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Mathematical Formulation and Structure of MESSAGE

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Mathematical Formulation of MESSAGE

Manfred Strubegger

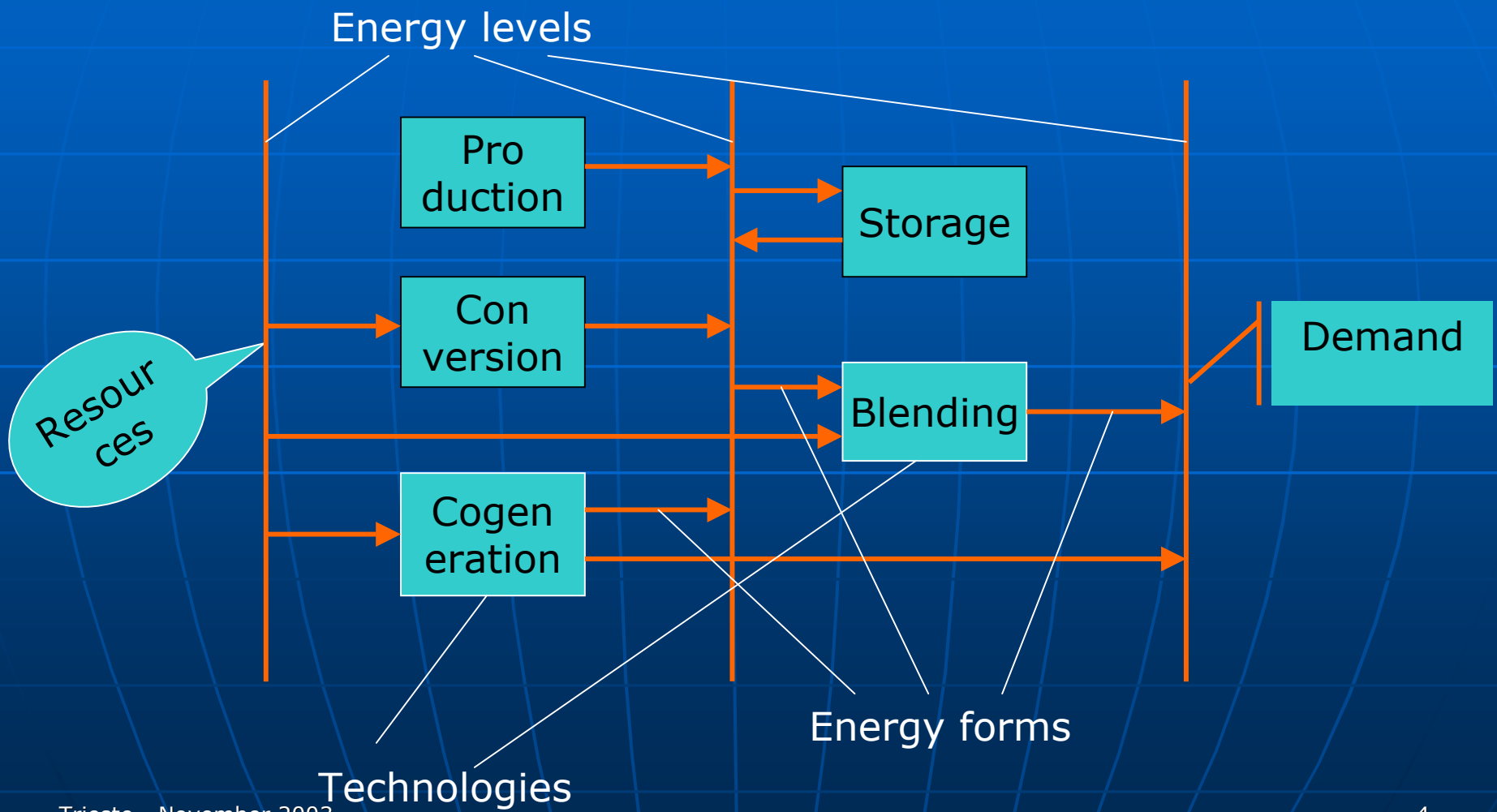
Introduction to MESSAGE

- What is MESSAGE:
  - Medium to long term model
  - Model for the overall energy system
  - Optimization model (LP/MIP)
- What is MESSAGE not:
  - Model for load scheduling
  - Simulation model

Introduction to MESSAGE

- Organization:
  - Time frame
  - Load regions
  - Energy levels
  - Energy forms
  - Technologies
  - Resources
  - Demands
  - Constraints (restrictions)

Introduction to MESSAGE



Introduction to MESSAGE

- Balance:

Σproduction – Σconsumption ≥ 0

$$\sum_{i=1}^{i=n} \eta_{i,t} \times x_{i,t} - \sum_{k=1}^{k=m} x_{k,t} \geq 0$$

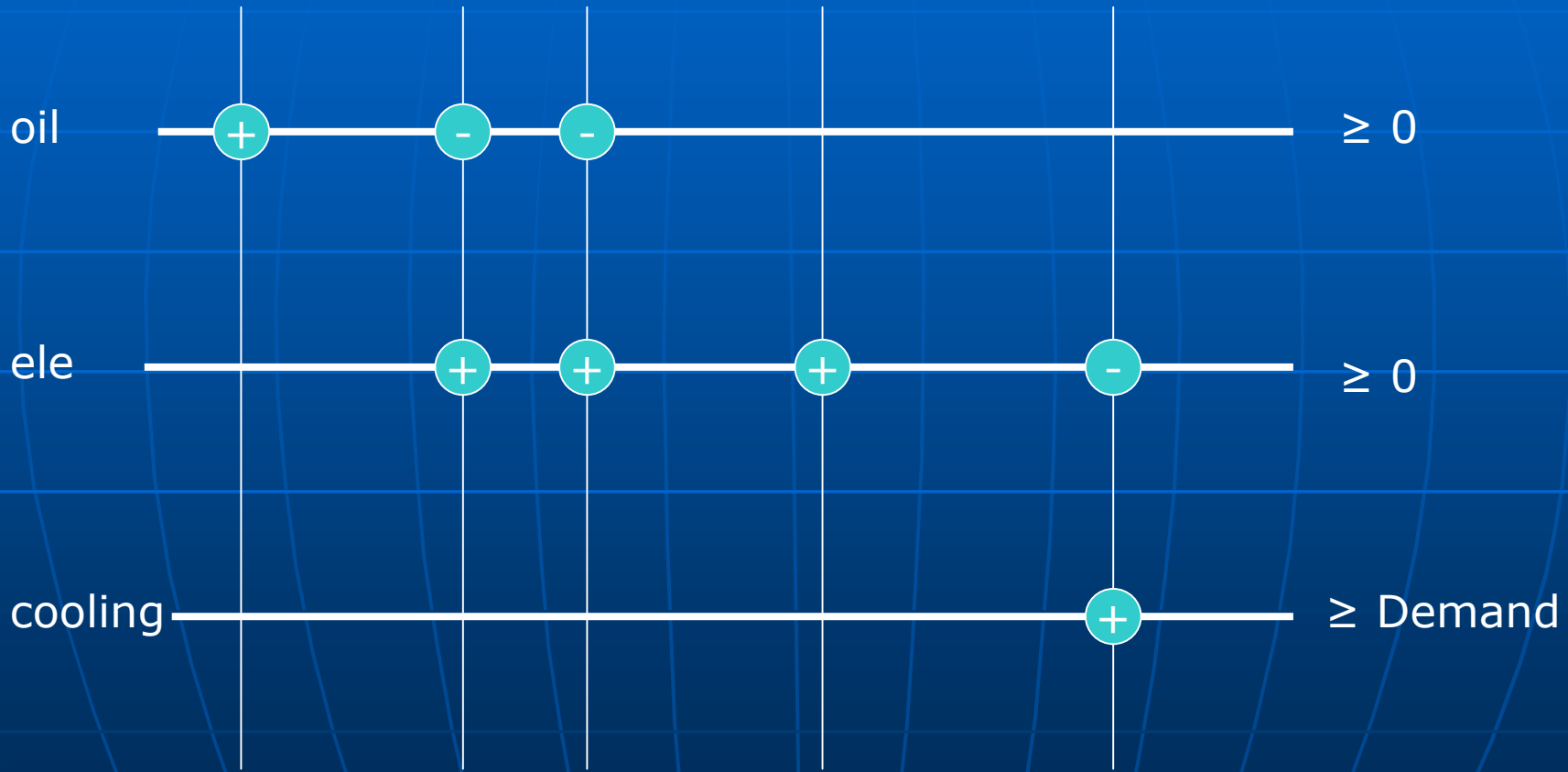
Introduction to MESSAGE

- Demand:

Σproduction ≥ Demand

$$\sum_{i=1}^{i=n} \eta_{i,t} \times x_{i,t} \geq D_{i,t}$$

Introduction to MESSAGE



Introduction to MESSAGE

- Technologies:

Represented by:

- activity variable
- investment variable

- linked by capacity equation

Representation of Elements

- Technologies
  - Installed capacity
  - Efficiency
  - Costs
    - Investment
    - Fixed O&M
    - Variable O&M
  - Availability factor
  - Plant life

Introduction to MESSAGE

- Technologies:
  - $X_{i,t}$ flow variable (consumption of fuel i in time step t)
  - $y_{i,t}$ investment variable (new installation of technology i in time step t)
  - η efficiency of technology
  - Π plant factor (availability factor)

Introduction to MESSAGE

- Capacity equation

$$\eta_T \times x_T - \sum_{t=1}^{t=T} \pi_t \times y_t \leq \pi_0 \times Y_0$$

Y_0 historic installations

- Bounds

$$x_{i,t} \geq l_{i,t}$$

$L_{i,t}$ lower limit

Representation of Elements

- Resources
  - Available amount
  - Annual extraction limits
  - Change of annually extracted amounts
  - Remaining resources
  - Different grades (cost categories)

Introduction to MESSAGE

- Resources
  - Total resource limit

$$\sum_{t=1}^{t=nt} x_t \leq R_0$$

R_0 resources in base year

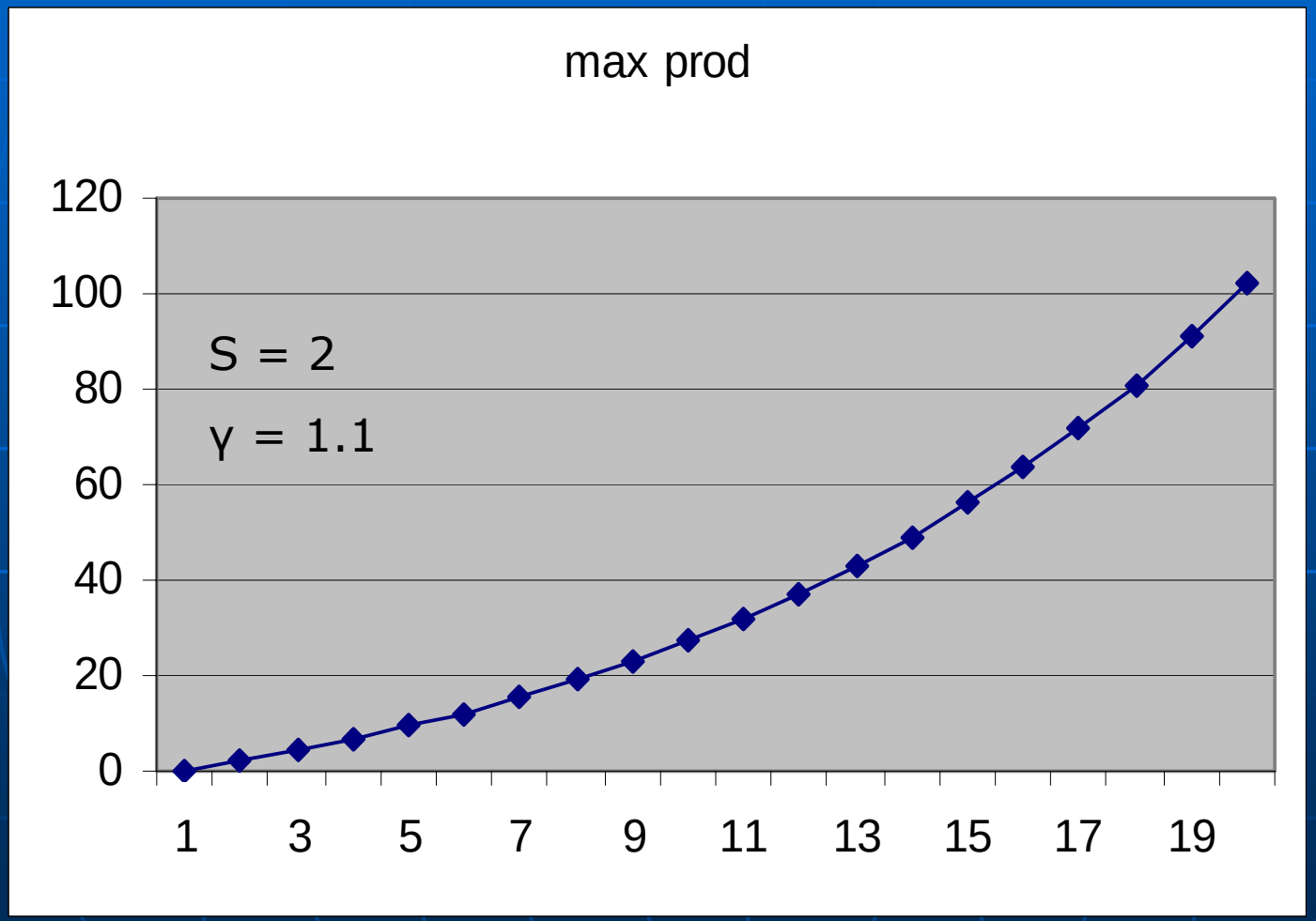
  - Annual change

$$x_t - \gamma x_{t-1} \leq s$$

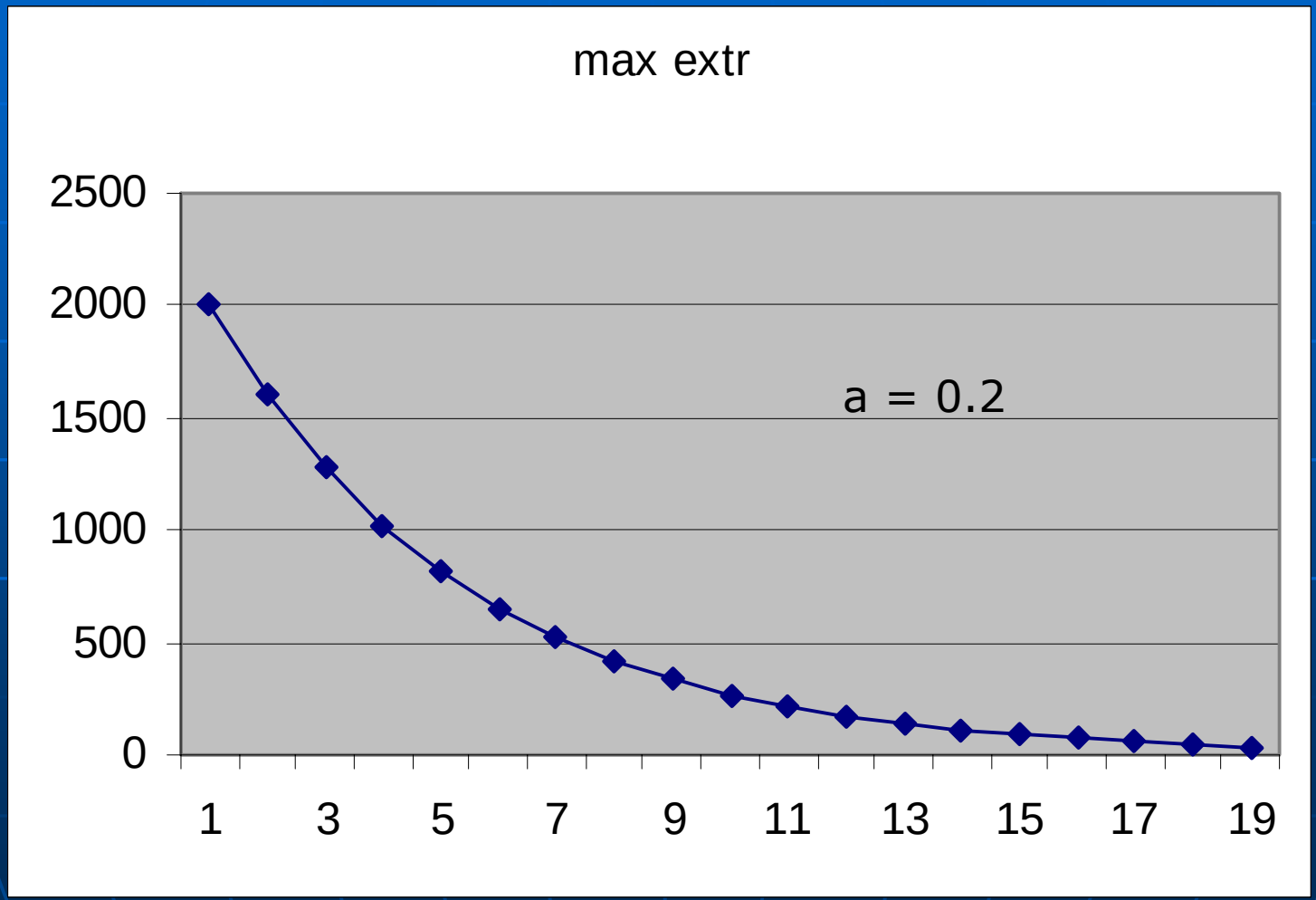
  - Remaining resources

$$x_T + a \sum_{t=1}^{t=T-1} x_t \leq aR_0$$

Growth restriction



Extraction restriction



Introduction to MESSAGE

- User defined equations
  e.g.: production share

$$(1-f)\sum_{i=1}^{i=n}\eta_{i,t}\times x_{i,t}-f\sum_{k=1}^{k=m}\eta_{k,t}\times x_{k,t}\leq 0$$

$$n\in k$$

e.g.: investment limit on groups of ppls

$$\sum_{i=1}^{i=n}c_{i,t}\times y_{i,t}\leq C_t$$

Optimization (1)

- Minimization/maximization of a goal function under constraints

$$A \times x \leq r$$

$$\sum c' \times x \rightarrow \min$$

$$Conditions:$$

$$l < x < u$$

Optimization (2)

- To optimize a linear problem with a standard package, the problem has to be transformed into a standard format
  $\Rightarrow$ MPS format (mathematical programming system)

  - Consists of a number of sections
  - Variables are 8 characters long (recent optimizers allow longer names)

Optimization (3)

- Sections
  - Name card
  - Rows section
  - Columns section
  - RHS section
  - Ranges section
  - Bounds section
  - Endata card

Optimization (4)

- Sections
  - Name card

```
NAME                lecture1_adb
```

  - Rows section

```
ROWS
 N   func
 G   Ls.we..a
 …
 L   Ms.we..a
 …
 E   Ssw1...b
```

Optimization (5)

- Sections
  - Columns section

COLUMNS

```
ys.we..a   Ms.we..b   -1.21000e+00
ys.we..a   Ls.we..b   -1.02010e+00
ys.we..a   Ms.we..a   +1.00000e+00
ys.we..a   Ls.we..a   +1.00000e+00
ys.we..a   func       +2.10873e+03
Xsw1...b   Ssw1...b   -1.00000e+00
```

Optimization (5)

- Sections
  - Rhs section

```
RHS
    rhs1          Ls.we..a   +1.02000e+00
    rhs1          Ms.we..a   +4.20000e+00
    rhs1          Ssw1...b   0.0
```

  - Ranges section

```
    rng1          Ssw1...b   +5.00000e+02
```

Optimization (6)

- Sections
  - Bounds section
    BOUNDS

```
UP bnd1        Ls.we..a   +5.00000e+00
UP bnd1        Ls.we..b   +5.00000e+00
LO …
FX …
FR …
```

  - Endata card
    ENDATA

Optimization (7)

- Variable names
  - Columns

sgae..3a gas_cc

s	output level identifier
g	main input fuel identifier
a	additional identifier
e	main output fuel identifier
.	place holder
.	region identifier
3	load region identifier
a	time step identifier

Optimization (8)

- Variable names
  - Rows

csgae.3a gas_cc

c capacity equation

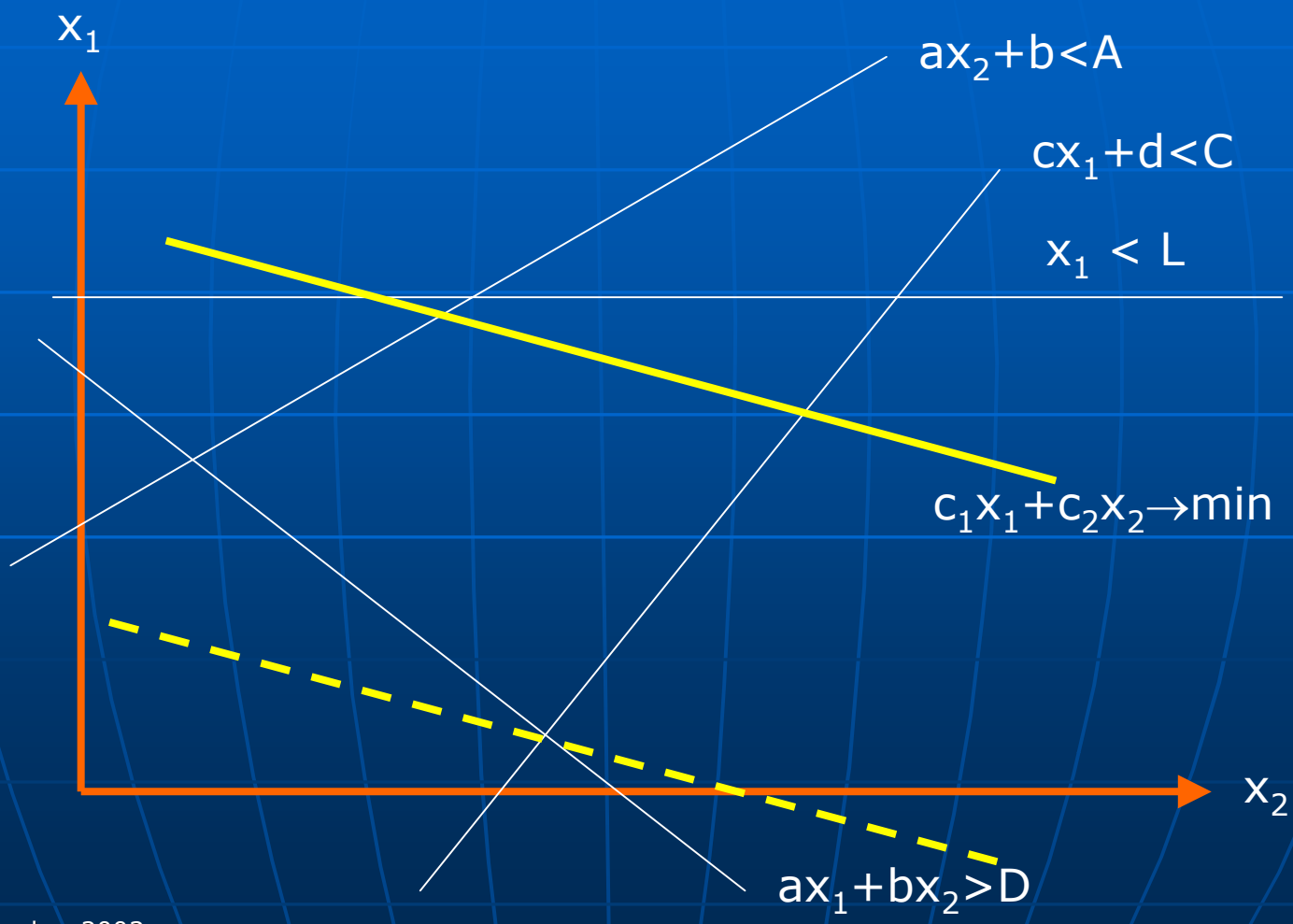
m upper market penetration (act)

l lower market penetration (act)

M upper market penetration (inv)

…

Linear Programming (1)



Linear Programming (2)

- Mixed integer programming
  - Columns section with integer variables

```
INT1      'MARKER'                    'INTORG'
    ys.we..a  cs.we..a  -4.50000e-01
    ys.we..b  cs.we..b  -4.50000e-01
INT1END   'MARKER'                    'INTEND‘
```

Each variable may only have integer values

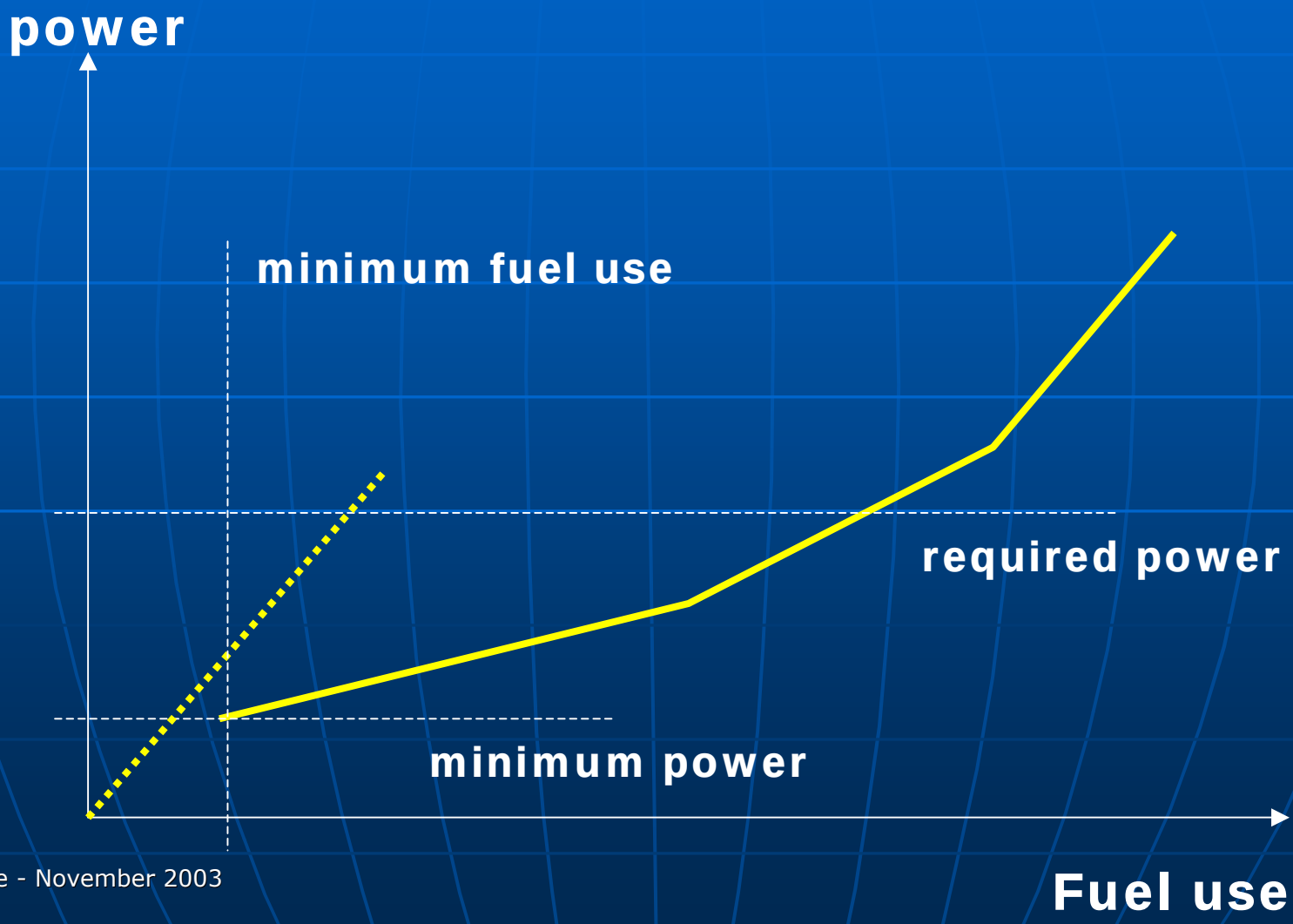
Linear Programming (3)

- Mixed integer programming
  - Columns section with SOS1 set

```
INT2      'MARKER'                    'INTORG'
    s.we1..a  cs.we..a  +4.50000e-01
    s.we2..b  cs.we..b  +5.00000e-01
    s.we3..b  cs.we..b  +5.40000e-01
INT2END   'MARKER'                  'INTEND‘
```

Only one of the variables may be active

SOS1 Set


power
minimum fuel use
required power
minimum power
Fuel use

Sample Energy System