



the
abdus salam
international centre for theoretical physics

ICTP 40th Anniversary

SMR.1555 - 32

**Workshop on
Nuclear Reaction Data and Nuclear Reactors:
Physics, Design and Safety**

16 February - 12 March 2004

Coupled-Channels

**Roberto M. CAPOTE NOY
International Atomic Energy Agency
Nuclear Data Section
Wagramerstrasse 5
P.O. Box 100
A-1400 Vienna
AUSTRIA**

These are preliminary lecture notes, intended only for distribution to participants

COUPLED CHANNEL vs SINGLE CHANNEL OMPs

Roberto Capote

IAEA, Nuclear Data Section

e-mail: Roberto.CapoteNoy@iaea.org

**Workshop on Nuclear Reaction Data
and Nuclear Reactors -
Physics, Design and Safety**

ICTP, Trieste, Italy, 16 February - 12 March 2004



OVERVIEW

- Spherical OMP for applications
- Coupled channel method
- EMPIRE optical model calculations

SINGLE CHANNEL EQUATION

$$\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + k^2 - \frac{2\mu}{\hbar^2} \left(U(r) + d_l^j U_{so}(r) \right) \right\} \psi_l^j(r) = 0$$

$$d_l^j = d_{so} (j(j+1) - l(l+1) - s(s+1))/2$$

Neutron-nucleus spherical OMP

$$U(r) = V(r) + i \cdot W(r)$$

$$V(r) = -V_V \cdot f_{WS}(r) - V_S \cdot g_{WS}(r)$$

$$W(r) = -W_V \cdot f_{WS}(r) - W_S \cdot g_{WS}(r)$$

$$\left(\frac{\hbar}{m_\pi c} \right)^2 (V_{SO} + iW_{SO}) \cdot \frac{1}{r} \cdot \frac{df_{WS}}{dr} (\vec{l} \cdot \vec{\sigma})$$

Spherical Optical Model

Energy dependence (I)

Functional form of the real and imaginary potentials:

$$V_{HF}(E) = V_0 \cdot \exp(-\alpha_{HF}(E - E_F))$$

$$W_V(E) = A_V \frac{(E - E_F)^4}{(E - E_F)^4 + (B_V)^4}$$

$$W_S(E) = A_S \frac{(E - E_F)^4}{(E - E_F)^4 + (B_V)^4} \cdot \exp(-C_S |E - E_F|)$$

Spherical Optical Model

Energy dependence (II)

$$W_V(E) = \begin{cases} 0 & E_F < E < E_P \\ A_V \frac{(E - E_P)^n}{(E - E_P)^n + (B_V)^n} & E > E_P \end{cases}$$

$$W_S(E) = \begin{cases} 0 & E_F < E < E_P \\ A_S e^{-C_S |E - E_P|} \frac{(E - E_P)^m}{(E - E_P)^m + (B_S)^m} & E > E_P \end{cases}$$

with the symmetry condition:

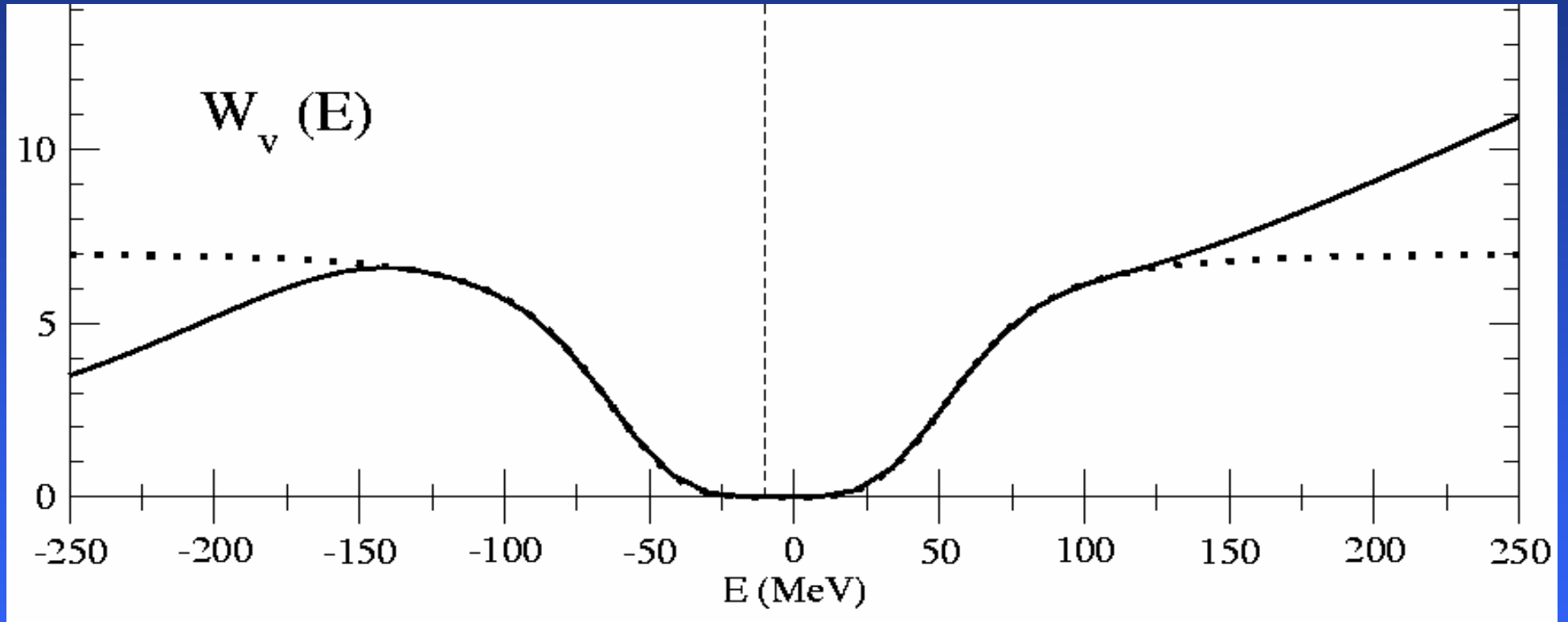
$$W(2E_F - E) = W(E)$$

Spherical Optical Model

Non-locality corrections

$$\left\{ \begin{array}{l} W_V(E) \left[1 - \frac{(E_F - E - E_a)^2}{(E_F - E - E_a)^2 + E_a^2} \right] \quad E < E_F - E_a \\ \\ W_V(E) + \alpha_V \cdot \left[\sqrt{E} + \frac{(E_F + E_a)^{3/2}}{2E} - \frac{3}{2} \sqrt{(E_F + E_a)} \right] \quad E > E_F + E_a \\ \\ W_S(E) + \alpha_S \cdot \left[\sqrt{E} + \frac{(E_F + E_a)^{3/2}}{2E} - \frac{3}{2} \sqrt{(E_F + E_a)} \right] \quad E > E_F + E_a \end{array} \right.$$

Imaginary potential extended to negative energies



Dispersive Optical Model

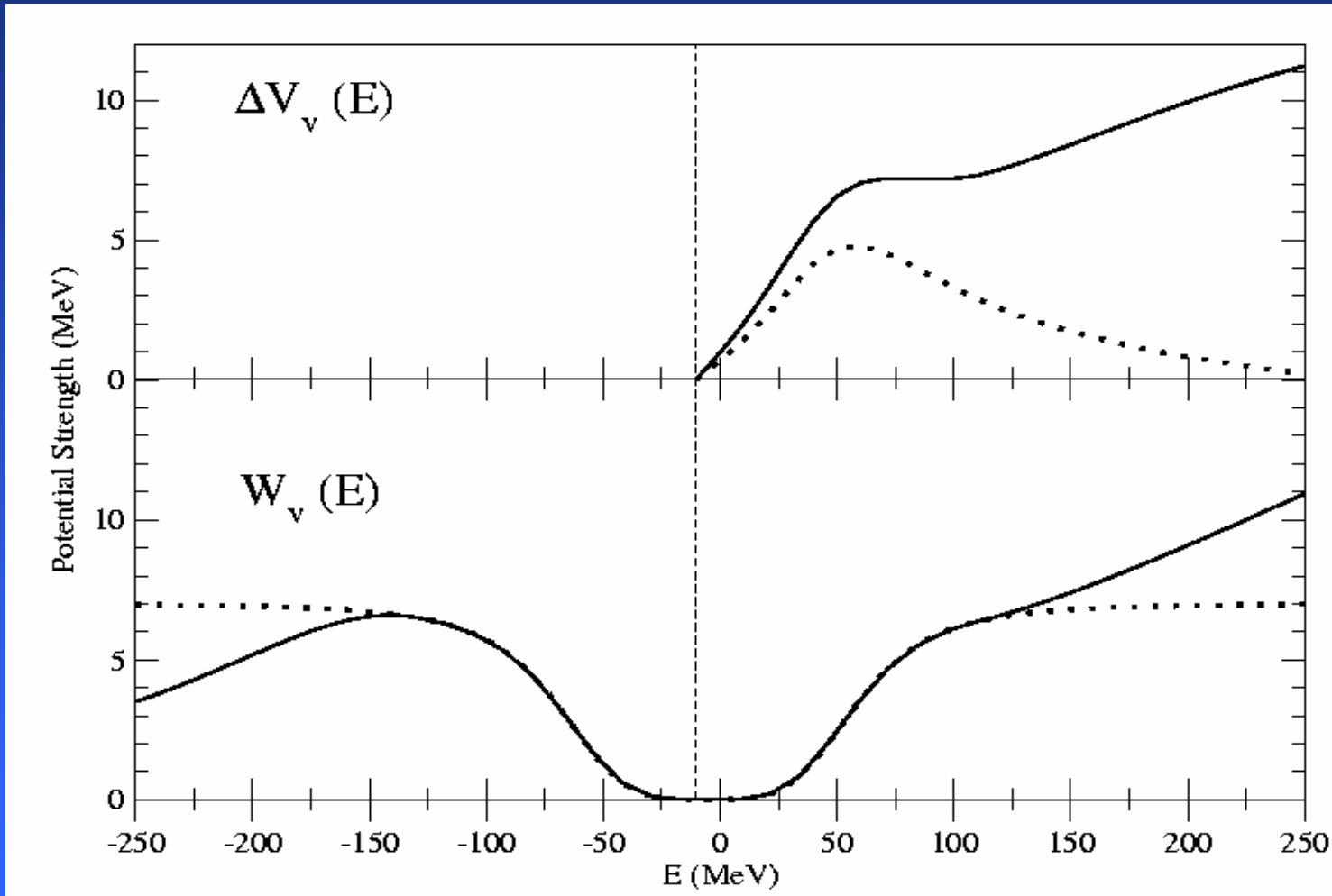
$$\Delta V(E) = \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} \frac{W(E')}{E' - E} dE'$$

$$V_V(E) = V_{HF}(E) + \Delta V_V(E) + \Delta V_{<}(E) + \alpha_V \Delta V_{>}(E)$$

$$V_S(E) = \Delta V_S(E) + \alpha_S \Delta V_{>}(E)$$

$W(E)$ must be defined in the interval $-\infty < E < \infty$

Imaginary potential W and dispersive contribution



Spherical Optical Model

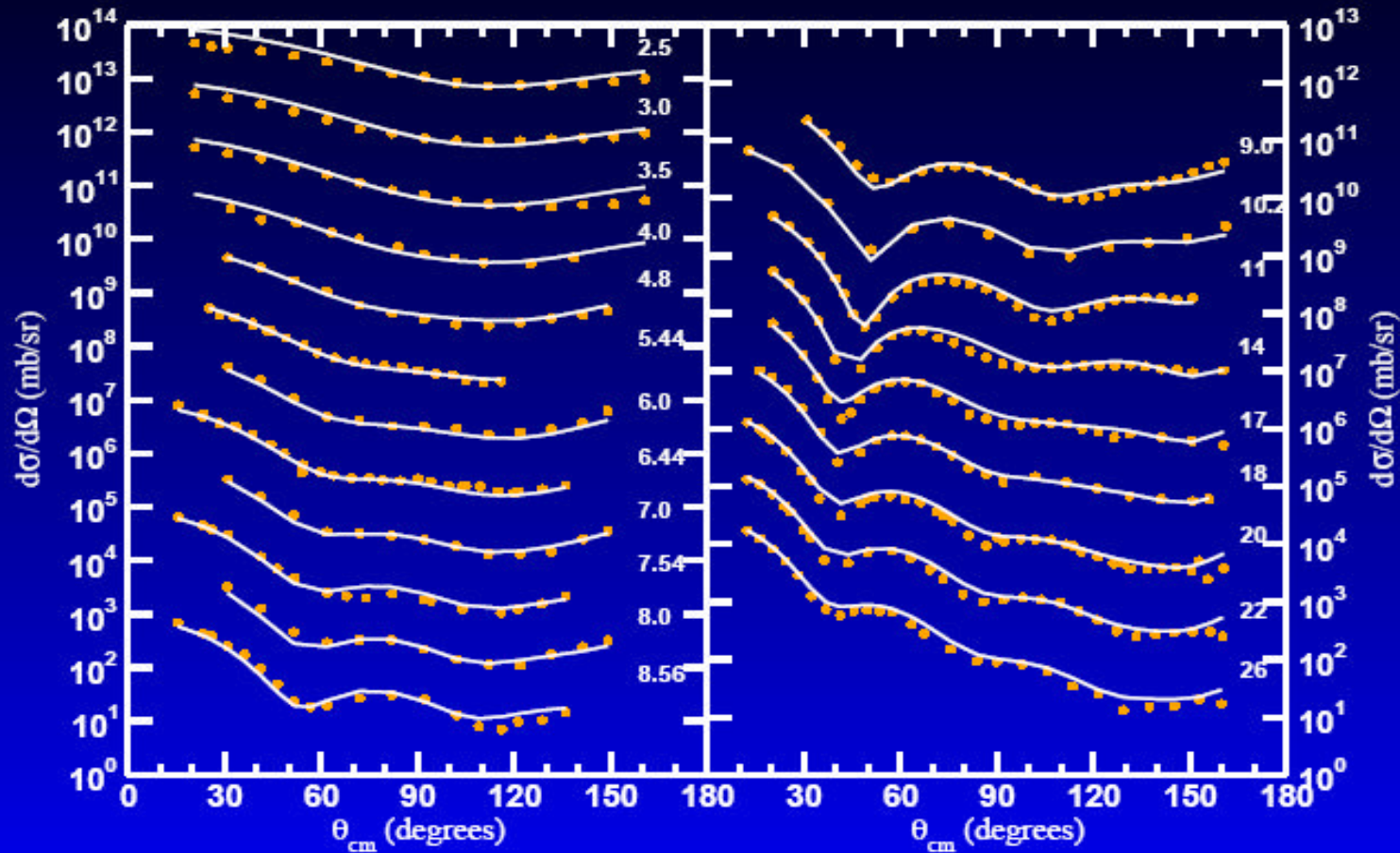
Energy dependence (III)

Functional form of the spin-orbit potentials:

$$V_{SO} = A_{SO} \cdot \exp(B_{SO}(E - E_F))$$

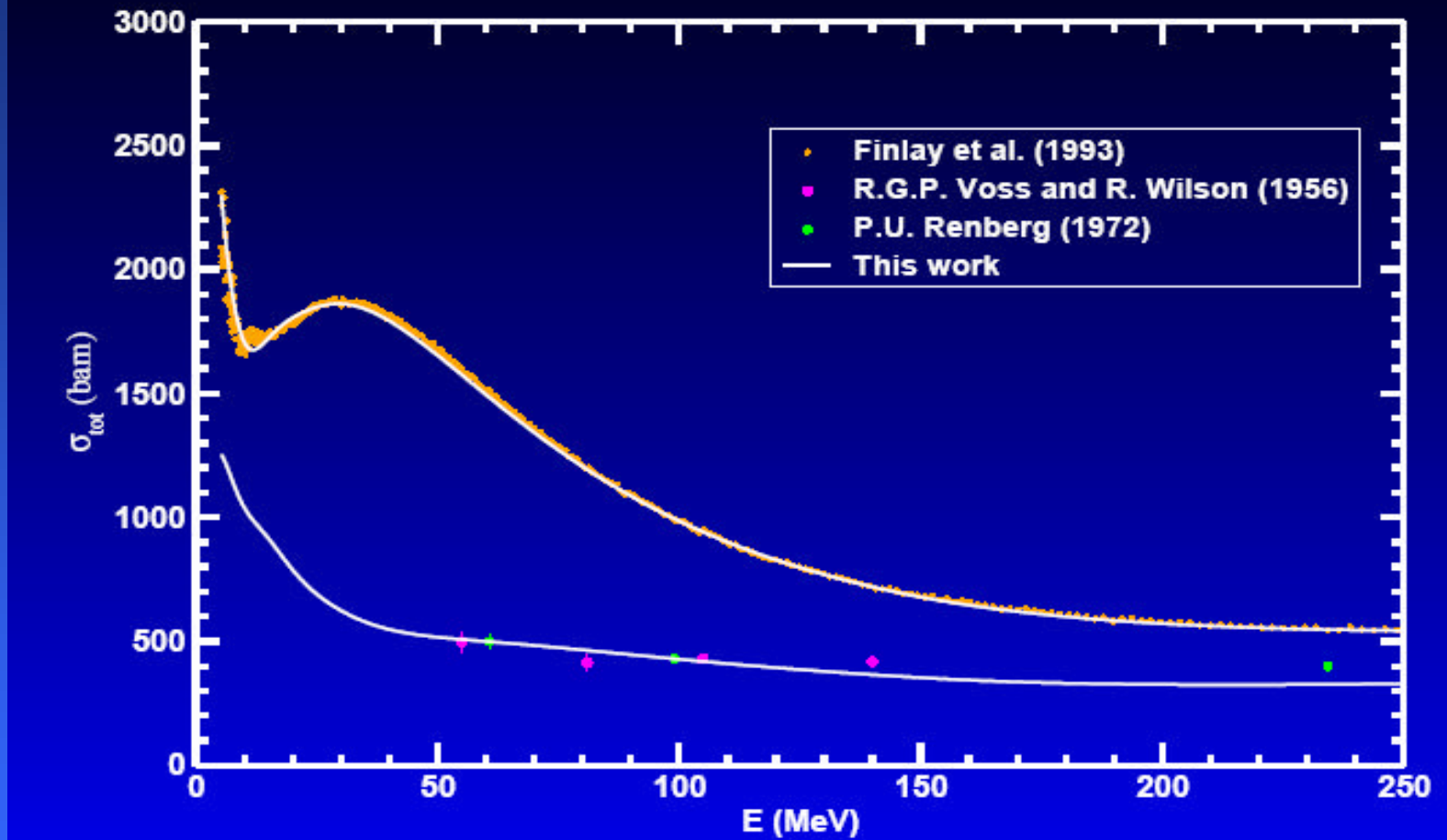
$$W_{SO} = C_{SO} \frac{(E - E_F)^2}{(E - E_F)^2 + D_{SO}^2}$$

$n+^{27}\text{Al}$ - Elastic scattering



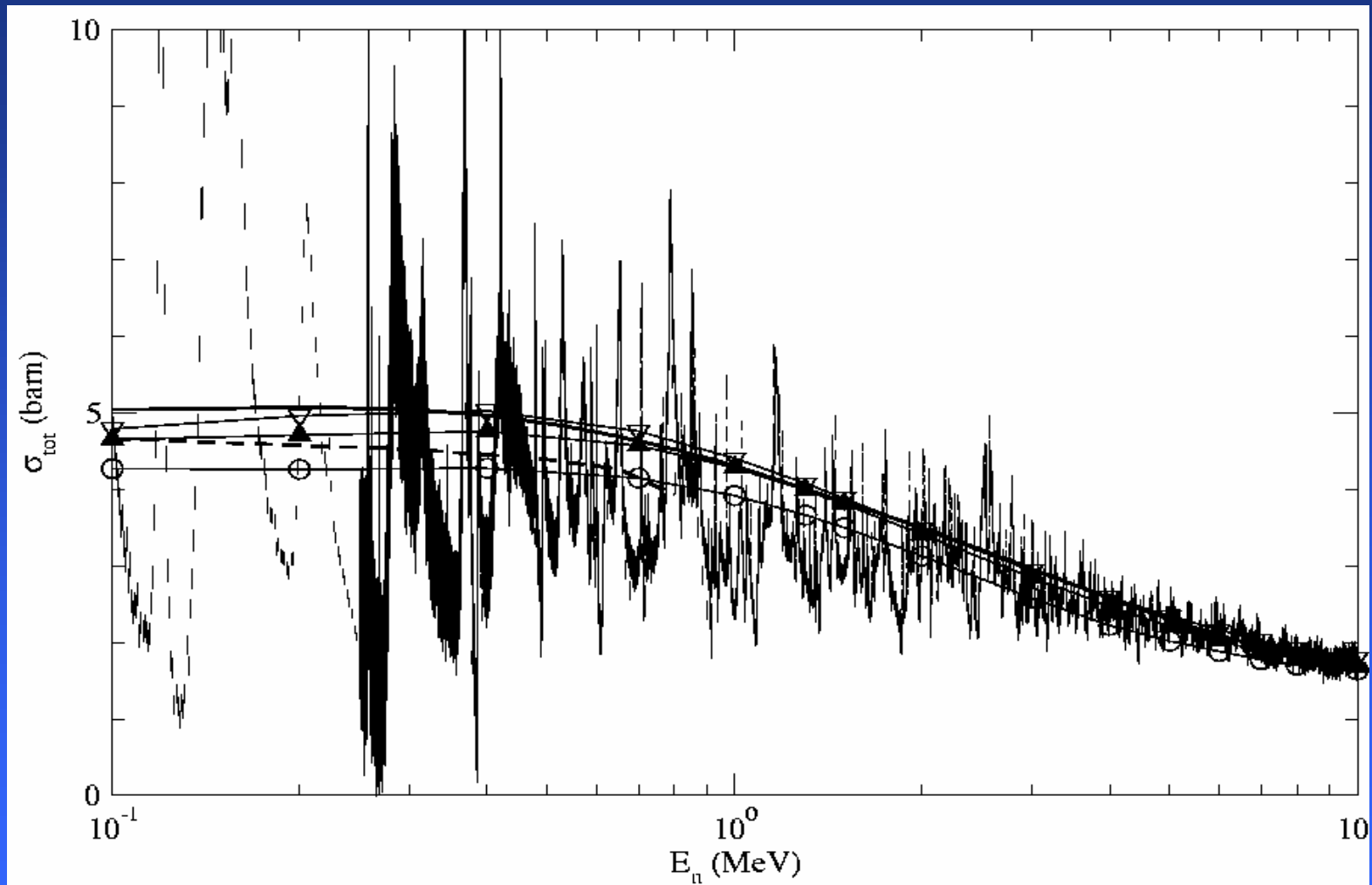
A.Molina Ph.D.Thesis, Univ.Sevilla 2003

$n+^{27}\text{Al}$ - Total and Reaction xs

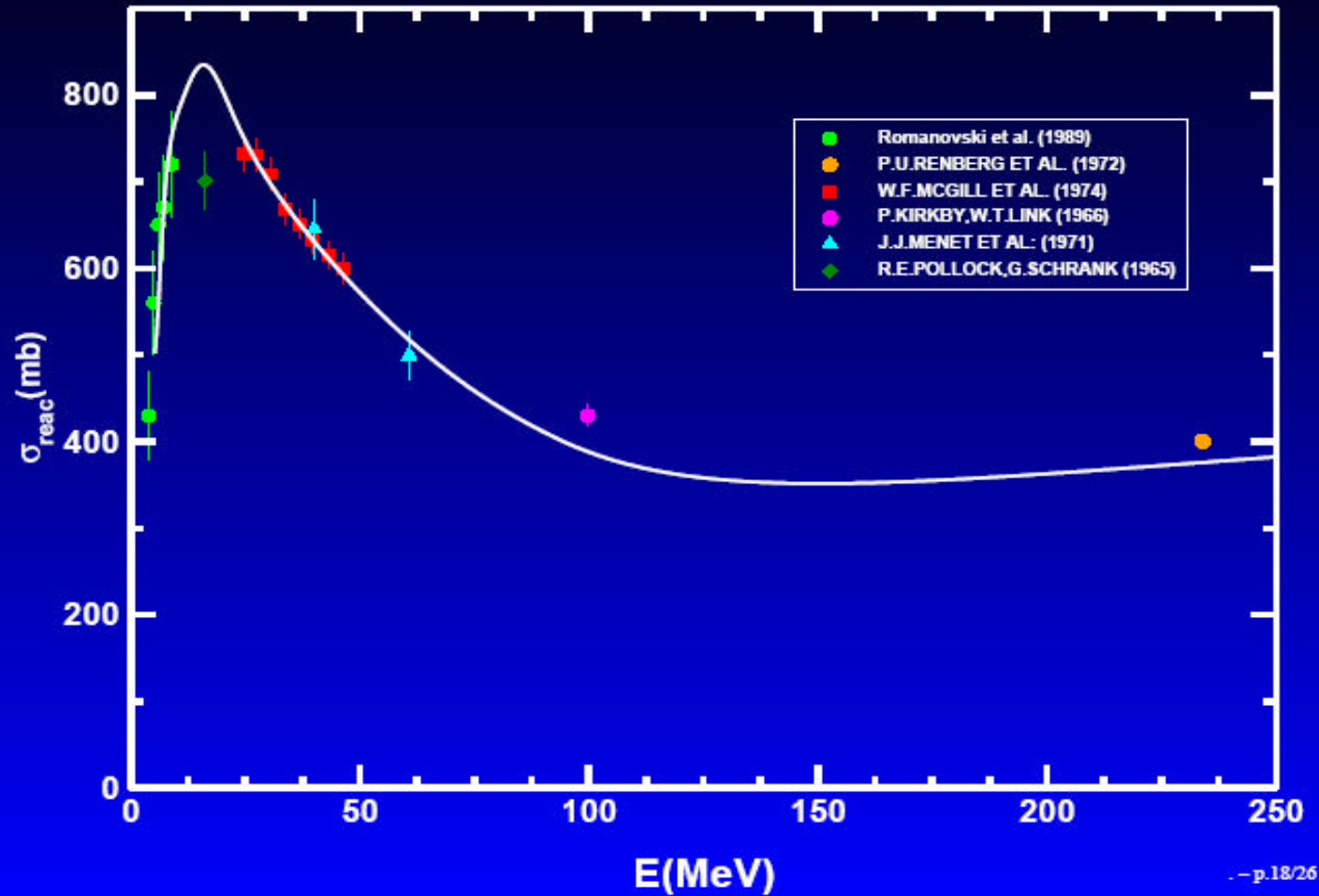


A.Molina Ph.D.Thesis, Univ.Sevilla 2003

OPTICAL MODEL AVERAGING

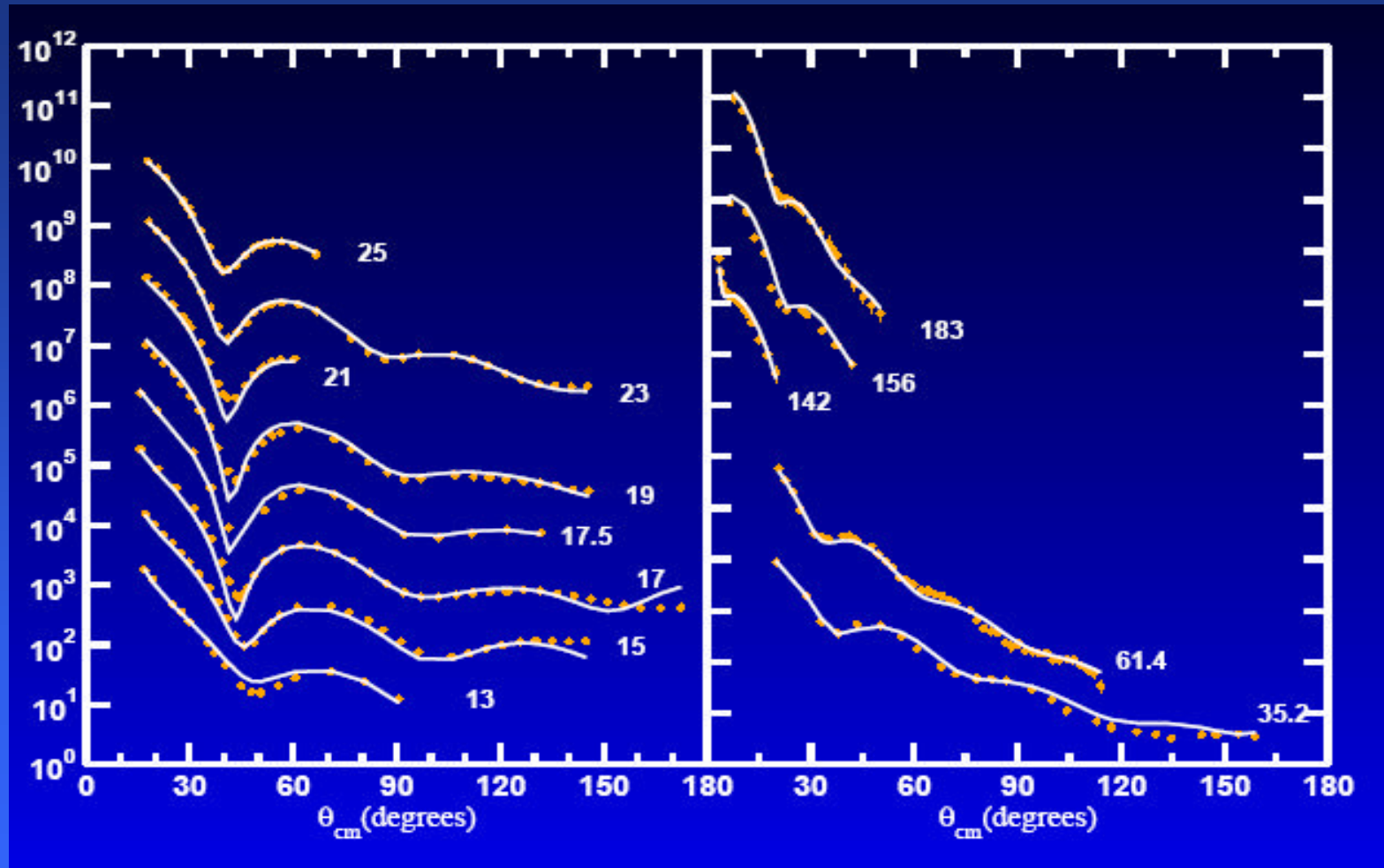


$p+^{27}\text{Al}$ - Reaction xs



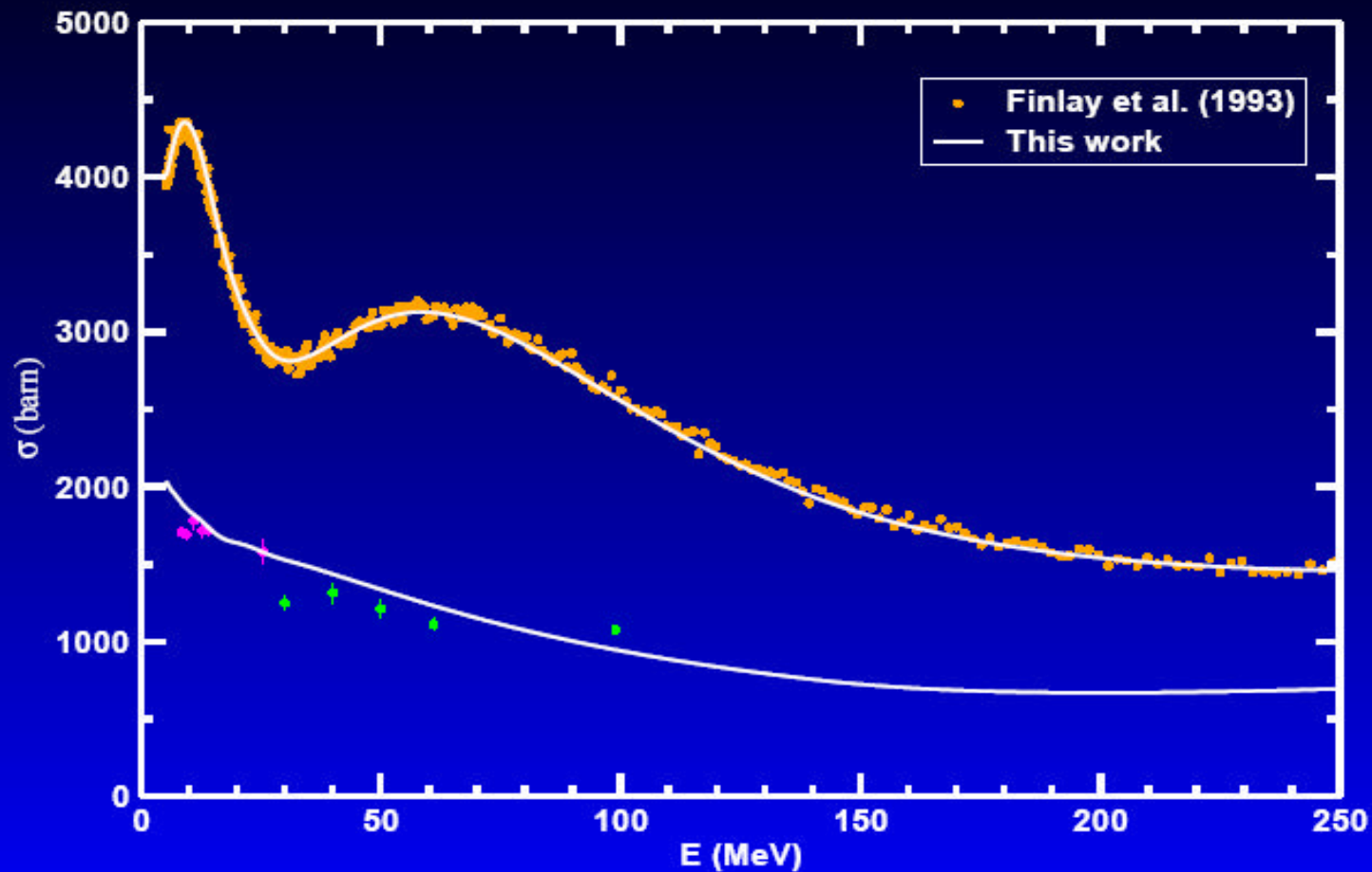
A.Molina Ph.D.Thesis, Univ.Sevilla 2003

$p+^{27}\text{Al}$ - Elastic scattering



A.Molina Ph.D.Thesis, Univ.Sevilla 2003

$n+^{90}\text{Zr}$ - Total and Reaction xs



A.Molina Ph.D.Thesis, Univ.Sevilla 2003

SINGLE CHANNEL POTENTIAL

The single-channel or spherical optical model treats the target nucleus as if it was spherical.

Even those that are spherical are often susceptible to shape oscillations  *deformed optical model*

COUPLED-CHANNEL POTENTIAL

T Tamura, "Coupled-Channel Approach to Nuclear Reactions"
Annual Review of Nuclear Science, Vol. 19 (1969) 99-138
(doi:10.1146/annurev.ns.19.120169.000531)

The expected deviation from spherical symmetry is considered by modifying the radii R_i of the OMP

VIBRATIONAL MODEL – SPHERICAL NUCLEI

$$\begin{aligned} R_i &= R_{0i} \left(1 + \sum_{\lambda\mu} a_{\lambda\mu} Y_{\lambda\mu}(\hat{r}) \right) \\ &= R_{0i} \left(1 + \sum_{\lambda} \frac{\beta_{\lambda}}{\sqrt{2\lambda+1}} \sum_{\mu} (b_{\lambda\mu}^{\dagger} + (-)^{\mu} b_{\lambda-\mu}) Y_{\lambda\mu}(\hat{r}) \right) \end{aligned}$$

RIGID ROTATOR – AXIAL SIMMETRY

$$R_i(\theta') = R_{0i} \left(1 + \sum_{\lambda} \beta_{\lambda} Y_{\lambda 0}(\theta') \right)$$

SOFT ROTATOR – TRIAXIAL SHAPE

$$\begin{aligned}
 R(\theta', \varphi') &= R_0 \left\{ 1 + \sum_{\lambda\mu} \beta_{\lambda\mu} Y_{\lambda\mu}(\theta', \varphi') \right\} \\
 &= R_0 \left\{ 1 + \beta_2 \left[\cos \gamma Y_{20}(\theta', \varphi') + \frac{1}{\sqrt{2}} \sin \gamma (Y_{22}(\theta', \varphi') + Y_{2-2}(\theta', \varphi')) \right] \right. \\
 &\quad \left. + \beta_3 \left[\cos \eta Y_{30}(\theta', \varphi') + \frac{1}{\sqrt{2}} \sin \eta (Y_{32}(\theta', \varphi') + Y_{3-2}(\theta', \varphi')) \right] \right. \\
 &\quad \left. + b_{40} Y_{40}(\theta', \varphi') + \sum_{\mu=2,4} b_{4\mu} (Y_{4\mu}(\theta', \varphi') + Y_{4-\mu}(\theta', \varphi')) \right\},
 \end{aligned}$$

where $0 \leq \beta_2 < \infty$, $-\infty < \beta_3 < \infty$, and $\frac{n\pi}{3} \leq \gamma \leq \frac{(n+1)\pi}{3}$.

$$b_{40} = \beta_4 \left(\sqrt{7/12} \cos \delta_4 + \sqrt{5/12} \sin \delta_4 \cos \gamma_4 \right),$$

$$b_{42} = \beta_4 \left(\sqrt{12} \sin \delta_4 \sin \gamma_4 \right),$$

$$b_{44} = \beta_4 \sqrt{1/2} \left(\sqrt{5/12} \cos \delta_4 + \sqrt{7/12} \sin \delta_4 \cos \gamma_4 \right)$$

SINGLE CHANNEL EQUATION

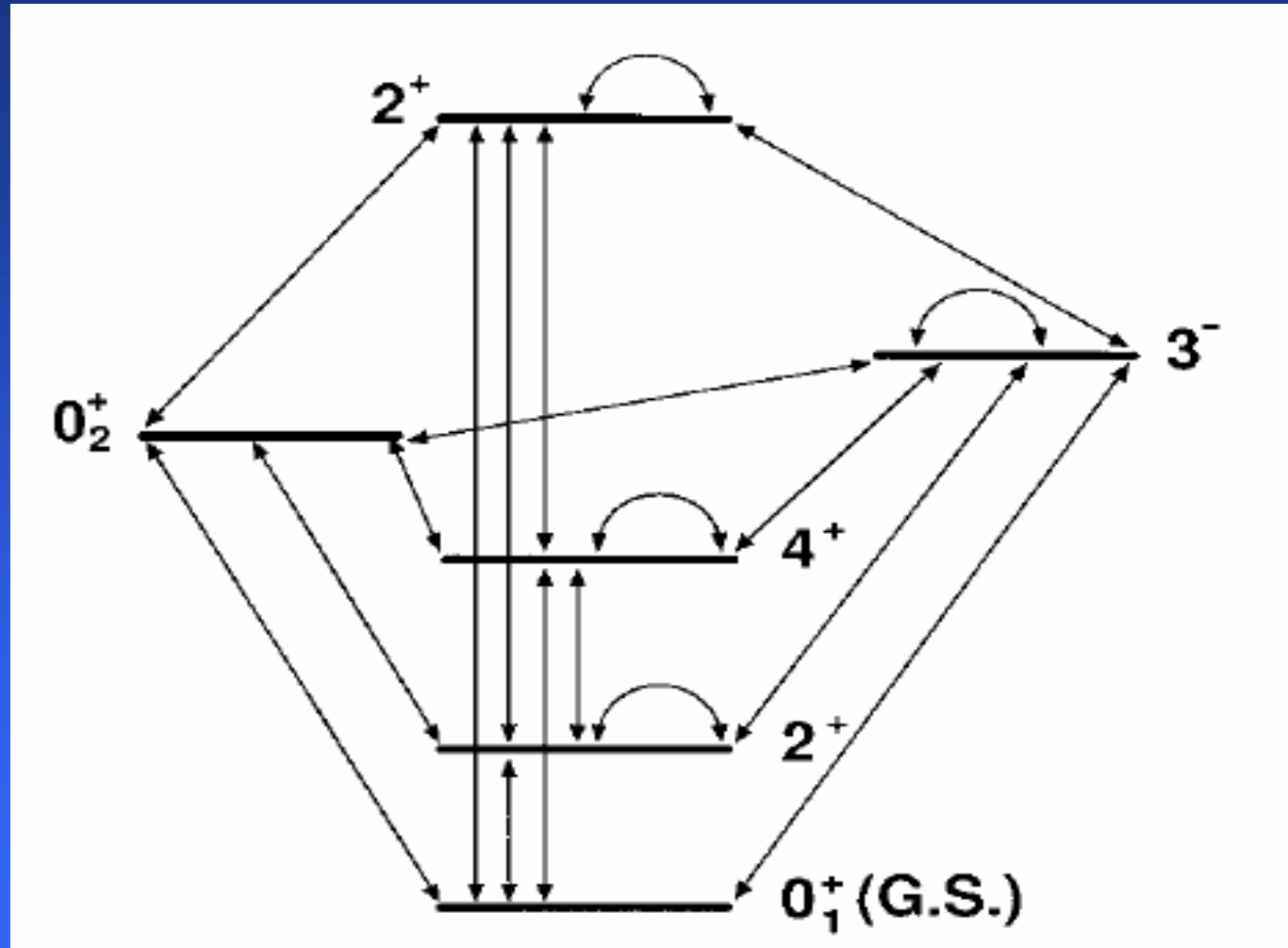
$$\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + k^2 - \frac{2\mu}{\hbar^2} \left(U(r) + d_l^j U_{so}(r) \right) \right\} \psi_l^j(r) = 0$$

COUPLED CHANNEL EQUATIONS

$$\frac{\hbar^2}{2\mu} \left\{ \frac{d^2}{dr^2} - \frac{l'(l'+1)}{r^2} + k_{c'}^2 \right\} \psi_{l'j'c',ljc}^J(r) - \sum_{l''j''c''} \mathcal{U}_{l'j'c',l''j''c''}^J(r) \psi_{l''j''c'',ljc}^J(r) = 0,$$

$$\mathcal{U}_{l'j'c',ljc}^J(r) = \int d^3r_{int} d\Omega \mathcal{Y}_{l'sj'c'}^{JM\dagger}(\hat{r}) U_{opt}(\vec{r}, \vec{r}_{int}) \mathcal{Y}_{lsjc}^{JM}(\hat{r}).$$

Coupling scheme for matrix element calculations



ORIGINAL PAPER

**Coupled-Channels Analysis of Nucleon Interaction Data of $^{28,30}\text{Si}$
up to 200 MeV Based on the Soft Rotator Model**

Weili SUN^{1,*}, Yukinobu WATANABE², Efrem Sh. SUKHOVITSKIĬ³, Osamu IWAMOTO⁴
and Satoshi CHIBA⁴

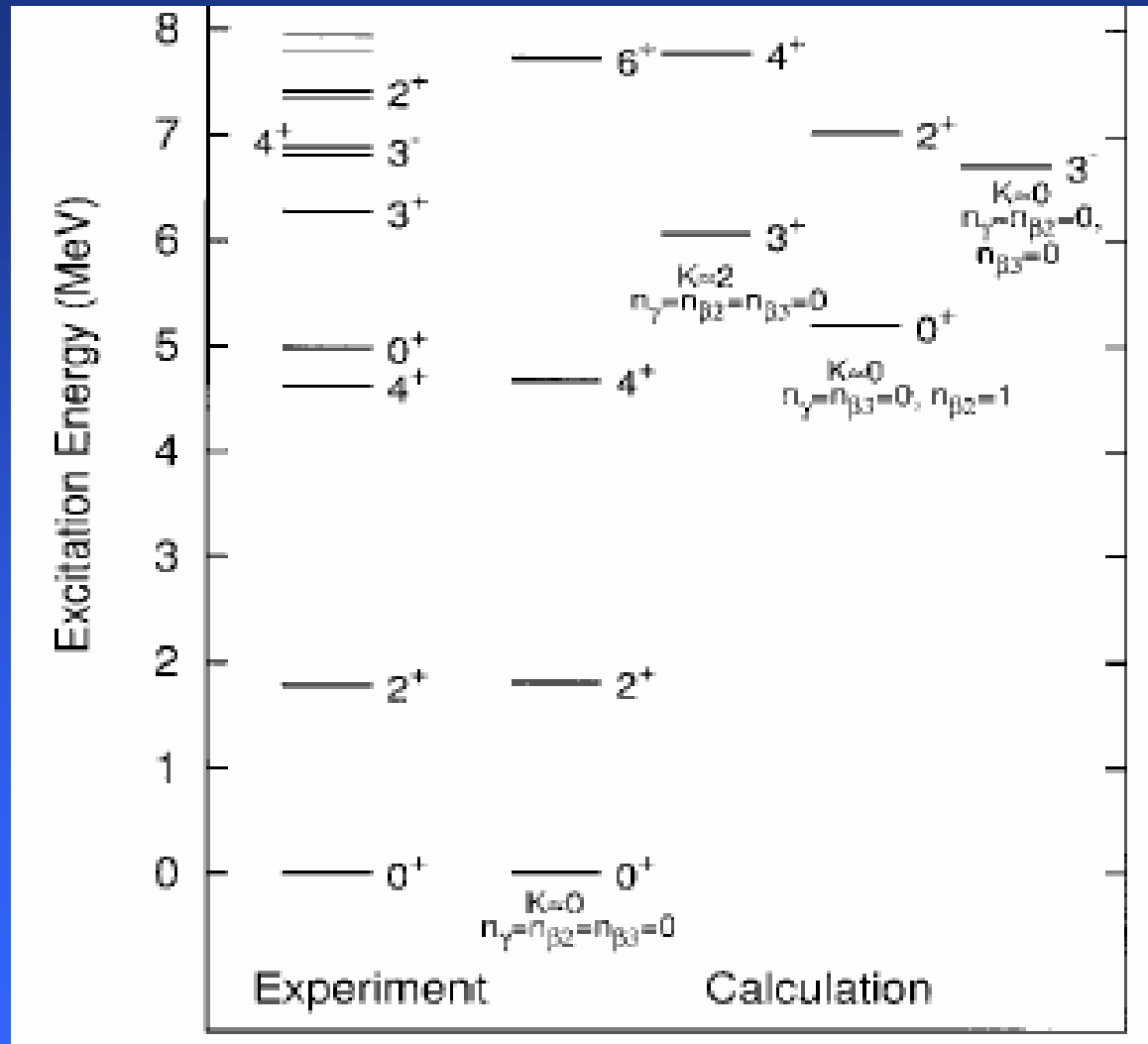
¹*Institute of Applied Physics and Computational Mathematics, No. 6, Huayuan Road, Hai-Dian District, Beijing 100088, China*

²*Department of Advanced Energy Engineering Science, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580*

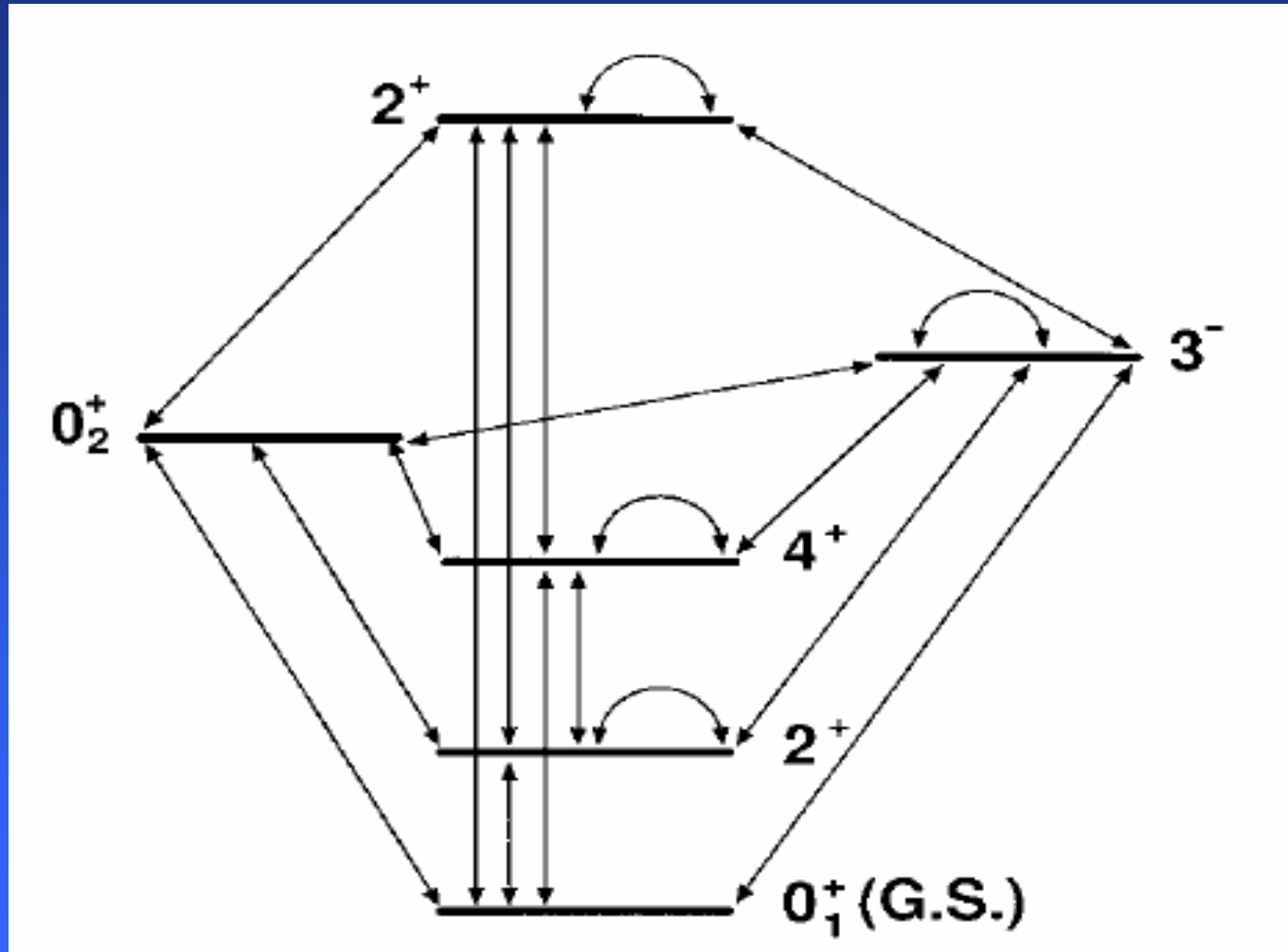
³*Joint Institute of Energy and Nuclear Research-Sosny, 220109, Minsk-Sosny, Belarus*

⁴*Japan Atomic Energy Research Institute, Tokai-mura, Naka-gun, Ibaraki 319-1195*

First step: Nuclear structure calculation



Second step: Define the coupling scheme

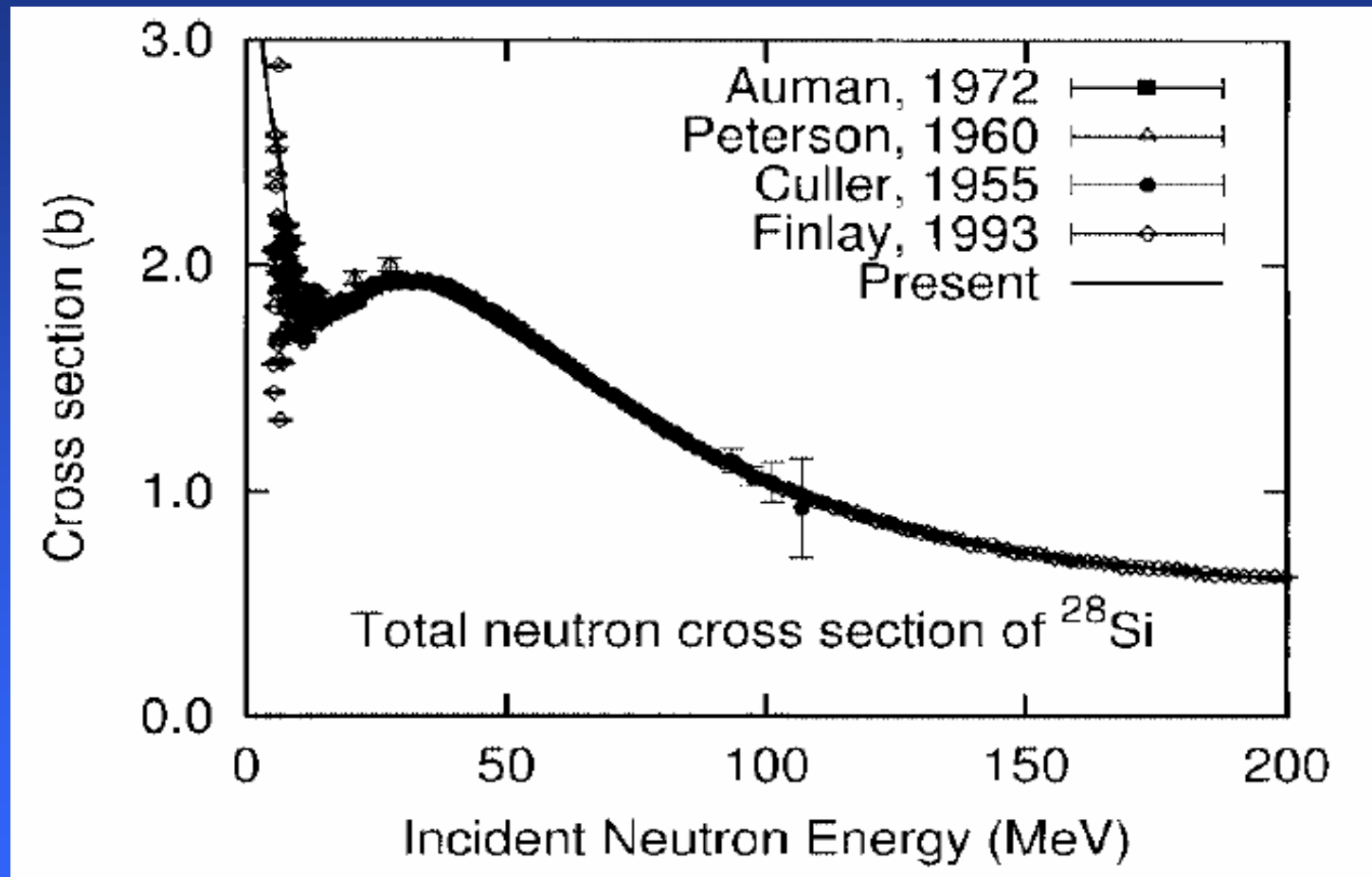


3rd step: Solve the CC equations

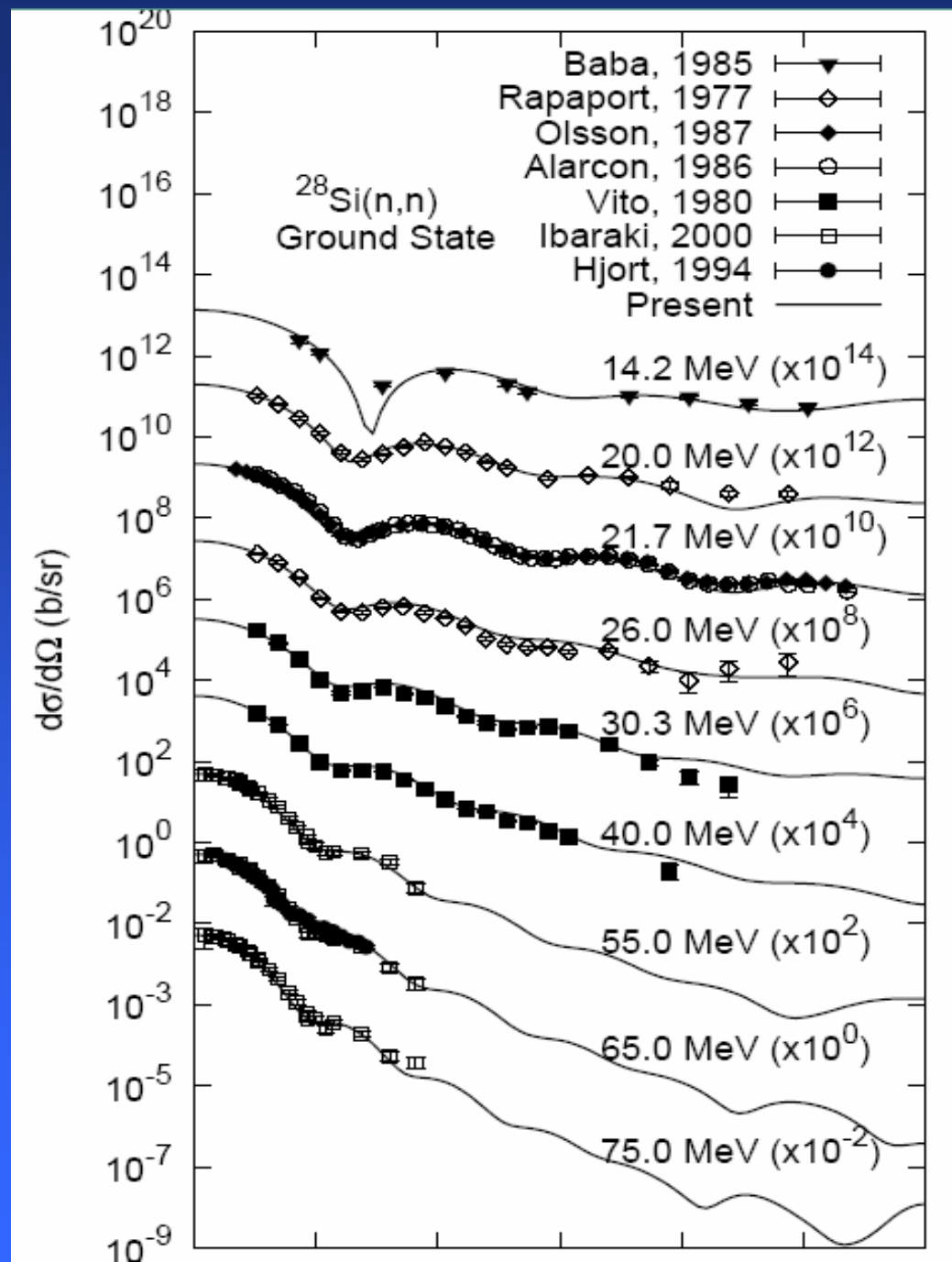
Calculate cross section, angular distribution and polarization observables for both, elastic and inelastic scattering on coupled levels.

(EMPIRE/ECIS ...)

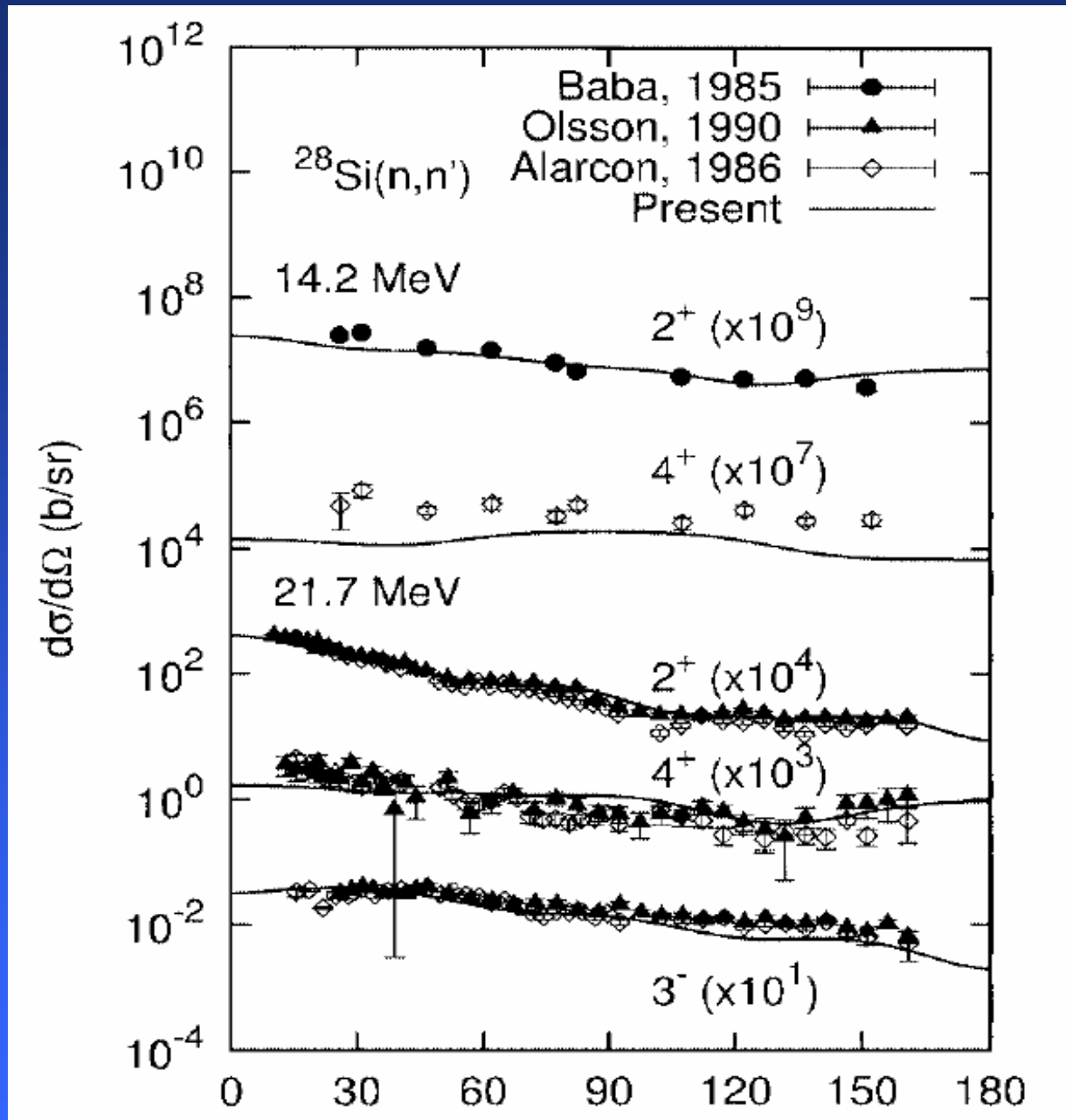
TOTAL xs (similar to spherical OMP)



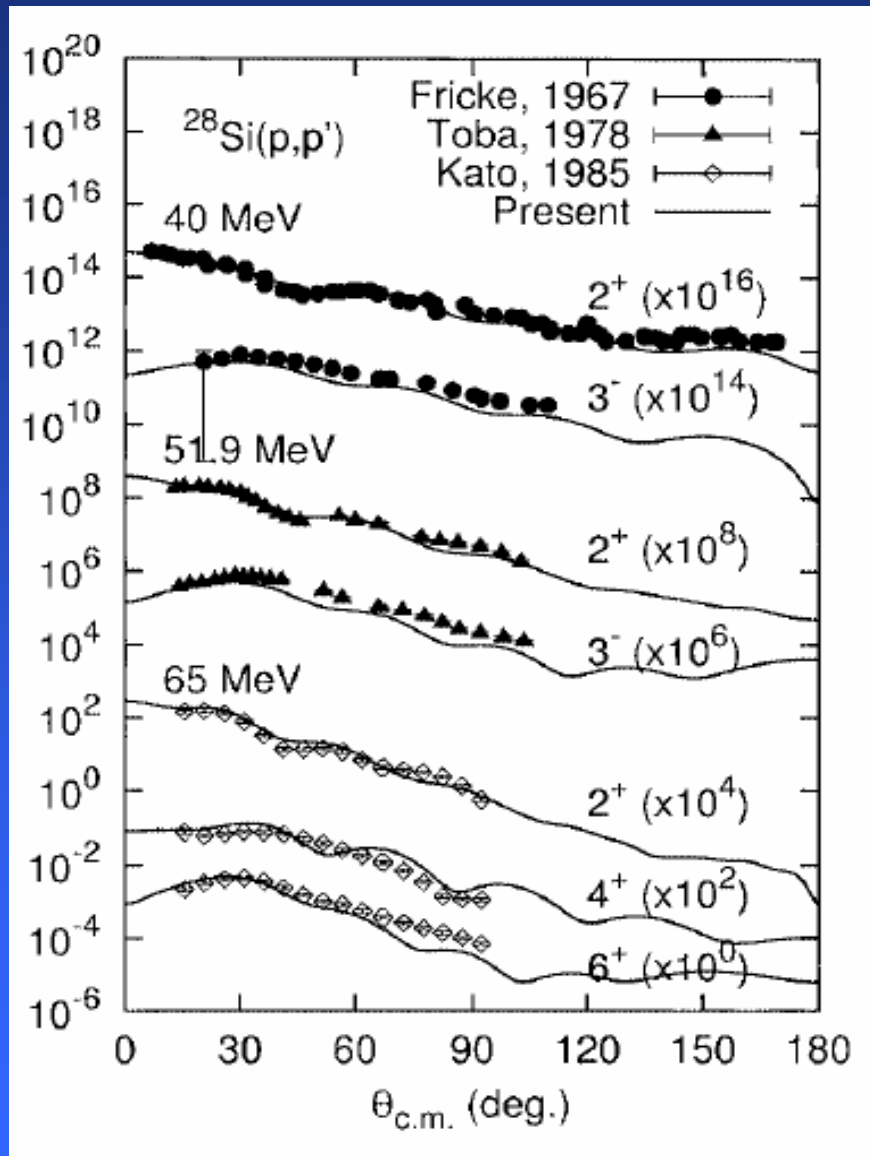
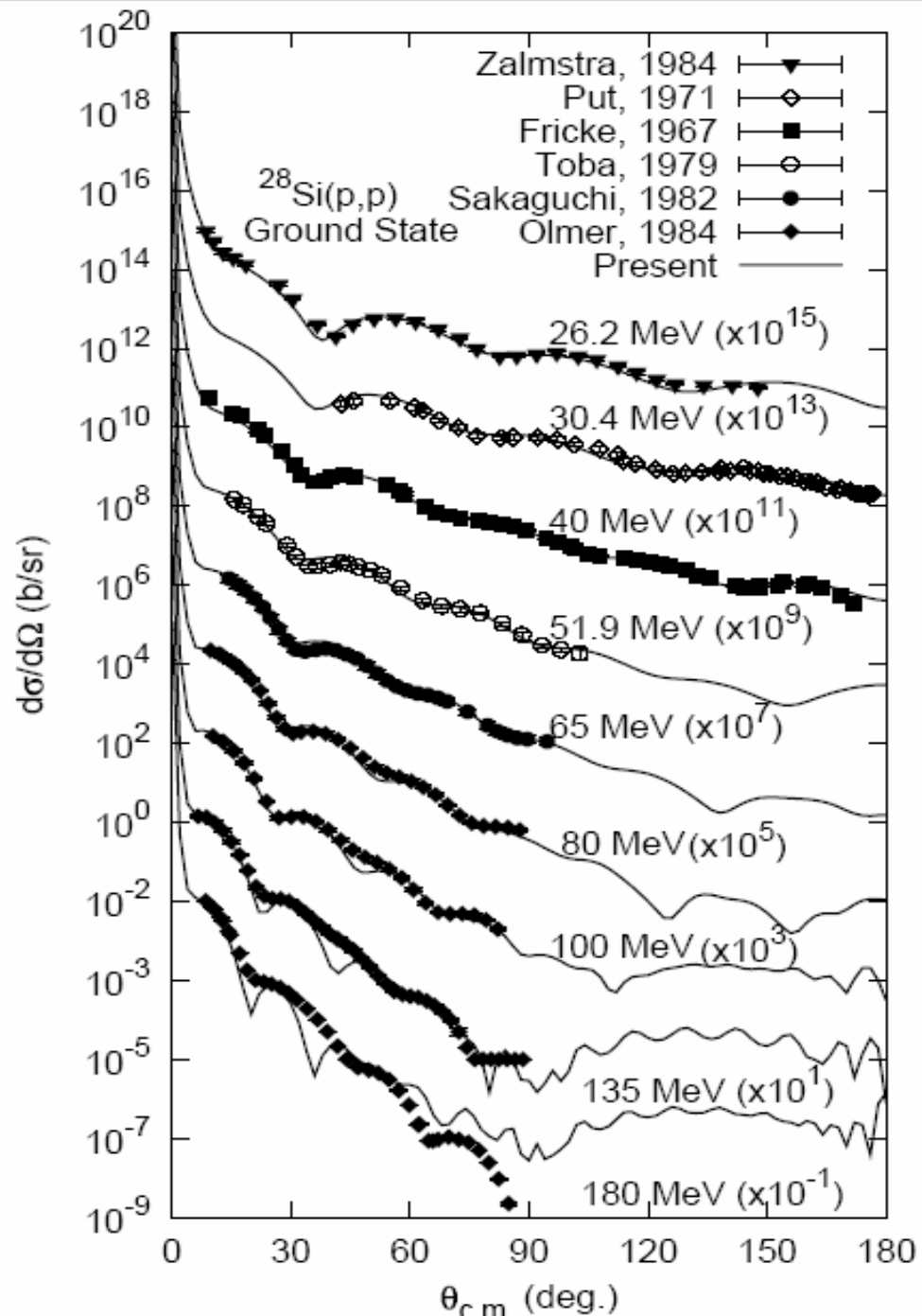
ELASTIC x_s (\sim spher.OMP)



PLUS
INELASTIC XS
for coupled levels !!



PROTON CHANNEL



Solve the CC equations (EMPIRE)

EMPIRE/ECIS highlights

- Spherical optical model (DIRECT = 0 *default*)
- Coupled-channel method (DIRECT = 1,2)
(vibrational, rotational, vibrational-rotational)
- DWBA calculations (DIRECT = 3)

RIPL-II optical model database

Lib. No.	Inc. Part.	Model Type	Disp Pot	Rel Pot	Z-Range	A-Range	E-Range (MeV)	Ref. No.	First Author
neutrons									
200	n	spher.	no	no	0-69	0-146	0.0- 20.0	1	S.Igarasi
240	n	spher.	no	no	0-69	0-146	0.0- 20.0	2	S.Igarasi
2001	n	spher.	no	yes	6-82	12-208	50.0-400.0	4	D.G.Madland
101	n	spher.	no	no	12-83	24-209	11.0- 11.0	5	J.C.Ferrer
116	n	spher.	no	no	20-83	40-209	0.0- 5.0	12	P.A.Moldauer
401	n	spher.	no	no	20-92	40-238	0.0- 25.0	13	D.Wilmore
2100	n	spher.	no	no	20-83	40-209	10.0- 26.0	14	R.L.Varner
100	n	spher.	no	no	20-92	40-238	10.0- 50.0	15	F.D.Becchetti
800	n	spher.	no	no	20-83	40-210	0.0-155.0	16	C.A.Engelbrecht
404	n	spher.	no	no	23-41	50- 95	0.0- 30.0	17	B.Strohmaier
600	n	CC rot.	no	no	90-103	227-260	0.0- 20.0	56	G.Vladuca

Part of the EMPIRE input file for CC calculation

```

DIRECT 1.
DIRPOT -600.
OMPOT -2002. 1
OMPOT -5405. 2
....
    
```

First run produces “collective level file”

```
Collective levels selected automatically from ...  
Dyn.deformations are not used in symm.rot.model  
94 240      nucleus is treated as deformed
```

```
      Ncoll  Lmax  IDef  Kgs  (Def(1,j),j=2,IDef,2)  
        6     4     4     .0   .223E+00   .000E+00
```

```
 N   E [MeV]   J   pi  Nph  L   K   Dyn.Def.  
 1   .0000     .0   1.   0   0   0   .100E-01  
 2   .0428     2.0   1.   0   0   0   .100E-01  
 3   .1417     4.0   1.   0   0   0   .100E-01  
 4   .2943     6.0   1.   0   0   0   .100E-01  
 5   .8600     0.0   1.   1   0   0   .08  
 6   .900      2.0   1.   1   2   0   .095
```

The “collective level file” should be edited to modify the parameters of the nuclear structure model

```
Collective levels selected automatically from ...  
Dyn.deformations are not used in symm.rot.model  
94 240      nucleus is treated as deformed
```

```
      Ncoll  Lmax  IDef  Kgs  (Def(1,j), j=2, IDef, 2)  
        6     4     4     .0   .223E+00   .000E+00
```

N	E [MeV]	J	pi	Nph	L	K	Dyn.Def.
1	.0000	.0	1.	0	0	0	.100E-01
2	.0428	2.0	1.	0	0	0	.100E-01
3	.1417	4.0	1.	0	0	0	.100E-01
4	.2943	6.0	1.	0	0	0	.100E-01
5	.8600	0.0	1.	1	0	0	.08
6	.900	2.0	1.	1	2	0	.095



Workshop on Nuclear Reaction Data and Nuclear Reactors - Physics, Design and Safety



Trieste, Febr.2004



ictp

