united nations educational, scientific and cultural organization

international atomic energy agency

the **abdus salam** international centre for theoretical physics

ICTP 40th Anniversary

SMR.1555 - 42

Workshop on
Nuclear Reaction Data and Nuclear Reactors:
Physics, Design and Safety

16 February - 12 March 2004

Reactor Dynamics
(Part I)

Piero RAVETTO
Politecnico di Torino
Dipartimento di Energetica
Corso Duca degli Abruzzi 24
10137 Torino
ITALY

These are preliminary lecture notes, intended only for distribution to participants

Reactor Dynamics

Piero Ravetto

Sandra Dulla, Matteo M. Rostagno

Politecnico di Torino
Dipartimento di Energetica
Corso Duca degli Abruzzi, 24
10129 Torino, Italy

Trieste, March 2004

Summary of lectures:

- General aspects of neutron time-dependent problems
- Models for neutron kinetics
- Special features of point kinetics
- Factorization methods and quasi-statics
- Extension of models to source-driven systems
- The modeling of fluid-fuel systems

Applications:

- Study of source and reactivity oscillations in a point system

- Use of a quasi-static code with feedback

- Neutron calculations for a fluid-fuel system

Time-scales appearing in the dynamics of nuclear reactors:

* prompt neutron (very fast) scale, connected to the lifetime of prompt neutrons (10^{-4}-10^{-6} s)

* delayed emission scale, connected to evolution of delayed neutron precursors (10^{-1}-10^{1} s)

* thermal-hydraulic scale (feedback), connected to the evolution of temperatures and hydraulic parameters (10^{-1}-10^{2} s)

* control scale, connected to the movement of masses in the system (control rods, poisons)

* nuclide transmutation scale, connected to neutron transmutation phenomena ($>10^{2}$ s)

Very different time-scales $\Rightarrow$ the physico-mathematical problem is **stiff**

Introduction to neutronic model

The neutron evolution is strongly affected by the delayed emissionsfrom fission

Many different precursors, grouped in 6 (8) families

Each family is characterized by:

* the fraction of fission neutrons appearing in the family β_i

* the dacay constant λ_i [s^{-1}]

* the emission spectrum χ_i

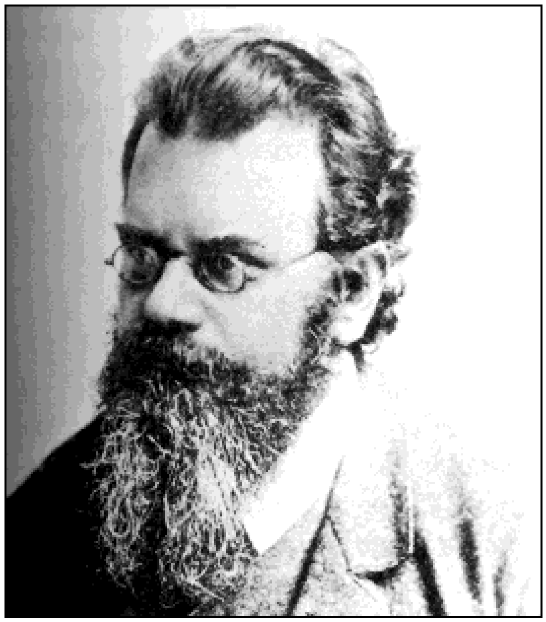
The determination of delayed parameters requires experiments and evaluations.

Time-dependent neutron **Boltzmann** transport equation with delayed neutrons:

$$
\begin{cases}
\dfrac{\partial n(\mathbf{r}, E, \Omega, t)}{\partial t} = \hat{\mathrm{B}}(t) n(\mathbf{r}, E, \Omega, t) + \\
\qquad \displaystyle\sum_{i=1}^{6} \lambda_i \frac{\chi_i(E)}{4\pi} C_i(\mathbf{r}, t) + S(\mathbf{r}, E, \Omega, t) \\
\dfrac{\partial (\chi_i(E) C_i(\mathbf{r}, t)/4\pi)}{\partial t} = \hat{\mathrm{M}}_i(t) n(\mathbf{r}, E, \Omega, t) - \\
\qquad \lambda_i \dfrac{\chi_i(E)}{4\pi} C_i(\mathbf{r}, t)
\end{cases}
$$

A tribute to a great man

Ludwig Boltzmann
(1844-1906)



Definition of operators:

⇒ balance operator

$$\hat{\mathrm{B}}(t) = \hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)$$

⇒ leakage operator

$$\hat{\mathrm{L}}(t) = -\Omega \cdot \nabla v(E) - \Sigma(r, E, t) v(E) +$$
$$\int dE' \oint d\Omega' v(E') \Sigma_s(r, E' \to E, \Omega' \cdot \Omega, t)$$

⇒ prompt multiplication operator

$$\hat{\mathrm{M}}_p(t) = \sum_j \frac{\chi_p^j(E)}{4\pi} \int dE' \oint d\Omega' v(E')(1 - \beta^j) \times$$
$$\nu^j(E') \Sigma_f^j(\mathbf{r}, E', t)$$

⇒ delayed multiplication operator

$$\hat{\mathrm{M}}_i(t) = \sum_j \frac{\chi_i(E)}{4\pi} \int dE' \oint d\Omega' v(E') \beta_i^j \times$$
$$\nu^j(E') \Sigma_f^j(r, E', t)$$

$\Rightarrow$ total multiplication operator

$$\hat{\mathrm{M}}(t) = \hat{\mathrm{M}}_p(t) + \sum_{i=1}^{6} \hat{\mathrm{M}}_i(t)$$

effective emission spectrum

$$\chi^j(E) = (1 - \beta^j)\chi_p^j(E) + \sum_{i=1}^{6} \beta_i^j \chi_i^j(E)$$

Operators can be time-dependent because of:

* effects independent of neutron flux
* non-linear effects (feedback)

Some considerations:

- the Boltzmann equation is a very challenging problem
- it yields too much physical detail
- in real systems only integral quantities can be observed

$$\Downarrow$$

1. need to construct simplified models (multigroup, diffusion...) based on physical assumptions
2. need of numerical algorithms (discretizations, expansions)

$\Longrightarrow$ approximations

important: to establish adequateness of approximations for the problem considered

The simplest model:

$\Longrightarrow$ Point kinetics

or, which is the same:

- only the fundamental eigenfunction of the operator appears in the neutron distribution at all instants
- the neutron distribution can be factorized in an amplitude (time-dependent) and a shape (time independent)

For source-driven system the eigenfunction interpretation fails: the distribution is dominated by the source injection and may involve a superposition of many eigenfunctions

The point model and its solution

How to derive point equations consistently?

Let us consider a simpler and easier problem: space one-group diffusion

Basic equations

one-group diffusion in homogeneous slab geometry with one delayed family and time-constant properties:

$$\begin{cases} \frac{1}{v}\frac{\partial \Phi(x,t)}{\partial t} = D\frac{\partial^2 \Phi(x,t)}{\partial x^2} - \Sigma_a \Phi(x,t) + \\ (1-\beta)\nu\Sigma_f \Phi(x,t) + \lambda C(x,t) + S(x,t), \\ \frac{\partial C(x,t)}{\partial t} = -\lambda C(x,t) + \beta\nu\Sigma_f \Phi(x,t). \end{cases}$$

boundary and initial conditions

$$\Phi(0, t>0) = \Phi(H, t>0) = 0,$$
$$\Phi(x, t=0) = \Phi_0(x),$$
$$C(x, t=0) = C_0(x).$$

Exact solution by eigenfunction expansion

Helmholtz eigenfunctions (complete and orthogonal, most suitable base for the diffusion problem)

$$\frac{d^2\varphi_n(x)}{dx^2} = -B_n^2\varphi_n(x),$$

$$\varphi_n(0) = \varphi_n(H) = 0.$$

Expansion of the solution:

$$\Phi(x,t) = \sum_{n=0}^{\infty} a_n(t)\varphi_n(x),$$

$$C(x,t) = \sum_{n=0}^{\infty} c_n(t)\varphi_n(x),$$

$$S(x,t) = \sum_{n=0}^{\infty} s_n(t)\varphi_n(x).$$

where

$$s_n(t) = \int_0^H dx S(x,t)\varphi_n(x) \equiv (\varphi_n, S(t)) \, .$$

and

$$\Phi_0(x) = \sum_{n=0}^{\infty} a_{n0}\varphi_n(x) \equiv \sum_{n=0}^{\infty} (\varphi_n, \Phi_0)\, \varphi_n(x),$$

$$C_0(x) = \sum_{n=0}^{\infty} c_{n0}\varphi_n(x) \equiv \sum_{n=0}^{\infty} (\varphi_n, C_0)\, \varphi_n(x).$$

Matrix form of the problem:

$$|x_n(t)\rangle = \left| \begin{array}{c} a_n(t) \\ c_n(t) \end{array} \right\rangle, \quad |s_n(t)\rangle = \left| \begin{array}{c} s_n(t) \\ 0 \end{array} \right\rangle,$$

$$\frac{d}{dt}|x_n(t)\rangle = M_n |x_n(t)\rangle + |s_n(t)\rangle \, ,$$

where

$$M_n = \begin{pmatrix} v\left[(1-\beta)\,\nu\Sigma_f - DB_n^2\right] & v\lambda \\ \beta\nu\Sigma_f & -\lambda \end{pmatrix}.$$

Solution is expressed in terms of eigenvectors:

$$M_n \left| u_n \right\rangle = \omega_n \left| u_n \right\rangle$$
$$\left\langle u_n \right| M_n = \omega_n \left\langle u_n \right|$$

eigenvalues (**generalized inhour equation**):

$$\det \left(M_n - \omega_n \Im \right) = 0,$$

eigenvectors:

$$\left| u_n \right\rangle = \left| \begin{array}{c} 1 \\ \dfrac{\beta \nu \Sigma_f}{\omega_n + \lambda} \end{array} \right\rangle,$$
$$\left\langle u_n \right| = \left\langle \; 1 \;\; \frac{v\lambda}{\omega_n + \lambda} \right|,$$

analytical full closed-form solution:

$$|x_n(t)\rangle = \sum_{j=1}^{2} \frac{1}{\left\langle u_n^{(j)} \middle| u_n^{(j)} \right\rangle} \Big[\left\langle u_n^{(j)} \right| x_n(0)\rangle \, e^{\omega_n^{(j)} t} + \int_0^t dt' \left\langle u_n^{(j)} | s_n(t') \right\rangle e^{\omega_n^{(j)}(t-t')} \Big] \left| u_n^{(j)} \right\rangle \equiv \sum_{j=1}^{2} \left[b_{n0}^{(j)} e^{\omega_n^{(j)} t} + \int_0^t dt' \sigma_n^{(j)}(t') e^{\omega_n^{(j)}(t-t')} \right] \left| u_n^{(j)} \right\rangle .$$

for the neutron flux:

$$\Phi(x,t) = \sum_{n=0}^{\infty} \left\{ \sum_{j=1}^{2} \left(1 + \frac{\beta \upsilon \Sigma_f \upsilon \lambda}{(\omega_n^{(j)} + \lambda)^2} \right)^{-1} \left[(\varphi_n, \Phi_0)\, e^{\omega_n^{(j)} t} + \frac{\upsilon \lambda}{\omega_n^{(j)} + \lambda} (\varphi_n, C_0)\, e^{\omega_n^{(j)} t} + \int_0^t dt' \, (\varphi_n, S(t'))\, e^{\omega_n^{(j)}(t-t')} \right] \right\} \varphi_n(x).$$

A **point** reactor evolves according to the fundamental eigenfunction φ_0 only

$\Longrightarrow$ no space distortion during the transient

$\Longrightarrow$ the evolution is space-time separable

$\Longrightarrow$ any point is representative of the whole system

$\Longrightarrow$ the source must be distributed according to the fundamental eigenfunction

Observations

$$\Phi(x,t) = \sum_{j=1}^{2} \left(1 + \frac{\beta \upsilon \Sigma_f \upsilon \lambda}{(\omega^{(j)} + \lambda)^2}\right)^{-1}$$

$$\Bigg[(\varphi_0, \Phi_0)\, e^{\omega^{(j)} t} + \frac{\upsilon \lambda}{\omega^{(j)} + \lambda} (\varphi_0, C_0)\, e^{\omega^{(j)} t} +$$

$$\int_0^t dt' \left(\varphi_0, S(t')\right) e^{\omega^{(j)}(t-t')}\Bigg] \varphi_0(x).$$

$$C(x,t) = \sum_{j=1}^{2} \left(1 + \frac{\beta \upsilon \Sigma_f \upsilon \lambda}{(\omega^{(j)} + \lambda)^2}\right)^{-1}$$

$$\Bigg[(\varphi_0, \Phi_0)\, e^{\omega^{(j)} t} + \frac{\upsilon \lambda}{\omega^{(j)} + \lambda} (\varphi_0, C_0)\, e^{\omega^{(j)} t} +$$

$$\int_0^t dt' \left(\varphi_0, S(t')\right) e^{\omega^{(j)}(t-t')}\Bigg] \frac{\beta \nu \Sigma_f}{\omega^{(j)} + \lambda} \varphi_0(x).$$

$\omega^{(j)}$ are solutions of the inhour equation (written in general form for 6 delayed families):

$$\det M_0 = \frac{\omega\Lambda}{1+\omega\Lambda} + \frac{\omega}{1+\omega\Lambda}\sum_{i=1}^{6}\frac{\beta_i}{\omega+\lambda_i} - \rho = 0,$$

where

$$\rho = \frac{k_{eff}-1}{k_{eff}} \qquad \text{reactivity}$$

$$k_{eff} = \frac{\upsilon\Sigma_f/\Sigma_a}{1+L^2B_0^2} \qquad \text{multiplication constant}$$

$$\Lambda = \frac{1}{\upsilon\Sigma_a\left(1+L^2B_0^2\right)} \qquad \text{prompt lifetime}$$

Features of the roots of the inhour equation $\omega^{(j)}$:

$\star$ all roots are real

$\star$ six of them are close to and approach each $-\lambda_i$ as subcriticality increases;

$\star$ the seventh one, $\omega^{(7)}$, is much larger in absolute value and negative and it determines the prompt response of the neutron population connected to the inverse of the prompt lifetime

$\star$ with a constant source, asymptotically the solution is driven by the exponential, associated to the dominant root

$$\Phi_{as}(x,t) = \left(1 + \frac{\beta v \Sigma_f v \lambda}{(\omega^{(j)} + \lambda)^2}\right)^{-1}$$

$$\left[(\varphi_0, \Phi_0)\, e^{\omega^{(0)}t} + \frac{v\lambda}{\omega^{(0)} + \lambda}(\varphi_0, C_0)\, e^{\omega^{(0)}t} + \right.$$

$$\left. \frac{(\varphi_0, S)}{\omega^{(0)}}\left(1 - e^{\omega^{(0)}t}\right)\right] \varphi_0(x).$$

$$C_{as}(x,t) = \left(1+\frac{\beta\upsilon\Sigma_f\upsilon\lambda}{(\omega^{(j)}+\lambda)^2}\right)^{-1}$$
$$\Big[(\varphi_0,\Phi_0)\,e^{\omega^{(0)}t} + \frac{\upsilon\lambda}{\omega^{(j)}+\lambda}(\varphi_0,C_0)\,e^{\omega^{(0)}t}+$$
$$\frac{(\varphi_0,S)}{\omega^{(0)}}\left(1-e^{\omega^{(0)}t}\right)\Big]\frac{\beta\nu\Sigma_f}{\omega^{(0)}+\lambda}\varphi_0(x).$$

Note:

⋆ the ratio of the precursor density to the neutron density is

$$\frac{C}{n} = \frac{vC}{\Phi} = \frac{v\beta\nu\Sigma_f}{\omega^{(0)}+\lambda} = \beta\frac{k_{eff}}{\Lambda}\frac{1}{\omega^{(0)}+\lambda},$$

which may assume values of the order of 10^3 - 10^4!

⋆ $-\lambda_1$ has a special role

⋆ the "averaging" of delayed families is a delicate task

Alternatively the point model can be derived assuming a factorization of the neutron flux in the product of an amplitude and a (constant) shape function

The derivation is carried out more generally later

Now: let us study the **problem of characterizing space and energy effects in transients**

The study can help us

* **To understand the physics of neutron evolution in multiplying systems**

* **To establish limits of simplified models**

New aspects for source-driven systems

$\Longrightarrow$ subcriticality

$\Longrightarrow$ dominance of the source

Scope of the study:

characterize spatial effects

* use analytical approach

$\Longrightarrow$ simplified configurations

* diffusion theory with delayed neutrons (one family)

Parameters to characterize transient:

asymptotic ratio

$$\mathcal{R}_{asy} = \left| \frac{\Phi - \Phi_{asy}}{\Phi_{asy}} \right|$$

dominance ratio

$$\mathcal{R}_{\mathcal{D}} = \left| \frac{\Phi - \Phi_{\mathcal{D}}}{\Phi_{\mathcal{D}}} \right|$$

spatiality parameter (difference between the full solution and a reference steady-state flux distribution)

$$\xi \equiv \frac{1}{\langle \Phi \rangle} \sqrt{\left\langle \left(\Phi - \frac{\langle \Phi \rangle}{\langle \Psi \rangle} \Psi \right)^2 \right\rangle}$$

Case 1. Initially critical reactor in absence of delayed emissions

$$\omega_n = v\left[\nu\Sigma_f - \left(DB_n^2 + \Sigma_a\right)\right].$$

$$\Phi(x,t) = \sum_{n=0}^{\infty} a_{n0} e^{\omega_n t} \varphi_n(x).$$

$$\Phi_{asy}(x,t) = a_{00} e^{\omega_0 t} \varphi_0(x),$$

$\Longrightarrow$

$$\mathcal{R}_{asy} \to e^{(\omega_1 - \omega_0)t},$$

Case 2. Initially critical reactor with delayed emissions

contribution of the fundamental eigenfunction:

$$\Phi_{fund}(x,t) = \left(b_{00}^{(1)} e^{\omega_0^{(1)} t} + b_{00}^{(2)} e^{\omega_0^{(2)} t} \right) \varphi_0(x).$$

Note:

$$\left|\omega_n^{(1)}\right| \ll \left|\omega_n^{(2)}\right|,$$

and $\omega_n^{(2)} < 0$,

$$\Phi_{asy}(x,t) = b_{00}^{(1)} e^{\omega_0^{(1)} t} \varphi_0(x).$$

$\Longrightarrow$

$$\mathcal{R}_{asy} \to e^{(\omega_1^{(1)} - \omega_0^{(1)})t}.$$

Case 3. Subcritical system

$\Longrightarrow$ source driven

source convolution term:

$$\int_0^t dt' \sigma_n^{(j)}(t') e^{\omega_n^{(j)}(t-t')} = -\frac{\sigma_n^{(j)}}{\omega_n^{(j)}} \left(1 - e^{\omega_n^{(j)} t}\right).$$

No delayed neutron case:

$$\Phi_{asy}(x,t) = -\sum_{n=0}^{\infty} \frac{\sigma_n}{\omega_n} \varphi_n(x),$$

The asymptotic behavior includes contribution from **all** eigenfunctions.

In the evolution towards the asymptotic behavior, also contribution of the dominant transient portion need to be taken into account $\Longrightarrow$ dominant flux

$$\Phi_{\mathcal{D}}(x,t) = \left(a_{00} + \frac{\sigma_0}{\omega_0}\right) e^{\omega_0 t} \varphi_0(x) + \Phi_{asy}(x,t).$$

$$\Longrightarrow \quad \mathcal{R}_{\mathcal{D}} \to \left| \frac{\left(a_{00} + \frac{\sigma_0}{\omega_0}\right) e^{\omega_0 t} \varphi_1(x)}{\Phi_{\mathcal{D}}} \right| .$$

Case with delayed neutrons

$$\Phi_{asy}(x,t) = \sum_{n=0}^{\infty} \left(\frac{\sigma_n^{(1)}}{\omega_n^{(1)}} + \frac{\sigma_n^{(2)}}{\omega_n^{(2)}} \right) \varphi_n(x),$$

$$\Phi_{\mathcal{D}}(x,t) = \sum_{j=1}^{2} \left(b_{00}^{(j)} + \frac{\sigma_0^{(j)}}{\omega_0^{(j)}} \right) e^{\omega_0^{(j)} t} \varphi_0(x) + \Phi_{asy}(x,t).$$

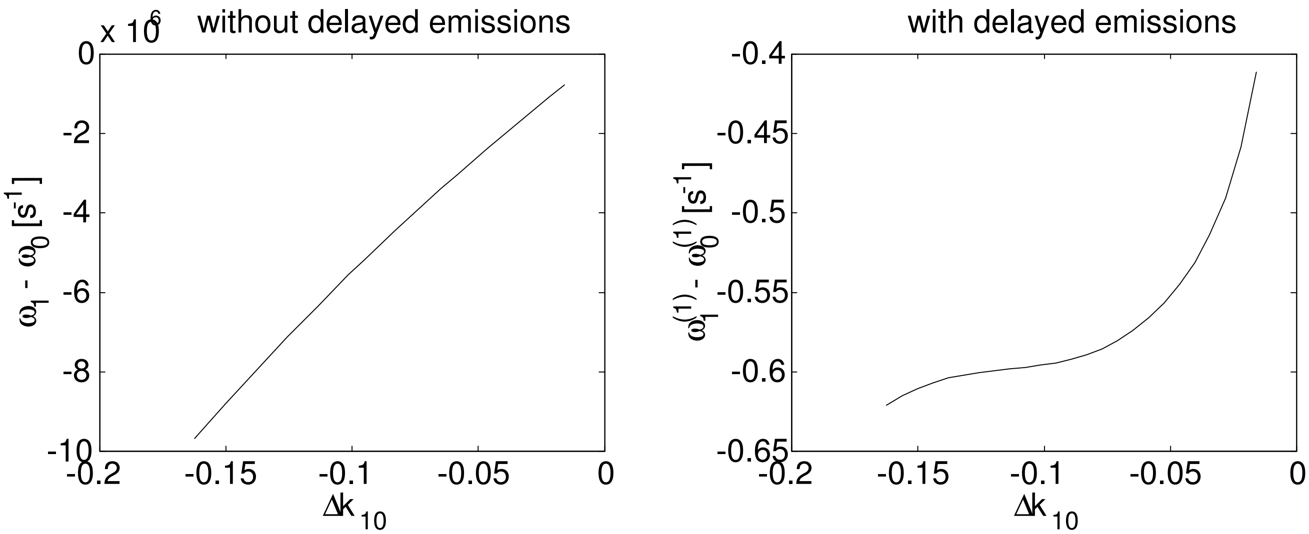
$\Longrightarrow$

$$\mathcal{R}_{\mathcal{D}} \to \left| \frac{\left(b_{10}^{(1)} + \frac{\sigma_1^{(1)}}{\omega_1^{(1)}} \right) e^{\omega_1^{(1)} t} \varphi_1(x)}{\Phi_{\mathcal{D}}} \right| .$$

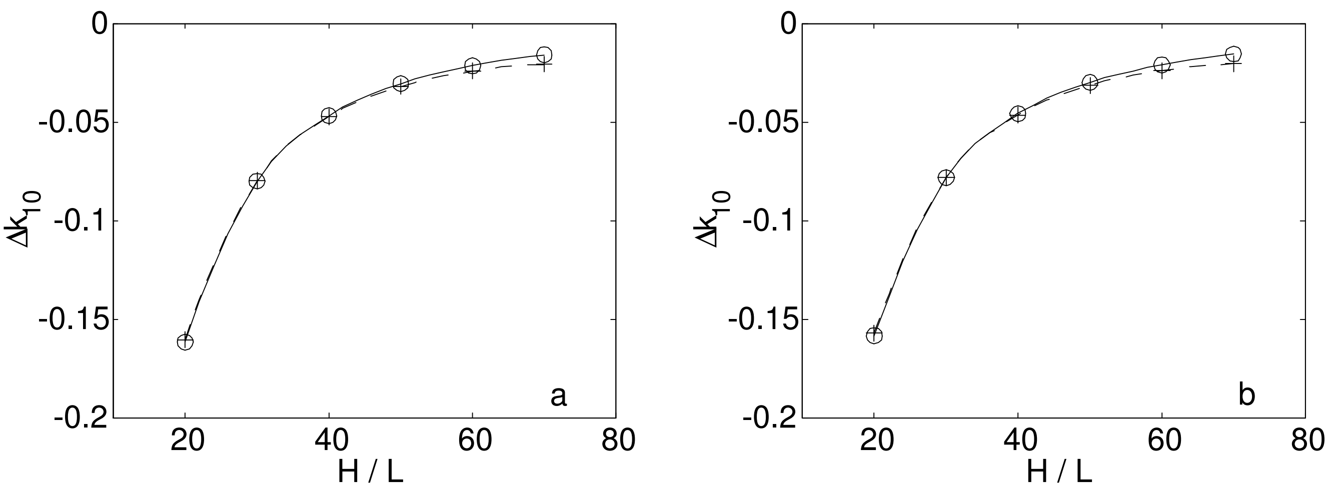
1. The response to a perturbation in a critical reactor is spatially more significant in a large system, i.e. R_{asy} is larger and takes longer to reduce to 0, and thus the contribution of higher order harmonics is more persistent;

2. The evolutions of both R_{asy} and $R_{\mathcal{D}}$ for subcritical systems show that the importance of higher-order harmonics increases with increasing subcriticality, as the systems are more source-dominated;

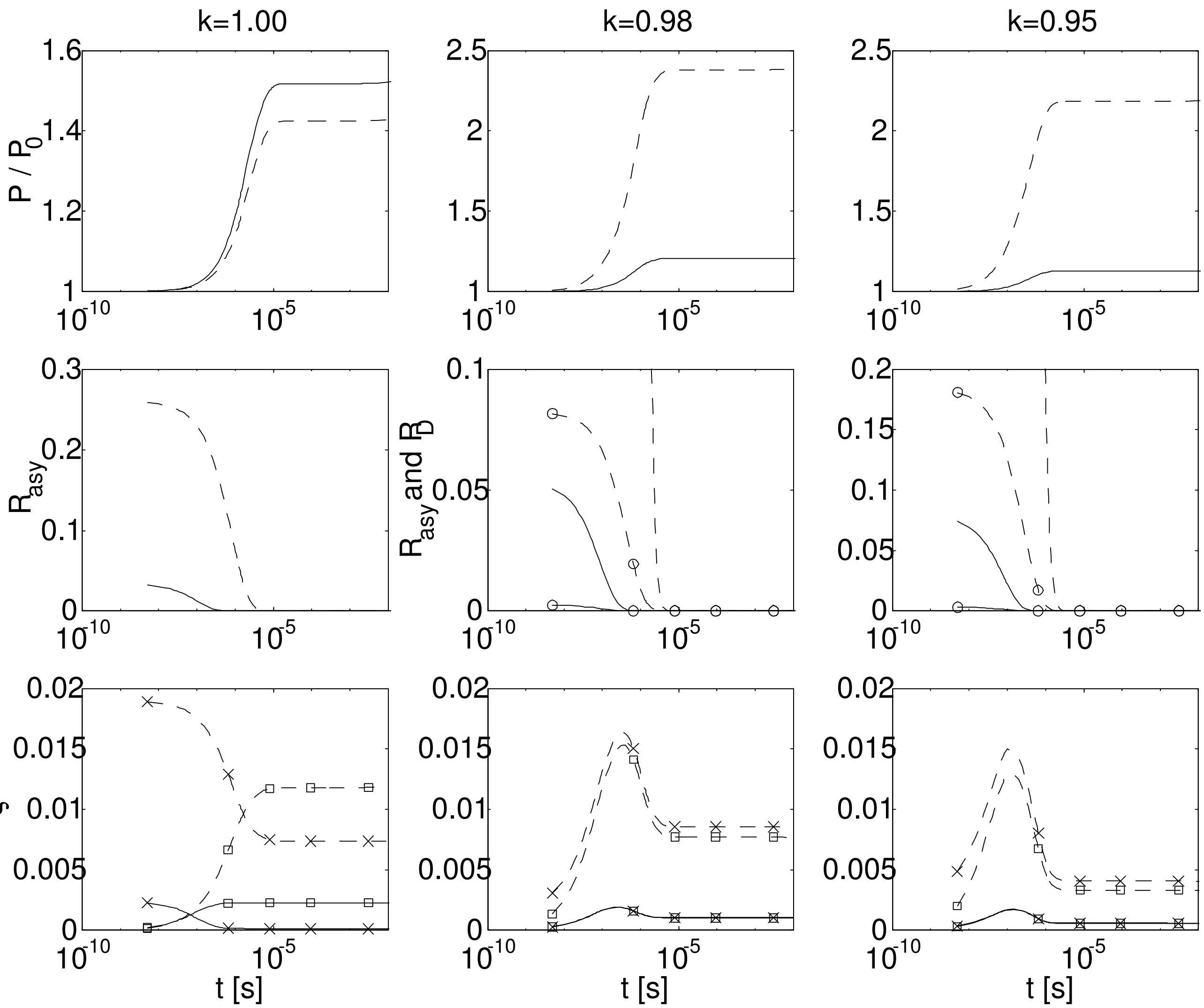
3. The comparison of initially critical and subcritical systems shows that the spatial feature of the transients is larger in systems departing from criticality; therefore, one can expect that the point model may have obvious limitations of applicability in these situations.



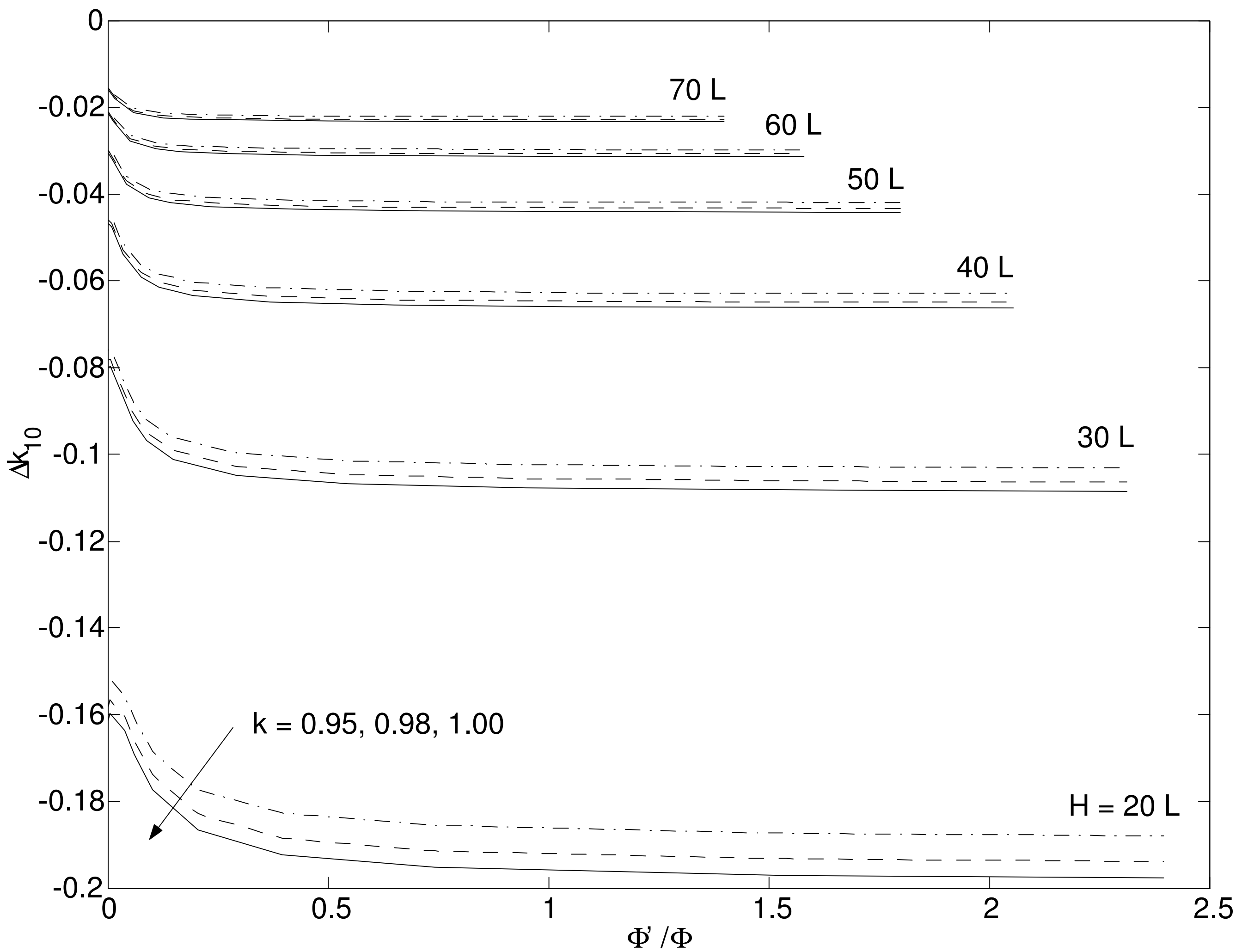
1.Relation between the separation of the first static multiplication eigenvalues and the separation of the first time eigenvalues.



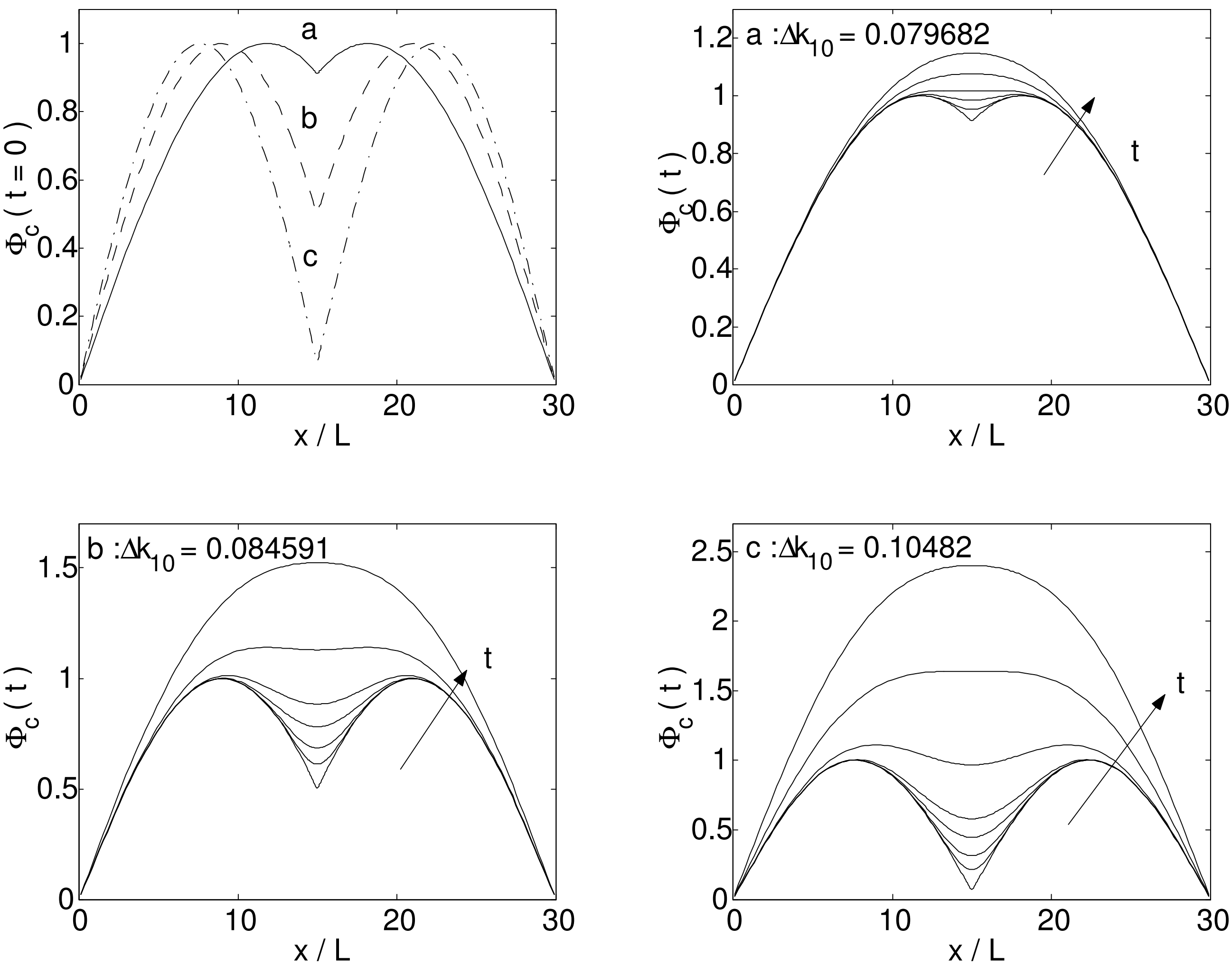
2.Eigenvalue separation for critical and subcritical systems. The circles indicate results for the homogeneous system and crosses for a system with an absorber that introduces a change in k of -500 pcm. Graph (a) on the left refers to the case in which the homogeneous system is critical, while graph (b) refers to the case in which the homogeneous system is subcritical ($k = 0.98$).



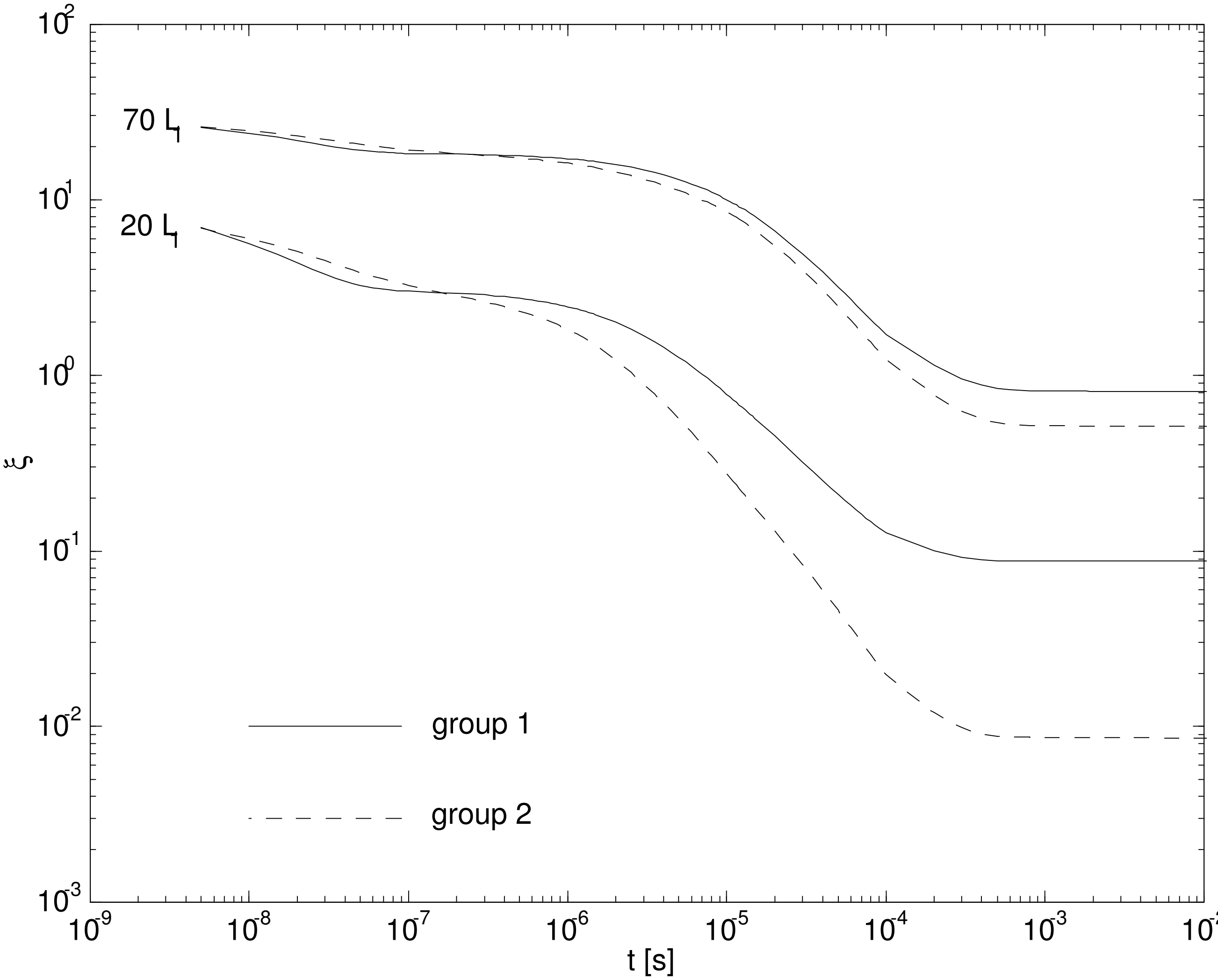
3.Spatiality of transients in critical and subcritical systems, for a fixed reactivity insertion of 500 pcm. The power evolution for the initially critical case is diverging and such behavior appears at longer times than shown in the graph. Solid line refers to a small system ($H = 20L$) and broken line to a large system ($H = 70L$). Crosses ($\times$) results for ξ are produced using as reference Ψ the final flux distribution, while squares ($\Box$) using the initial state. For subcritical systems



4.Eigenvalue separation for different values of the greyness of the control device and of the physical dimensions of the system.



5.Spatial evolution in initially critical systems with different eigenvalue separations. The top left graph reports the initial flux distributions for three values of the greyness of the control device.



6.Evolution of the spatiality parameter ξ for two subcritical systems having $\Delta k/k = -1.4114\beta$. The reference Ψ is the final flux distribution.