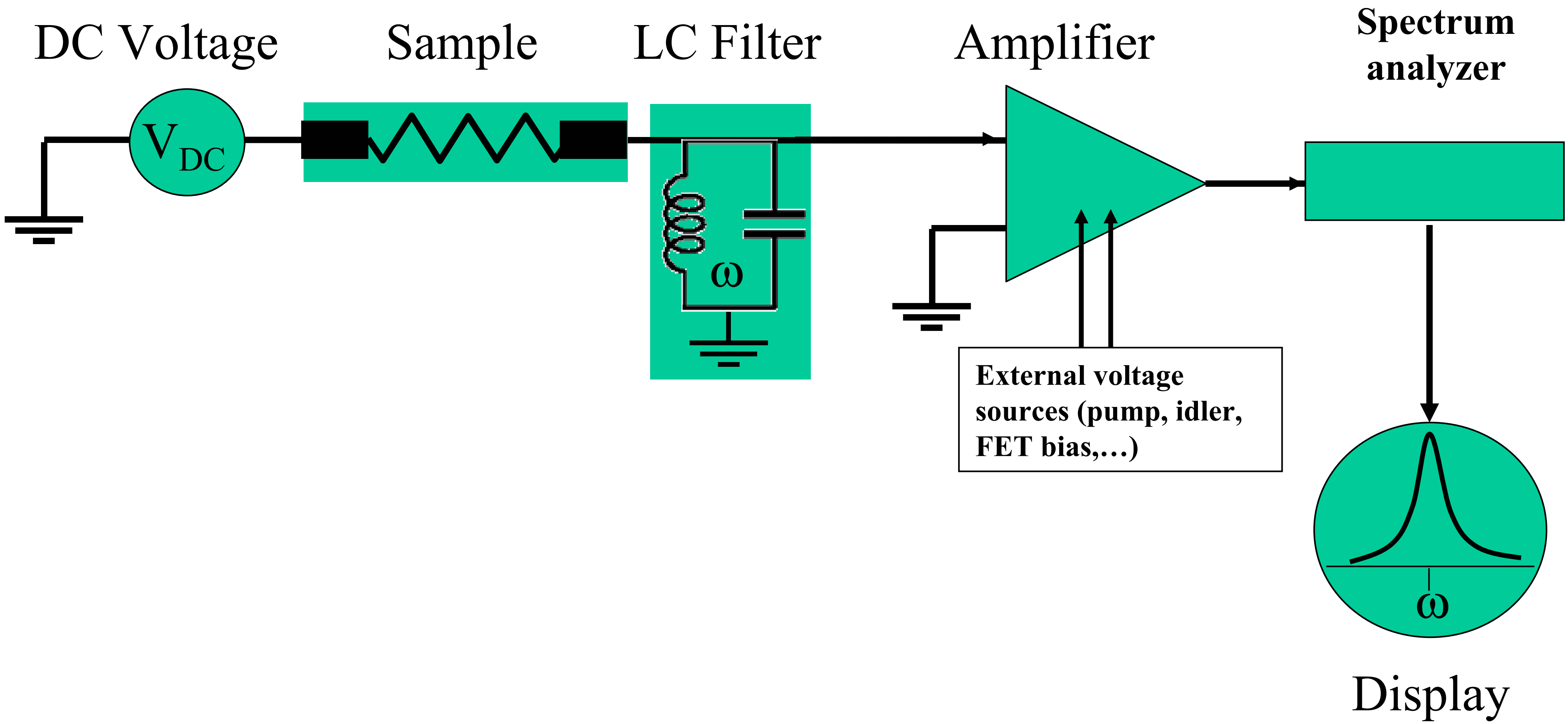
  FDT: NET absorption from clasical field.

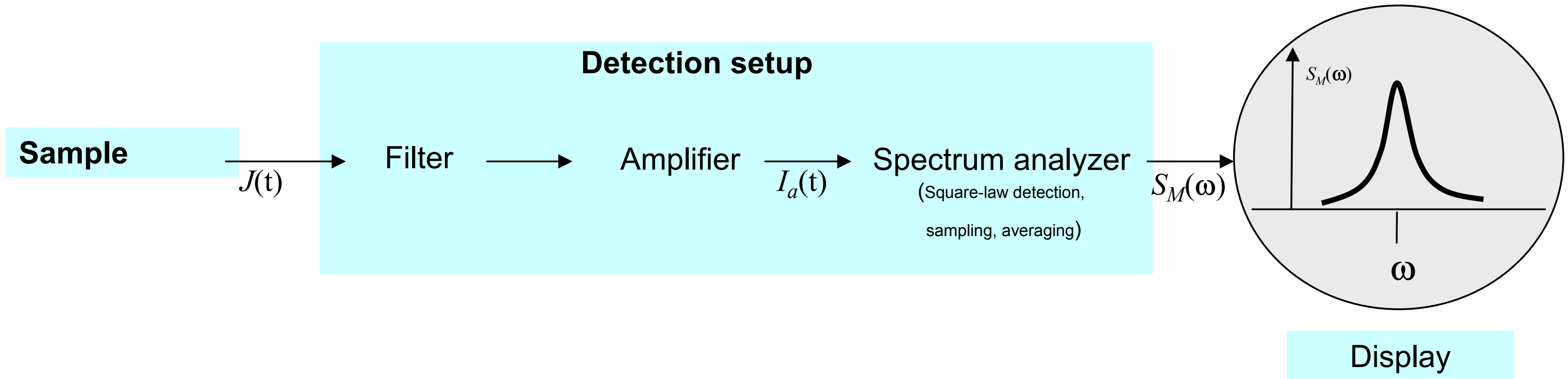
  (Valid also in steady nonequilibrium States)
- **Nothing is emitted from a T = 0 sample,**

  but it may absorb…
- **Noise power depends on final state filling.**

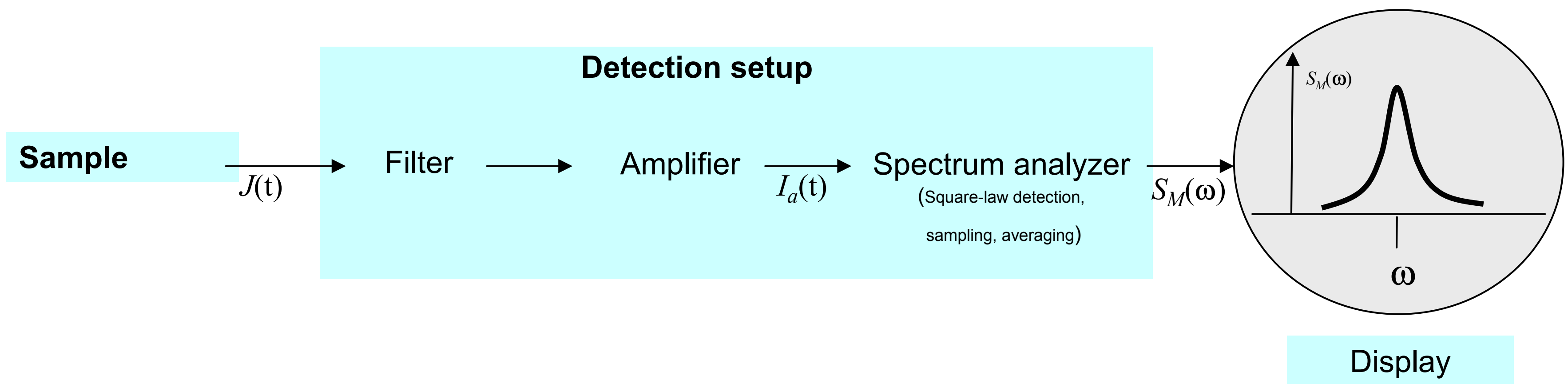
Full Noise Measurement Chain

Typical experimental setup:





Problem: Amp + Filter add their own stray noise to measured result, NONUNIVERSAL!!!


Detection setup
Sample
J(t)
Filter
Amplifier
I_a(t)
Spectrum analyzer
(Square-law detection,
sampling, averaging)
S_M(ω)
S_M(ω)
ω
Display

What Quantity is Obtained in an *Excess* Quantum Noise Measurement?

nonequilibrium quantum Measuring noise

- Problem. Other types of noise exist in the system. Thermal noise, amplifier noise, etc…
- Solution. Make an *excess* noise measurement:

1. Measure $S_M(V,\omega)$

   Turn on the voltage and make a noise measurement.

2. Measure $S_M(0\ ,\omega)$

   Turn off the voltage and make another noise measurement.

3. Subtract the results.

$$S_{M,excess}(\omega) \equiv S_M(V\ ,\omega) - S_M(0,\omega)$$

Examples

A cooled and a warm linear Amplifier,
a phase sensitive or insensitive linear Amplifier
will give different results for $S_M(V,\omega)$.

These differences can be quantum mechanical.

Yurke and Denker, PRA, '84

However, What about Excess Noise?

Can nonuniversal portion cancel?

It Does, in linear conductance regime!

This is our main result

$$S_{M,excess}(V,\omega) = G \times S_{excess}(V,\omega)$$

$$S(V,\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \left\langle \hat{J}_e(0)\hat{J}_e(t) \right\rangle$$

G = (amplification)^2

$$S_{M,excess}(V,\omega) = G \times S_{excess}(V,\omega)$$

Physical meaning of the result

$$S_{M,excess}(V,\omega) = G \times S_{excess}(V,\omega)$$

What is obtained in an excess noise measurement is the excess ***power-flow*** from the sample into the detector. This is the reason for the universality of the result.

Filter and Amplifier strongly coupled to their baths

(=>Amplifier noise does not change with sample voltage)

$$S_M(\Omega) = G \times \langle \Delta I_f^2 \rangle / \Delta\Omega + S_N(\Omega)$$

$S_N(\Omega)$= Amp Noise (**independent of sample**)

Ω = Center filter frequency, L = its self-inductance

$\Delta\Omega$ = filter bandwidth, N_Ω = no of its quanta

$$< \Delta I_f^2 > / \quad L = \gamma^2 [S(\Omega) - 2N_\Omega \hbar\Omega\, G_D(\Omega)]$$

($\gamma \propto$ sample-filter coupling)

$G_D(\Omega)$ = differential sample conductance

When is it valid?

As long as differential conductance does not change – backflow into sample is

INDEPENDENT OF VOLTAGE

i.e. in linear conductance regime

(also necessary to keep impedance matching!)

How to verify the result?

- Make a high frequency measurement and change the amplifier type.

  High frequency is required to distinguish the nonsymmetrized and symmetrized correlators.

- Make a high frequency, $\omega \approx V$, measurement and change the amplifier temperature without changing the sample temperature.

$$S_{M,excess}(V,\omega) = G \times S_{excess}(V,\omega)$$

Conclusion

- Current noise measurement is setup dependent

$$S_{M,excess}(V,\omega) = G \times S_{excess}(V,\omega)$$

Conclusion

- Current noise measurement is setup dependent
- However, nonequilibrium excess noise is to a large extent setup independent since it is basically a measurement of power flow from the sample into the detector.

$$S_{M,excess}(V,\omega) = G \times S_{excess}(V,\omega)$$

Conclusion

- Current noise measurement is setup dependent
- However, nonequilibrium excess noise is to a large extent setup independent since it is basically a measurement of power flow from the sample into the detector.
- At T=0, An excess noise measurement yields the nonsymmetrized correlator, does not contain ZPF.
- Since power, not accumulated charge, is measured → can get fractional charges in spite of leads!

Work in Progress:

Fundamental Limitations Imposed by the Heisenberg Principle on Noise and Back-Action in Nanoscopic Transistors

Koch van Harlingen and Clarke '82

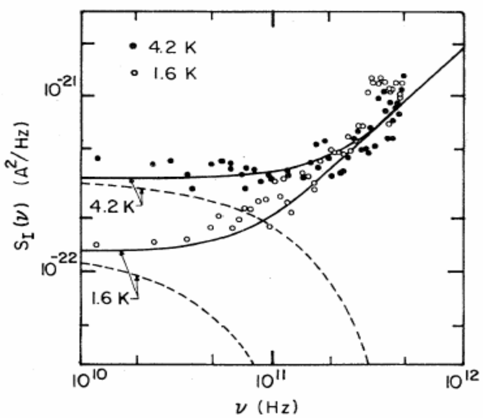


FIG. 6. Measured spectral density of current noise in shunt resistor of junction 2 at 4.2 K (solid circles) and 1.6 K (open circles). Solid lines are prediction of Eq. (1.4), while dashed lines are $(4h\nu/R)[\exp(h\nu/k_BT)-1]^{-1}$.

values of $\nu=2eV/h$, R, and T. The slight increase of the data above the theory at the highest voltages may reflect the presence of a resonance on the I-V characteristic. The agreement between the data and the predictions is rather good, bearing in mind that, once again, no fitting parameters are used. By contrast, the dashed lines represent the theoretical prediction in the absence of the zero-point term,

$$(4h\nu/R)[\exp(h\nu/k_BT)-1]^{-1} ,$$

and fall far below the data at the higher frequencies. The existence of zero-point fluctuations in the measured spectral density of the current noise is rather convincingly demonstrated.

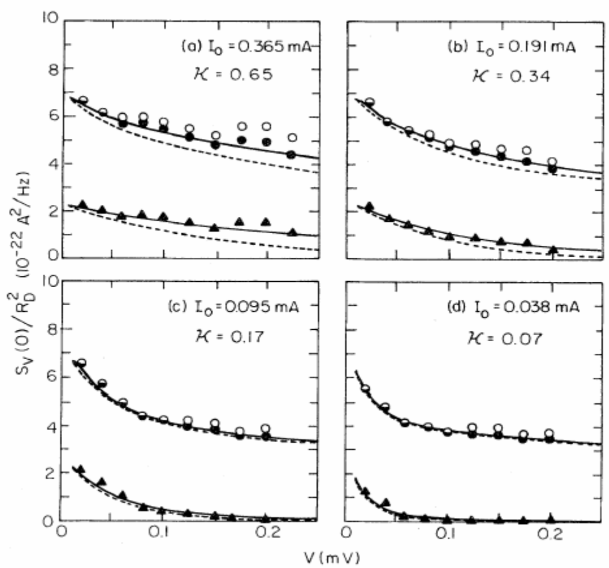


FIG. 7. $S_v(0)$ at 183 kHz vs V for junction 3 at 4.2 K for four values of I_0. Notation is as for Fig. 4.

somewhat above the prediction of Eq. (1.5). Apart from this discrepancy, the measured total noise and the measured mixed-down noise are in very good agreement with the predictions. For $\kappa=0.65$, the data lie convincingly above the theory that does not include the mixed-down zero-point fluctuations, while for $\kappa=0.07$ the contribution of the zero-point term is less than our experimental error. Once again, the correct observed dependence of the noise on I_0 demonstrates the absence of any significant extraneous noise.

Koch van Harlingen and Clarke '82

$$S_I(\nu) = \frac{2h\nu}{R}\coth\left[\frac{h\nu}{2k_BT}\right]$$

$$= \frac{4h\nu}{R}\left[\frac{1}{\exp(h\nu/k_BT)-1}+\frac{1}{2}\right] . \quad (1.4)$$

In the limit[1,2] $0 < \beta_c \equiv 2\pi I_0 R^2 C/\Phi_0 \ll 1$ ($\Phi_0 \equiv h/2e$), the first term on the left-hand side of Eq. (1.3) can be neglected, and the equations can then be solved analytically using the techniques of Likharev and Semenov.[8] At frequencies much less than ν_J and in the limit $I/I_0 > 1$, in which noise rounding can be neglected, the spectral density of the voltage noise $S_v(0)$ is given by

$$\frac{S_v(0)}{R_D^2} = \frac{4k_BT}{R} + \frac{2eV}{R}\left[\frac{I_0}{I}\right]^2 \coth\left[\frac{eV}{k_BT}\right] . \quad (1.5)$$

The first term on the right-hand side is noise-generated at the measurement frequency, while the second term is noise-generated near the Josephson frequency that is mixed down to the measurement frequency by the nonlinearity of the junction. The contribution of noise-generated near frequencies

Koch van Harlingen and Clarke '82

B. Measurement procedures

Each junction was immersed in liquid ^{4}He, and surrounded by a superconducting can and a *mu*-metal can. We measured the I-V characteristic and dynamic resistance and obtained the shunt resistance by reducing the critical current nearly to zero. The circuit for measuring the noise across a junction is shown in Fig. 1(b). The low-pass filters for the bias current consisted of a cooled 1.5-$k\Omega$

resistor R_F and the cable capacitance C_c. The two cooled LC-resonant circuits with inductors L_{t1}, L_{t2} and capacitors C_{t1}, C_{t2} had resonant frequencies of 70 and 183 kHz. Each tank circuit was connected in turn to a Brookdeal 5004 preamplifier; in addition, by connecting together the tank circuit leads we could measure the noise at a third, intermediate frequency, about 106 kHz. After further amplification, the noise was mixed-down to frequencies below 500 Hz and its spectral density was measured with a typical averaging time of 10 min. The system gain was calibrated against the Nyquist noise of a resistor R_c(5.1 $k\Omega$) to $\pm 2\%$. The noise produced by the junction across the tank circuit was $Q^2 S_v(0) = \omega^2 L_t^2 [S_v(0)/R_D^2]$, so that the required quantity $S_v(0)/R_D^2$ was independent of Q. We note that the predicted value of $S_v(0)/R_D^2$ is virtually independent of β_c in the range $0 < \beta_c \leq 0.5$, while the value of $S_v(0)$ does increase significantly as β_c is increased in this range.[7,9] Thus, for β_c appreciably greater than zero (junctions 2 and 3), it is more appropriate to compare experimental and theoretical values of $S_v(0)/R_D^2$ rather than $S_v(0)$.

We now discuss the various measured corrections to the noise spectral density.

(i) Corrections were made for the measured preamplifier voltage and current noise. The preamplifier noise was comparable with the junction noise at 4.2 K, and the corresponding error introduced by the correction was about $\pm 5\%$.

(ii) Noise due to losses in the tank circuit was negligible for the 70- and 183-kHz tank circuits, but not for the 106-kHz tank circuit, which contained leads that were partially at room temperature. In the last case, the error in the correction was $\pm 5\%$.

(iii) From measurements at three frequencies we determined that some junctions (2 and 4) generated a small amount of $1/f$ noise.[21] For example, for junction 2 at 183 kHz the $1/f$ noise was typically 10% or less of the white-noise spectral density at the higher voltages; even if the uncertainty in the noise was as high as $\pm 30\%$, the error introduced was no more than $\pm 3\%$.

(iv) The noise measurements were performed at bias voltages well below the sum of the gaps of the two superconductors. The quasiparticle current I_{qp} contributes a noise with a current spectral density[13] $2eI_{qp}\coth(eV/2k_BT)$. Thus, the ratio of the spectral densities of the quasiparticle and mixed-down noises is of order $I_{qp}/(V/R)$, which we estimate to be $\lesssim 10^{-2}$ at 4.2 K.

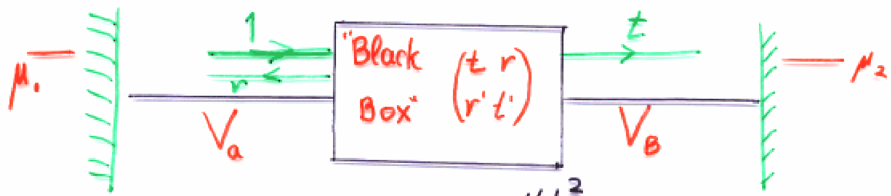
(v) The power dissipation in the shunt resistor caused a significant temperature rise at the high bias voltages in some junctions. We determined the rise ΔT by reducing the critical current almost to zero and measuring the Nyquist noise of the shunt as a function of power dissipation. At low bias voltages the measured noise agreed with the Nyquist formula to within $\pm 3\%$. For most junctions the heating effect was important only at bias voltages $V \gg k_BT/e$, where the mixed-down term in Eq. (1.5) is nearly independent of the shunt temperature. Thus, it was sufficient to correct the data by subtracting $4k_B\Delta T/R$ from the measured noise; the maximum error introduced was $\pm 3\%$. However, for junction 3, where the heating correction was particularly large, it was necessary to correct the mixed-down term was well.

(vi) We took considerable care to shield the experiment from extraneous noise sources, and to avoid coupling significant 300-K noise into the low-temperature circuitry. Measurement of the Nyquist noise in cooled resistors were within $\pm 3\%$ of the predicted value, and measurements on junctions in the classical limit $eV \ll k_BT$ showed the correct temperature dependence and were in excellent agreement with theory (see Secs. III A, III B, and III C). Thus, we believe our measurements were not significantly influenced by extraneous noise sources.

"Conductance = Transmission"

$$\mu_1 - \mu_2 = eV$$

$|\ell k\rangle$ scatt. state from left



$$T \equiv |t|^2$$

$$I = \frac{e^2 \cancel{v}}{\pi\hbar \cancel{v}} T V \Rightarrow \boxed{G = \frac{e^2}{\pi\hbar} T}$$

Two-terminal Conductance

Obtained from Kubo (Lee + Fisher, Economou and Soukoulis)

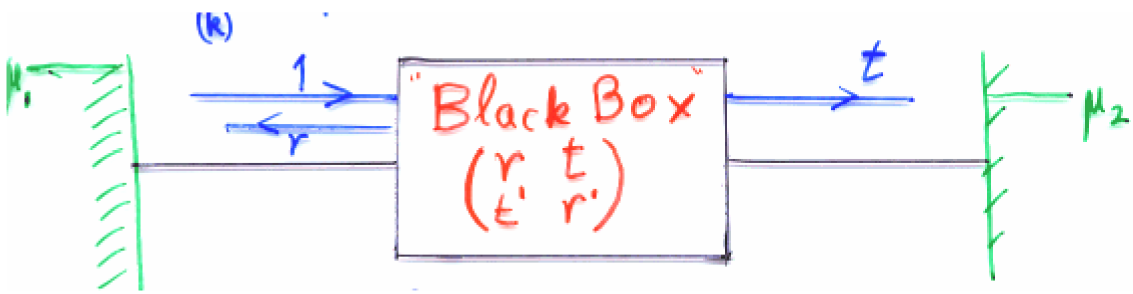
$$\sigma = \frac{1}{\omega} \times DOS)_{e\text{-}h} \langle |m.e.|^2 \rangle$$

$$|\langle \ell | \hat{j}_0 | \ell \rangle|^2 = e^2 v^2 T^2 = |\langle r | \hat{j}_0 | r \rangle|^2 ; |\langle \ell | j_0 | r \rangle|^2 = e^2 T(1-T)$$

$$\boxed{\sigma \propto T}$$

Four-terminal: $\dfrac{I}{V_a - V_b} = \dfrac{e^2}{\pi\hbar} \dfrac{T}{1-T}$ (Landauer)

"SELF-CONSISTENT SCREENING"



(k)
1
r
"Black Box"
$\begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$
t
μ_1
μ_2

Partial Conclusions

1. Correlations in Quantum Domain are obtainable by F.T of full emission/absorption ($\omega \gtrless 0$) spectra (or from either by detailed-balance, in eq.)

   - Symmetrization is BAD
   - $\langle I \rangle(t)$ does not give $\langle I(0) I(t) \rangle$

2. ZPF NOT MEASURABLE PASSIVELY

   (need amp, driving, ...)

3. Conduction electron $\frac{1}{T_\varphi}$ vanishes as $T \to 0$

   (unless $\exists$ ground-state degeneracy) !!!

4. Shot-Noise requires knowledge of full many-body state.

   (Can interpolate between vac and Fermi...)

5. Fermionic HBT NOT OBSERVED YET!

   Interaction Effects ???