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Reactor Dynamics (Part II)

The General Neutron Kinetic Problem Fluid-fuel Systems

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THE GENERAL NEUTRON KINETIC PROBLEM

Analysis of nuclear reactor dynamics may require spatial and spectral neutron kinetics

Multidimensional evaluations require large computational effort for a direct numerical solution

$\Longrightarrow$ Quasi-statics is an attracting method

The subcriticality of ADS requires the development of ad-hoc numerical methods

Classic **Quasi-Static Method**

Steady-state problem (source-free critical system, reference reactor)

$$(\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_0) N_0(r, E, \Omega) = 0, \qquad N_0(r_S, E, \Omega_{in}) = 0$$

steady-state multiplication operator

$$\hat{\mathrm{M}}_0 = \sum_j \frac{\chi^j(E)}{4\pi} \int dE' \oint d\Omega' v(E') \nu^j(E') \Sigma_f^j(r, E', 0)$$

Adjoint problem (source-free critical system, reference reactor)

$$(\hat{\mathrm{L}}_0^+ + \hat{\mathrm{M}}_0^+) N_0^+(r, E, \Omega) = 0, \qquad N_0^+(r_S, E, \Omega_{out}) = 0$$

Factorization formula:

$$n(r, E, \Omega, t) = P(t) \varphi(r, E, \Omega; t)$$

* $P(t)$ is the amplitude function;
* $\varphi(r, E, \Omega; t)$ is the shape function;
* Two-scales in time are introduced: the evolution of the amplitude may be much faster than the evolution of the shape;
* As such the factorization is not unique;

Introduce the factorization into the balance equations (shape equations)

$$\begin{cases} P\dfrac{\partial \varphi}{\partial t} + \varphi\dfrac{dP}{dt} = P\hat{\mathrm{B}}\varphi + \displaystyle\sum_{i=1}^{6} \lambda_i \left(\frac{\chi_i}{4\pi}C_i\right) + S \\ \dfrac{\partial(\chi_i C_i/4\pi)}{\partial t} = P\hat{\mathrm{M}}_i\varphi - \lambda_i \left(\dfrac{\chi_i}{4\pi}C_i\right) \end{cases}$$

solve for the delayed neutron precursor concentrations

$$\frac{\chi_i(E)}{4\pi}C_i(\mathbf{r}, t) = \frac{\chi_i(E)}{4\pi}C_i(\mathbf{r}, t = t_0)e^{-\lambda_i(t-t_0)} + \\ + \int_{t_0}^{t} P(t')\hat{\mathrm{M}}_i\varphi(\mathbf{r}, E, \Omega; t')e^{-\lambda_i(t-t')}dt'$$

Project on the solution to the adjoint problem

$$\begin{cases} P\frac{\partial}{\partial t}\left\langle N_0^+\right|\varphi\rangle + \frac{dP}{dt}\left\langle N_0^+\right|\varphi\rangle = \\ \qquad P\left\langle N_0^+\right|\hat{\mathrm{B}}\varphi\Big\rangle + \sum_{i=1}^{6}\lambda_i\left\langle N_0^+\right|\frac{\chi_i}{4\pi}C_i\Big\rangle + \left\langle N_0^+\right|S\rangle \\ \frac{\partial}{\partial t}\left\langle N_0^+\right|\frac{\chi_i}{4\pi}C_i\Big\rangle = P\left\langle N_0^+\right|\hat{\mathrm{M}}_i\varphi\Big\rangle - \lambda_i\left\langle N_0^+\right|\frac{\chi_i}{4\pi}C_i\Big\rangle \end{cases}$$

Require a normalization condition for the shape function

$$\frac{\partial}{\partial t}\left\langle N_0^+\right|\varphi\rangle = 0$$

Definitions:

$$\hat{\mathrm{B}}(t) = \hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t) = (\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_0) + \delta\hat{\mathrm{B}}(t) - \\ \sum_{i=1}^{6}\hat{\mathrm{M}}_i(0) - \sum_{i=1}^{6}\delta\hat{\mathrm{M}}_i(t) + \sum_{i=1}^{6}\delta\hat{\mathrm{M}}_i(t) = \\ = (\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_0) + \delta\left[\hat{\mathrm{B}}(t) + \sum_{i=1}^{6}\hat{\mathrm{M}}_i(t)\right] - \sum_{i=1}^{6}\hat{\mathrm{M}}_i(t)$$

Perturbation operator:

$$\delta\hat{\mathrm{K}} = \delta\hat{\mathrm{B}}(t) + \sum_{i=1}^{6} \delta\hat{\mathrm{M}}_i(t)$$

Equations for the amplitude are obtained (**point-like equations**):

$$\begin{cases} \dfrac{dP(t)}{dt} = \dfrac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S}(t) \\ \dfrac{\partial \tilde{C}_i(t)}{\partial t} = \dfrac{\tilde{\beta}_i}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t) \end{cases}$$

Note: if shape is kept constant, the standard point model is obtained.

Kinetic parameters:

$\Rightarrow$ reactivity

$$\rho(t) = \frac{\left\langle N_0^+ \right| \delta\hat{\mathrm{K}}\varphi \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}}\varphi \Big\rangle}$$

$\Rightarrow$ effective mean prompt-neutron generation time

$$\Lambda = \frac{\left\langle N_0^+ \right| \varphi \rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}}\varphi \Big\rangle}$$

$\Rightarrow$ effective delayed neutron fractions

$$\tilde{\beta}_i = \frac{\left\langle N_0^+ \right| \hat{\mathrm{M}}_i\varphi \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}}\varphi \Big\rangle}$$

Effective delayed neutron precursor concentrations

$$\tilde{C}_i(t) = \frac{\left\langle N_0^+ \right| \frac{\chi_i}{4\pi} C_i \Big\rangle}{\left\langle N_0^+ \right| \varphi \rangle} = \frac{1}{\Lambda} \frac{\left\langle N_0^+ \right| \frac{\chi_i}{4\pi} C_i \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}}\varphi \Big\rangle}$$

Effective external source

$$\tilde{S}(t) = \frac{\langle N_0^+ | S \rangle}{\langle N_0^+ | \varphi \rangle} = \frac{1}{\Lambda} \frac{\langle N_0^+ | S \rangle}{\left\langle N_0^+ \middle| \hat{\mathrm{M}} \varphi \right\rangle}$$

For the numerical solution of the problem two time intervals are introduced:

⇒ Shape interval (slow phenomena) Δt_φ

⇒ Amplitude interval (fast phenomena) Δt_P

Discrete equations for the the shape:

$$T = t_0 + \Delta t_\varphi$$

$$P(T)\frac{\varphi(T) - \varphi(t_0)}{\Delta t_\varphi} + \varphi(T)\dot{P}(T) = P(T)\hat{\mathrm{B}}\varphi(T) +$$

$$\sum_{i=1}^{6} \lambda_i \left[\frac{\chi_i(E)}{4\pi} C_i(t_0) e^{-\lambda_i \Delta t_\varphi} + \right.$$

$$\left. \int_{t_0}^{T} P(t')\hat{\mathrm{M}}_i \varphi(t_0) e^{-\lambda_i (T-t')} dt' + S(T) \right]$$

$$\Longrightarrow \varphi(T)$$

In general, $\varphi(T)$ will not satisfy the required normalization condition, namely:

$$\gamma(T) = \left\langle N_0^+ \middle| \varphi(T) \right\rangle \neq \gamma(t_0) = \left\langle N_0^+ \middle| \varphi(t_0) \right\rangle$$

An iterative process is necessary.

$$P(T)\frac{\varphi^{(n)}(T)-\varphi(t_0)}{\Delta t_\varphi}+\varphi^{(n)}(T)\dot{P}^{(n)}(T)=$$

$$P(T)\hat{\mathrm{B}}\varphi^{(n)}(T)+\sum_{i=1}^{6}\lambda_i\left[\frac{\chi_i(E)}{4\pi}C_i(t_0)e^{-\lambda_i\Delta t_\varphi}\right.$$

$$\left.+\int_{t_0}^{T}P(t')\hat{\mathrm{M}}_i\varphi(t_0)e^{-\lambda_i(T-t')}dt'+S(T)\right]$$

$$\Longrightarrow \varphi^{(n)}(T)$$

$$\gamma^{(n)}(T)=\left\langle N_0^{+}\right|\left.\varphi^{(n)}(T)\right\rangle$$

$$\varphi^{(n+1/2)}(T)=\frac{\gamma(t_0)}{\gamma^{(n)}(T)}\varphi^{(n)}(T)$$

The derivative of the amplitude function is allowed to be discontinuous; it is updated according to:

$$\dot{P}^{(n+1)}(T) = P(T)\frac{\left\langle N_0^+ \right| \hat{\mathrm{B}}\varphi^{(n+1/2)} \left.\right\rangle}{\left\langle N_0^+ \right| \varphi^{(n+1/2)}(T) \left.\right\rangle} + \sum_{i=1}^{6} \lambda_i \frac{\left\langle N_0^+ \right| \frac{\chi_i}{4\pi} C_i \left.\right\rangle}{\left\langle N_0^+ \right| \varphi^{(n+1/2)}(T) \left.\right\rangle} + \frac{\left\langle N_0^+ \right| S(T) \rangle}{\left\langle N_0^+ \right| \varphi^{(n+1/2)}(T) \left.\right\rangle}$$

Source-Driven Problems:
Quasi-static needs to be adapted

The reference reactor is driven by an external source $\Rightarrow$ the initial shape is assumed as solution of the steady-state equation:

$$(\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_0)N_0(r, E, \Omega) + S_0 = 0,$$

$$N_0(r_S, E, \Omega_{in}) = 0$$

How to define the adjoint?

a) introduce a multiplication eigenvalue in the adjoint equation

$$(\hat{\mathrm{L}}_0^+ + \frac{1}{k_0}\hat{\mathrm{M}}_0^+)N_{0,cr}^+(r, E, \Omega) = 0,$$

$$N_{0,cr}^+(r_S, E, \Omega_{out}) = 0$$

b) introduce an adjoint source

$$(\hat{\mathrm{L}}_0^+ + \hat{\mathrm{M}}_0^+)N_{0,s}^+(r, E, \Omega) + S^+ = 0,$$

$$N_{0,s}^+(r_S, E, \Omega_{out}) = 0$$

Problem: how to define the adjoint source?

Possible solution:

$\Rightarrow$ if importance is defined as the number of fission neutrons to be produced per neutron injected at a point in phase space, adjoint source is

$$S^{+} = \nu\Sigma_f$$

Consequence on the reactivity of the system:

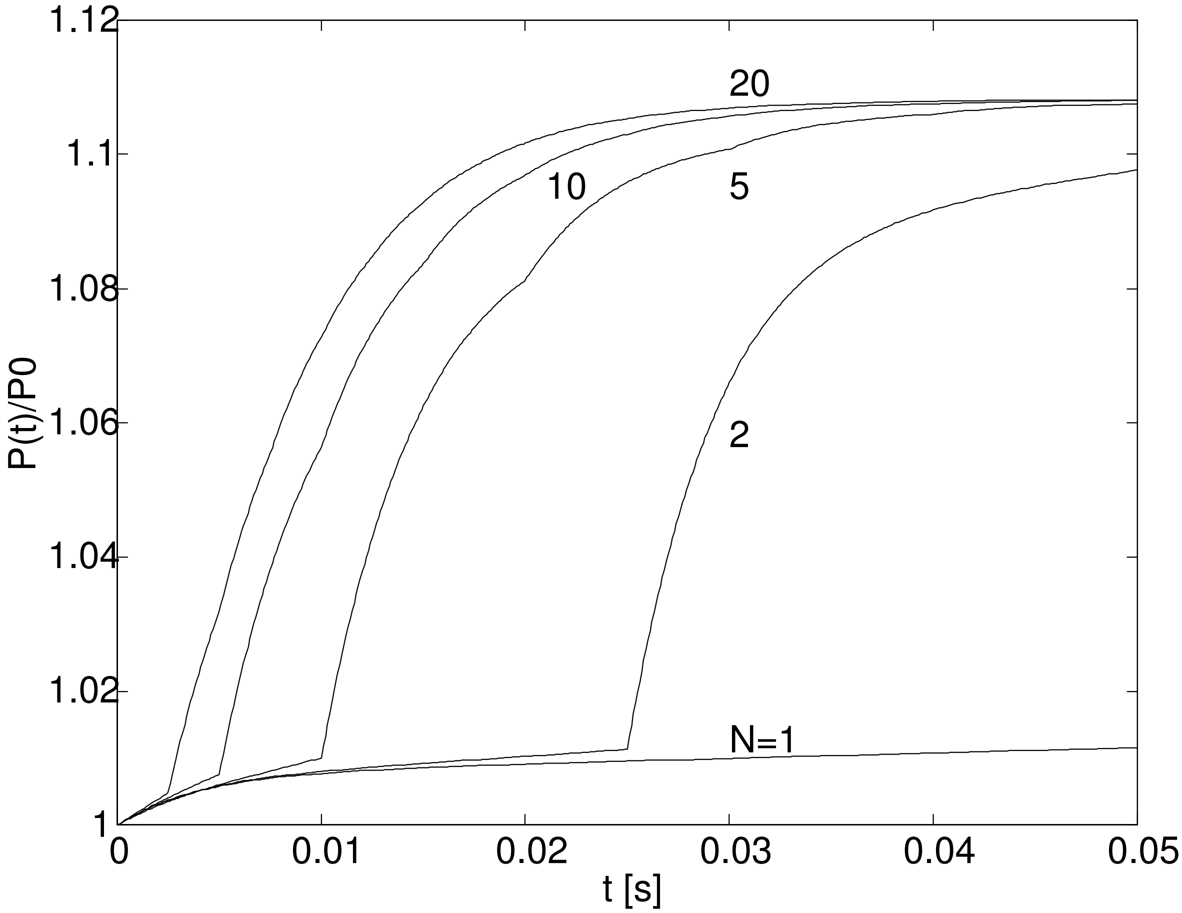
$$\rho = \rho_0 + \rho_p$$

ρ_p is the pertubation reactivity

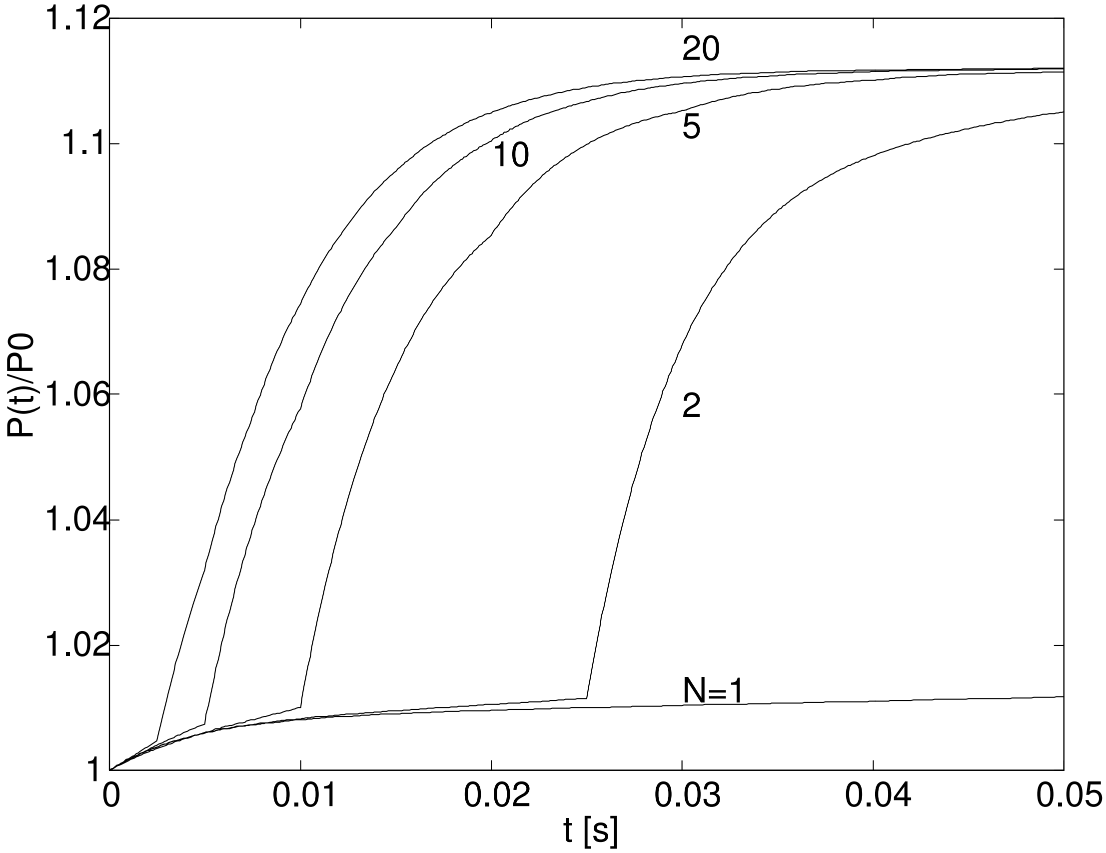
ρ_0 is connected to the initial subcriticality level

$$\rho_0 = \frac{\left\langle N^{+}_{0,cr} \middle| \hat{\mathrm{M}}_0 \varphi \right\rangle}{\left\langle N^{+}_{0,cr} \middle| \hat{\mathrm{M}} \varphi \right\rangle} \left(\frac{k-1}{k} \right)$$

$$\rho_0 = -\frac{\left\langle S^{+} | \varphi \right\rangle}{\left\langle N^{+}_{0,s} \middle| \hat{\mathrm{M}} \varphi \right\rangle}$$



Power: weight function as source adjoint.



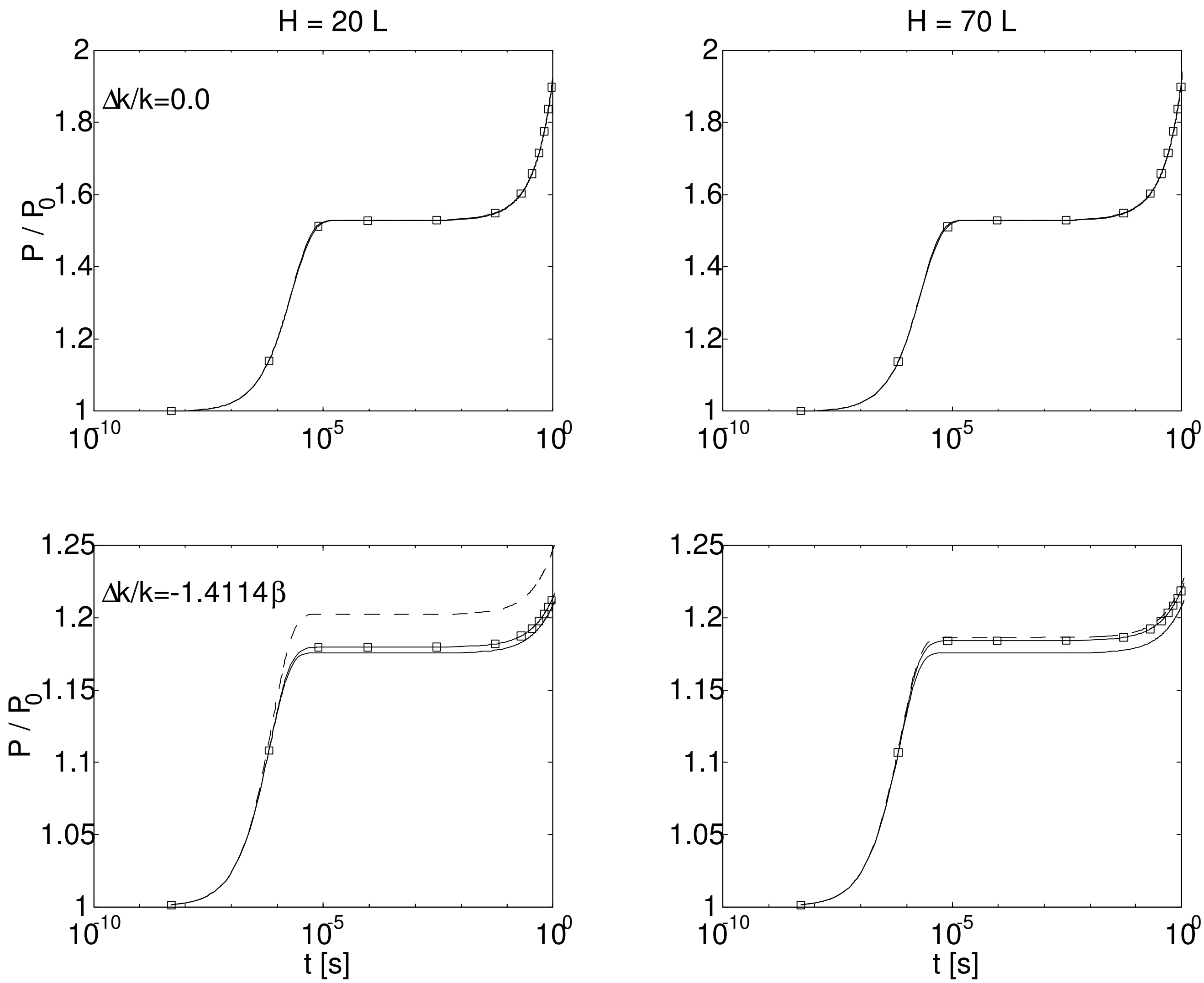
Power: weight function as critical adjoint.

DISCUSSION ON THE CHOICE OF THE WEIGHTING FUNCTION IN SEPARATION SCHEMES

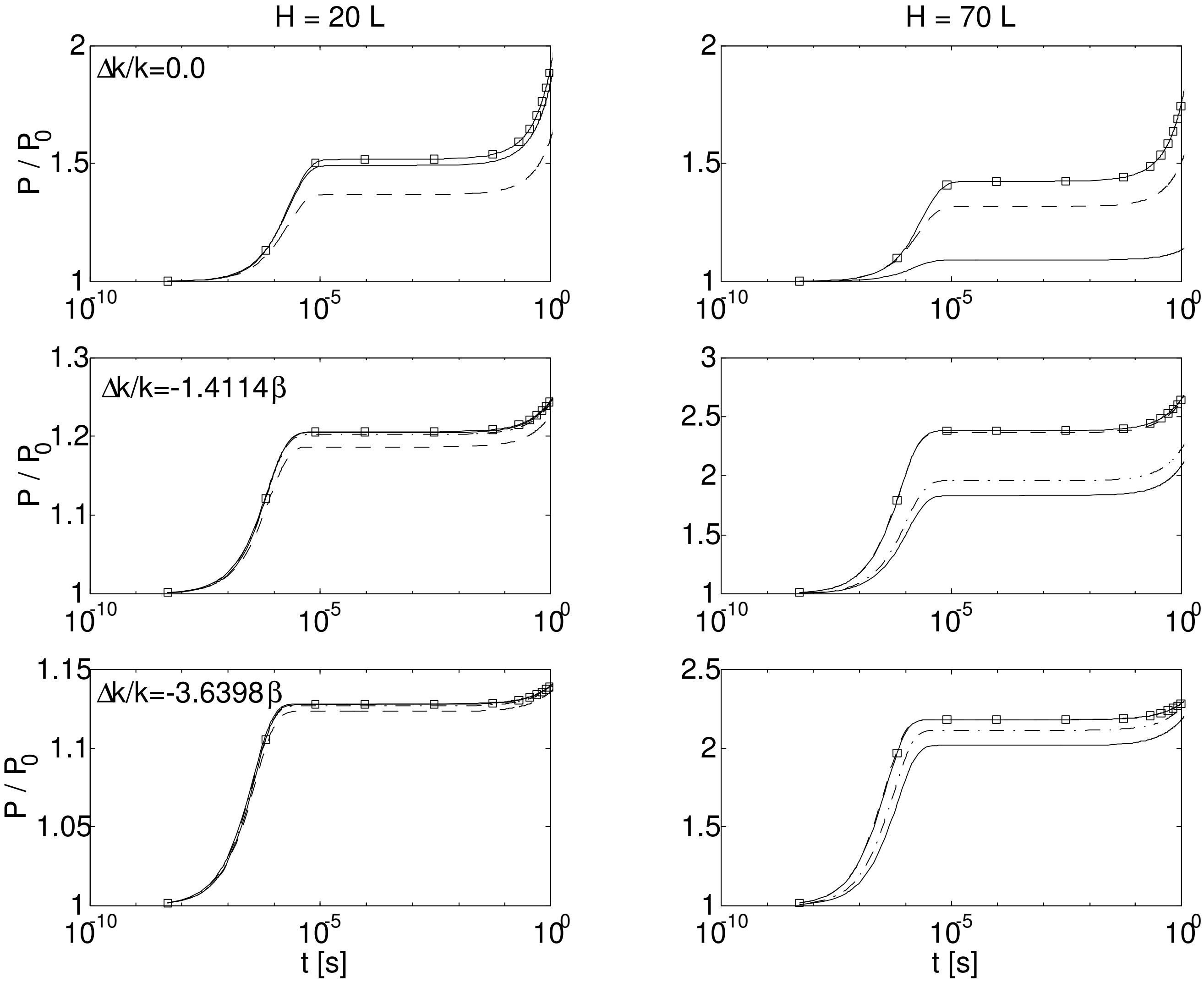
It is an open problem for subcritical systems:

$\Longrightarrow$ Evidence problem, rather then propose solutions;

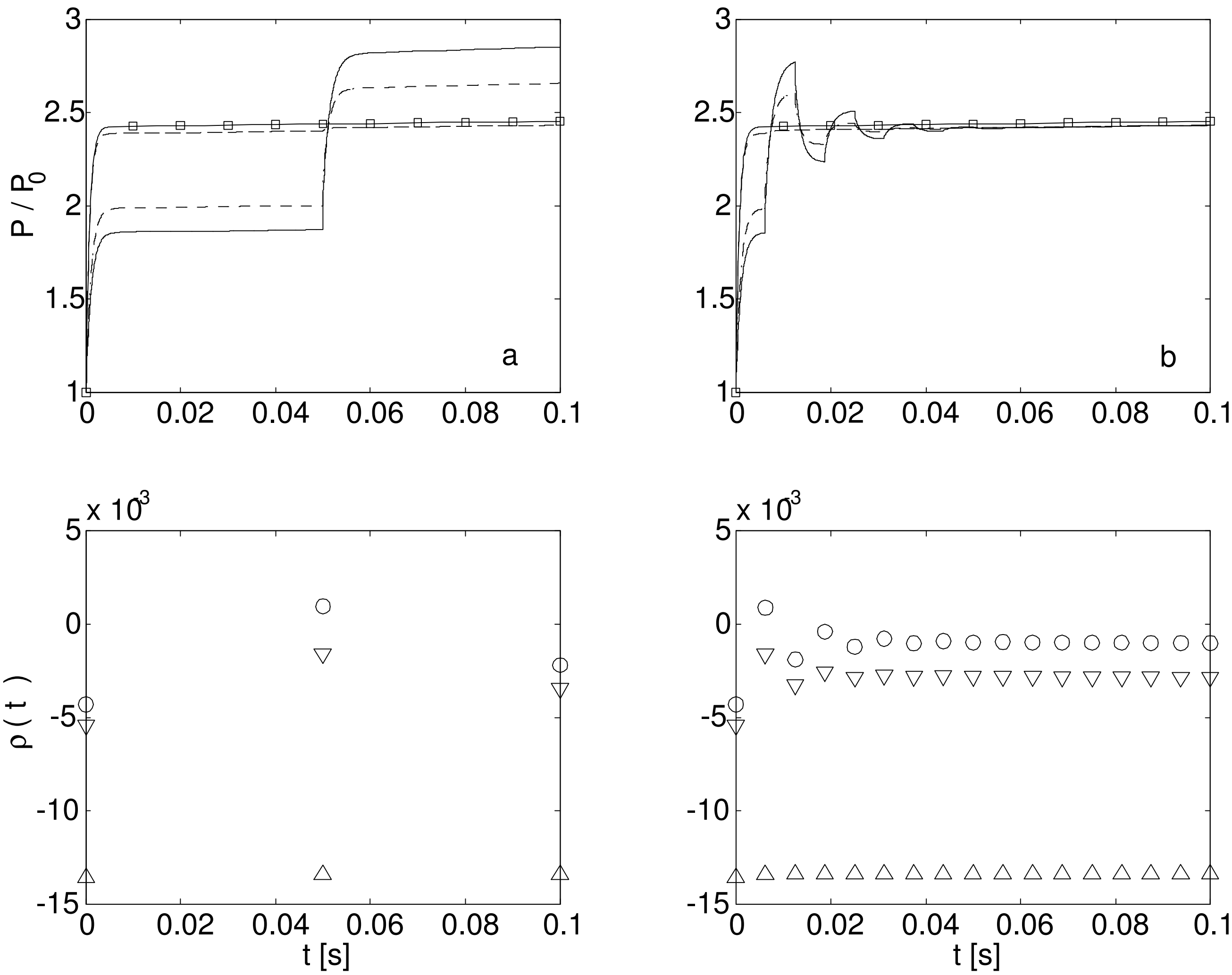
$\Longrightarrow$ Reasonable possibilities: critical adjoint or the adjoint driven by the fission cross-section as a source.



Homogeneous perturbation, systems with different k. Top: initial critical system; bottom: initial subcritical system, $k = 0.98$. Squares: reference; broken line: constant adjoint; solid line: critical adjoint; dot-point line: source-driven adjoint.



Comparison of the power evolutions calculated using different weighting functions in response to a localized perturbation. Squares: reference results; broken line: constant adjoint; solid line: critical adjoint; dot-point line: source-driven adjoint.



Quasi-static calculations with $H = 70L$ and $\Delta k/k = -1.4114\beta$, compared to the full spatial results. One shape update for graphs a), 15 recalculations for case b). Reactivity at the bottom; $\triangledown$: source adjoint; $\triangle$: constant; $\circ$: critical adjoint.

DEVELOPMENTS

The quasi-static method proves to be a powerful and computationally efficient tool for the analysis of source-driven systems

Accurate predictions of the transient behavior in non-linear conditions dominated by thermal feed-back can be attained

Development:
$\Longrightarrow$ An improvement: the **multipoint** method

Balance equations in discretized form:

$$\begin{cases} \frac{1}{v_m}\frac{d\phi_{nm}}{dt} = \sum_{n'}\sum_{m'} k_{nm,n'm'}\phi_{n'm'} + \\ \qquad \sum_{i=1}^{6} \lambda_i \chi_{i,m} C_{i,n} + S_{nm} \\ \frac{dC_{i,n}}{dt} = \beta_i \sum_{m'} f_{nm'}\phi_{nm'} - \lambda_i C_{i,n} \qquad i = 1, 2, ..., 6 \end{cases}$$

$$\phi_{nm}(t) = \phi(\mathbf{r}_n, V_m, t) \qquad C_{i,n}(t) = C_i(\mathbf{r}_n, t)$$

$$\phi_{nm}(t) = A_{NM}(t)\varphi_{nm}(t) \qquad \mathbf{r}_n, V_m \in \Gamma_{NM}$$

definition of a regionwise inner product

$$\langle w|\, g\rangle = \left[\sum_n \sum_m\right]_{NM} w_{nm} g_{nm}$$

Introduce factorization into the balance equations:

$$\begin{cases} \frac{1}{v_m}\varphi_{nm}\frac{dA_{NM}}{dt}+\frac{1}{v_m}A_{NM}\frac{d\varphi_{nm}}{dt} = \\ \sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}A_{N'M'}+ \\ \qquad \sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n}+S_{nm} \\ \\ \frac{dC_{i,n}}{dt}=\beta_i\sum_{M'}\left[\sum_{m'}\right]_{M'} f_{nm'}\varphi_{nm'}A_{NM'}-\lambda_i C_{i,n} \\ \qquad i=1,2,...,6, \quad \mathbf{r}_n, V_m \in \Gamma_{NM} \end{cases}$$

Projection: multiply by w_{nm} and sum on NM

Normalization condition

$$\frac{d}{dt}\left[\sum_{n}\sum_{m}\right]_{NM} w_{nm}\frac{1}{v_m}\varphi_{nm}(t)=\frac{d}{dt}\gamma_{NM}=0$$

Point-to-point transfer term

$$\left[\sum_n \sum_m\right]_{NM} w_{nm} \sum_{N'} \sum_{M'} \left[\sum_{n'} \sum_{m'}\right]_{N'M'} \cdots$$

$$k_{nm,n'm'} \varphi_{n'm'} A_{N'M'} =$$

$$\sum_{N'} \sum_{M'} \left[\sum_n \sum_m\right]_{NM} \cdots$$

$$\left(w_{nm} \left[\sum_{n'} \sum_{m'}\right]_{N'M'} k_{nm,n'm'} \varphi_{n'm'} \right) A_{N'M'} =$$

$$\sum_{N'} \sum_{M'} K^*_{NM,N'M'} A_{N'M'}$$

Multipoint equations:

$$\begin{cases} \dfrac{dA_{NM}}{dt} = \displaystyle\sum_{N'} \sum_{M'} K_{NM,N'M'} A_{N'M'} + \\ \qquad \displaystyle\sum_{i=1}^{6} \lambda_i C_{i,NM} + S_{NM} \\ \dfrac{dC_{i,NM}}{dt} = \beta_i \displaystyle\sum_{M'} F_{i,NM,M'} A_{NM'} - \lambda_i C_{i,NM} \\ \quad i = 1, 2, ..., 6, \end{cases}$$

point-to-point coupling coefficients

$$K_{NM,N'M'} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m\right]_{NM} \left(w_{nm} \left[\sum_{n'} \sum_{m'}\right]_{N'M'} k_{nm,n'm'} \varphi_{n'm'} \right)$$

effective multipoint source

$$S_{NM} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m\right]_{NM} w_{nm} S_{nm}$$

effective multipoint delayed precursor concentrations

$$C_{i,NM} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m\right]_{NM} w_{nm} \chi_{i,m} C_{i,n}$$

delayed neutron production coefficients

$$F_{i,NM,M'} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m\right]_{NM} w_{nm} \chi_{i,m} \beta_i \left[\sum_{m'}\right]_{M'} f_{nm'} \varphi_{nm'}$$

Multipoint can be included into a quasi-static scheme

Solution of the ”slow” shape equation

$$T = t_0 + \Delta t_\varphi$$

$$\frac{1}{v_m}\dot{A}_{NM}(T)\varphi_{nm}(T) + \frac{1}{v_m}A_{NM}(T)\frac{\varphi_{nm}(T) - \varphi_{nm}(t_0)}{\Delta t_\varphi} =$$

$$\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}(T)\varphi_{n'm'}(T)A_{N'M'}(T)$$

$$+\sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n}(T) + S_{nm}(T)$$

$$C_{i,n}(T) = C_{i,n}(t_0)e^{-\lambda_i\Delta t_\varphi} +$$

$$\int_{t_0}^{T}\beta_i\sum_{M'}\left[\sum_{m'}\right]_{M'} f_{nm'}(T)\varphi_{nm'}(t_0)A_{N'M'}(t')\times$$

$$e^{-\lambda_i(T-t')}dt'$$

Iteration scheme

$$\frac{1}{v_m}\dot{A}_{NM}^{(l)}(T)\varphi_{nm}^{(l)}(T) + \frac{1}{v_m}A_{NM}(T)\frac{\varphi_{nm}^{(l)}(T) - \varphi_{nm}(t_0)}{\Delta t_\varphi} =$$

$$\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}(T)\varphi_{n'm'}^{(l)}(T)A_{N'M'}(T)+$$

$$\sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n}(T) + S_{nm}(T)$$

$$\gamma_{NM}^{(l)}(T) = \left[\sum_{n}\sum_{m}\right]_{NM} w_{nm}\frac{1}{v_m}\varphi_{nm}^{(l)}(T)$$

$$\varphi_{nm}^{(l+1/2)}(T) = \frac{\varphi_{nm}^{(l)}(T)}{\gamma_{NM}^{(l)}(T)}\gamma_{NM}(t_0)$$

$$\dot{A}_{NM}^{(l+1)}(T) = \gamma_{NM}^{(l+1/2)}(T) \sum_{N'} \sum_{M'} \left[\sum_{n} \sum_{m} \right]_{NM} \dots$$

$$\left(w_{nm} \left[\sum_{n'} \sum_{m'} \right]_{N'M'} k_{nm,n'm'}(T) \varphi_{n'm'}^{(l+1/2)}(T) \right) A_{N'M'}+$$

$$+ \frac{\left[\sum_{n} \sum_{m} \right]_{NM} w_{nm} \chi_{i,m} C_{i,n}(T)}{\gamma_{NM}^{(l+1/2)}(T)} +$$

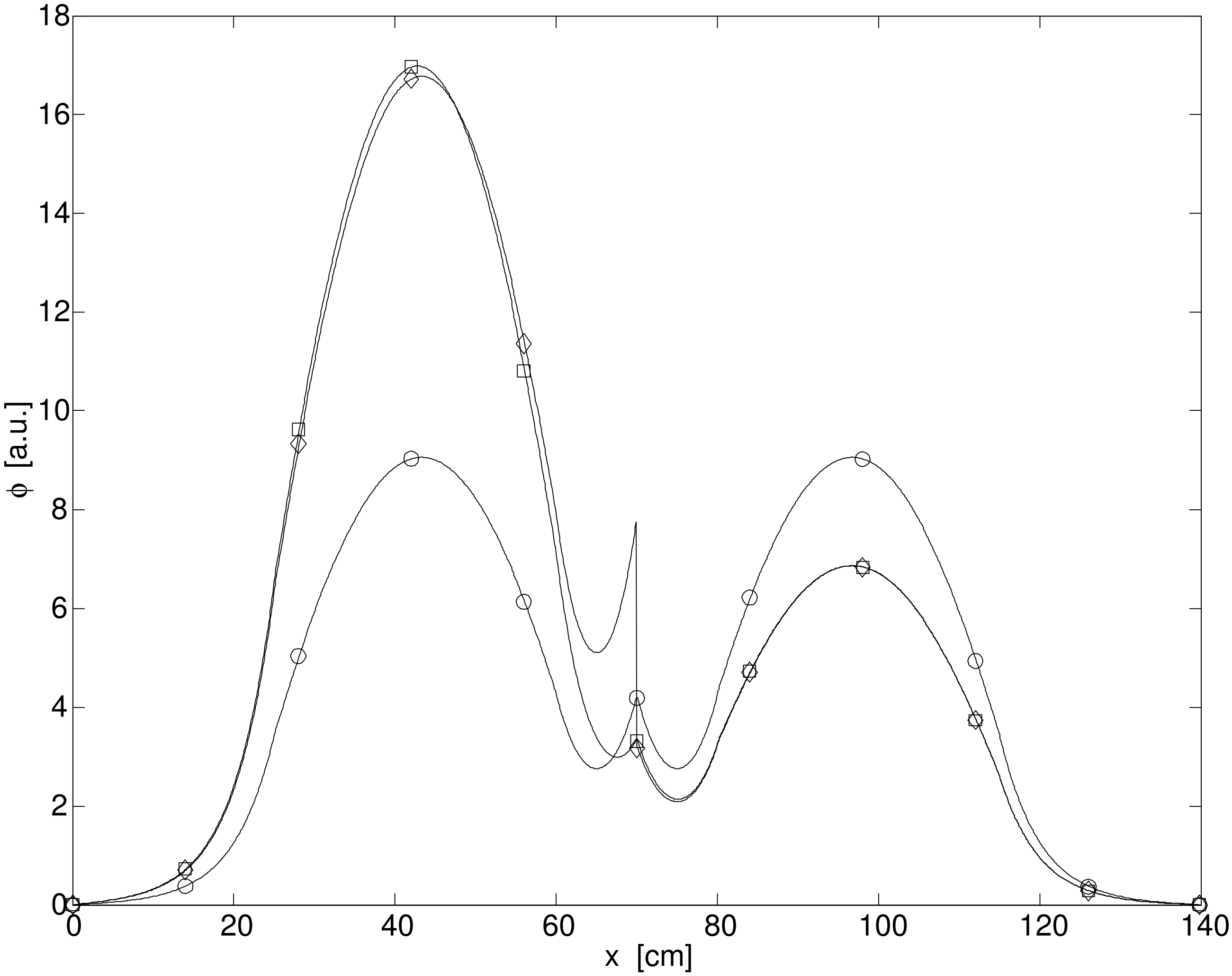
$$\frac{\left[\sum_{n} \sum_{m} \right]_{NM} w_{nm} S_{nm}(T)}{\gamma_{NM}^{(l+1/2)}(T)}$$

Concluding remarks

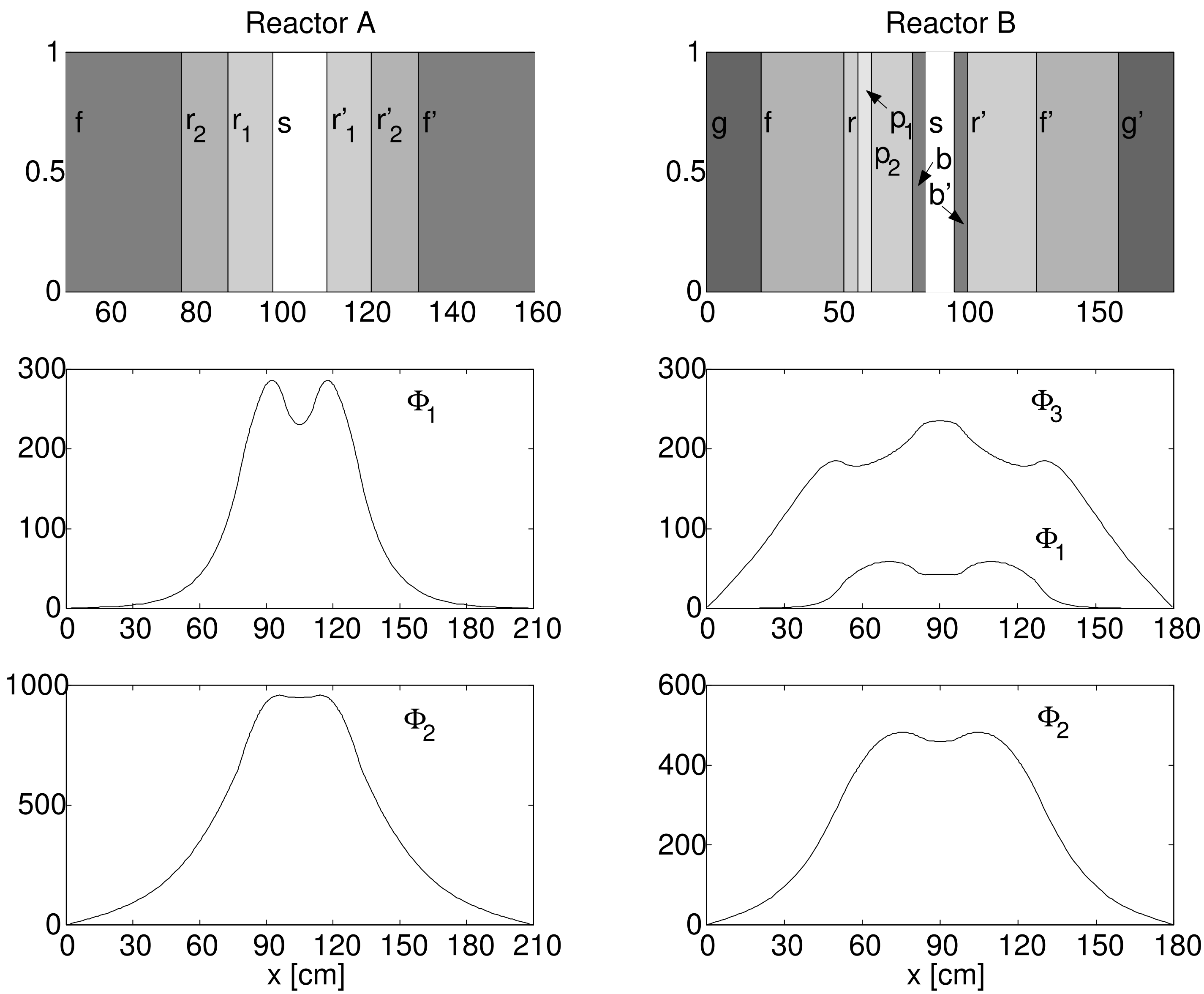
- multipoint is effective in many reactor kinetics problems

- the method can easily be included within quasi-statics, greatly enhancing its performance

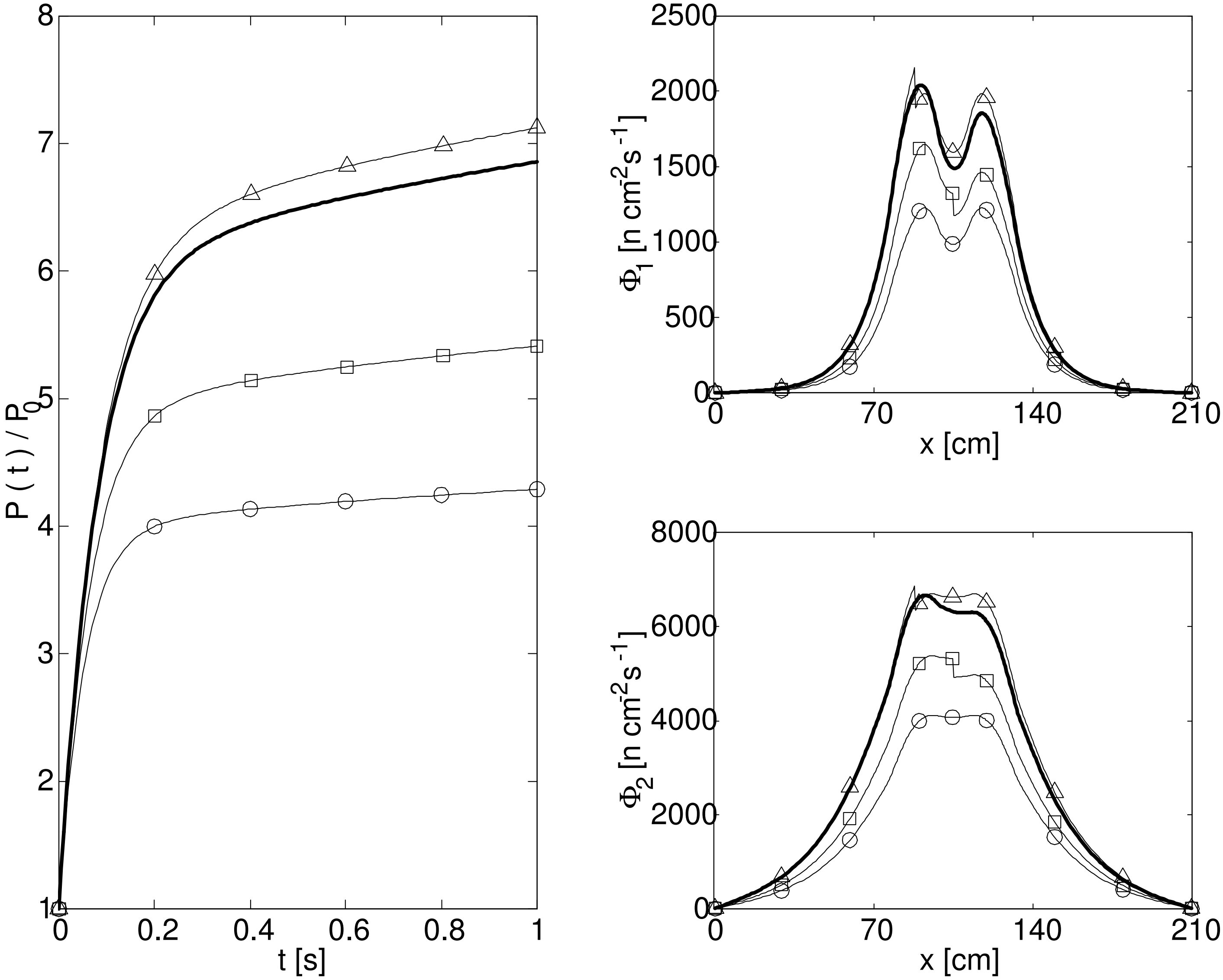
- development: apply multipoint to angular schemes in transport calculations



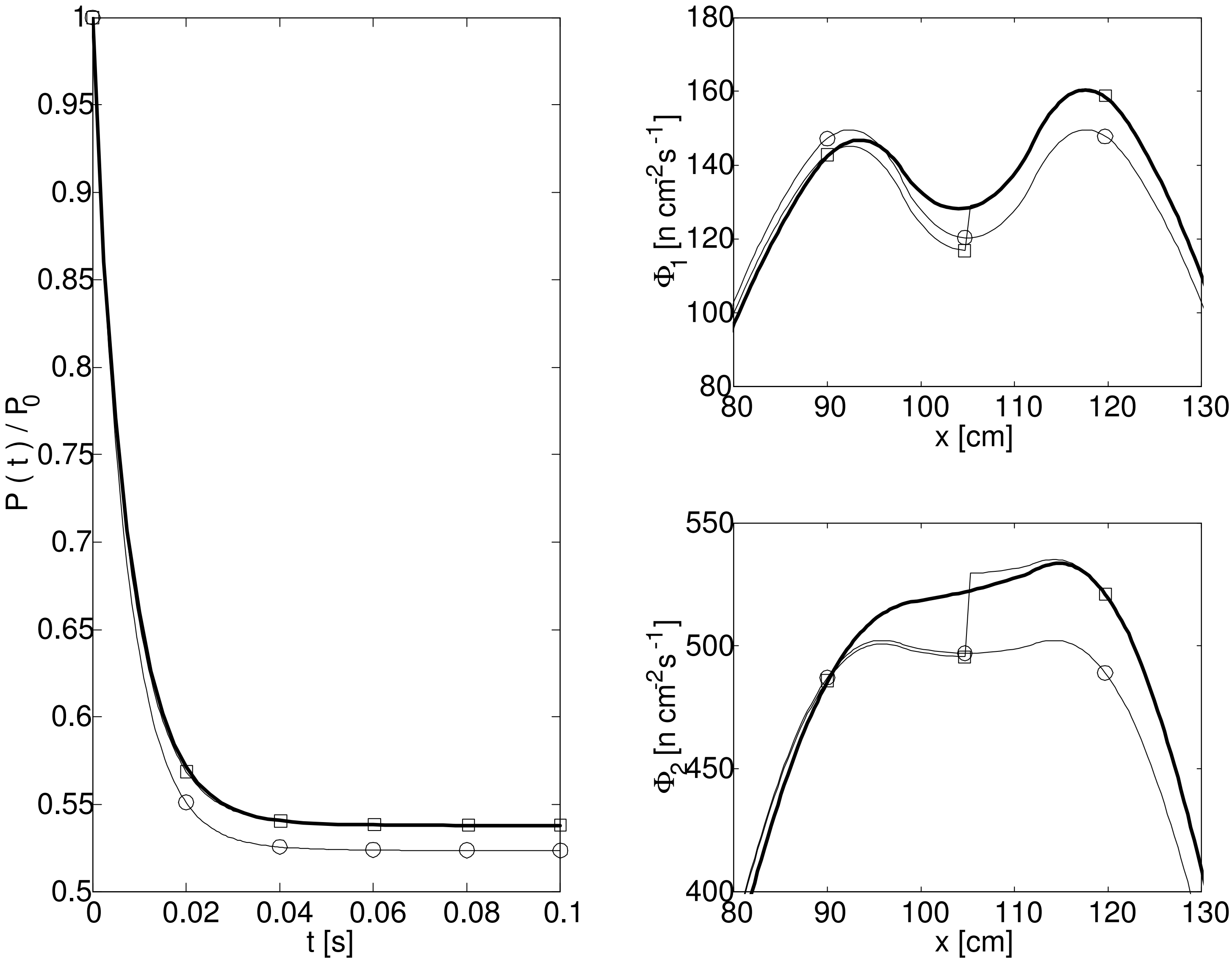
1.Flux at end of transient: ○ : point kinetics; ◇ : 2-point kinetics; □ : exact solution.



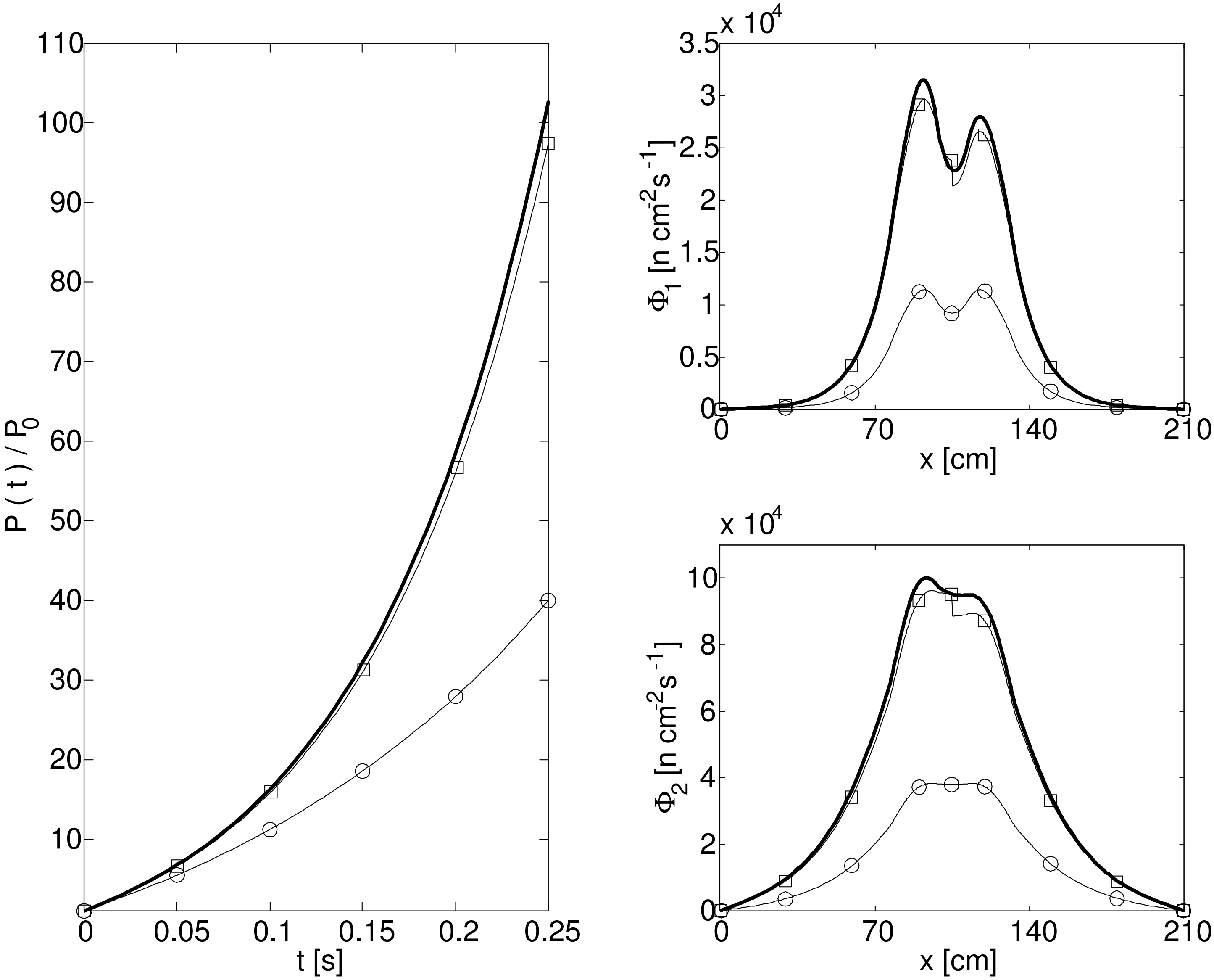
2.Material configurations. s: source channel; r, r': multiplying zones, Masurca type; r_1, r_1': multiplying zones, Myrrha type (low enrichment); r_2, r_2': multiplying zone, Myrrha type (high enrichment); f, f': reflectors; b,b': lead buffers; p_1, p_2: perturbed zones; g, g': shields. Initially, r, p_1 and p_2 have the same properties as r'. Flux in steady state, 2-group diffusion for A and



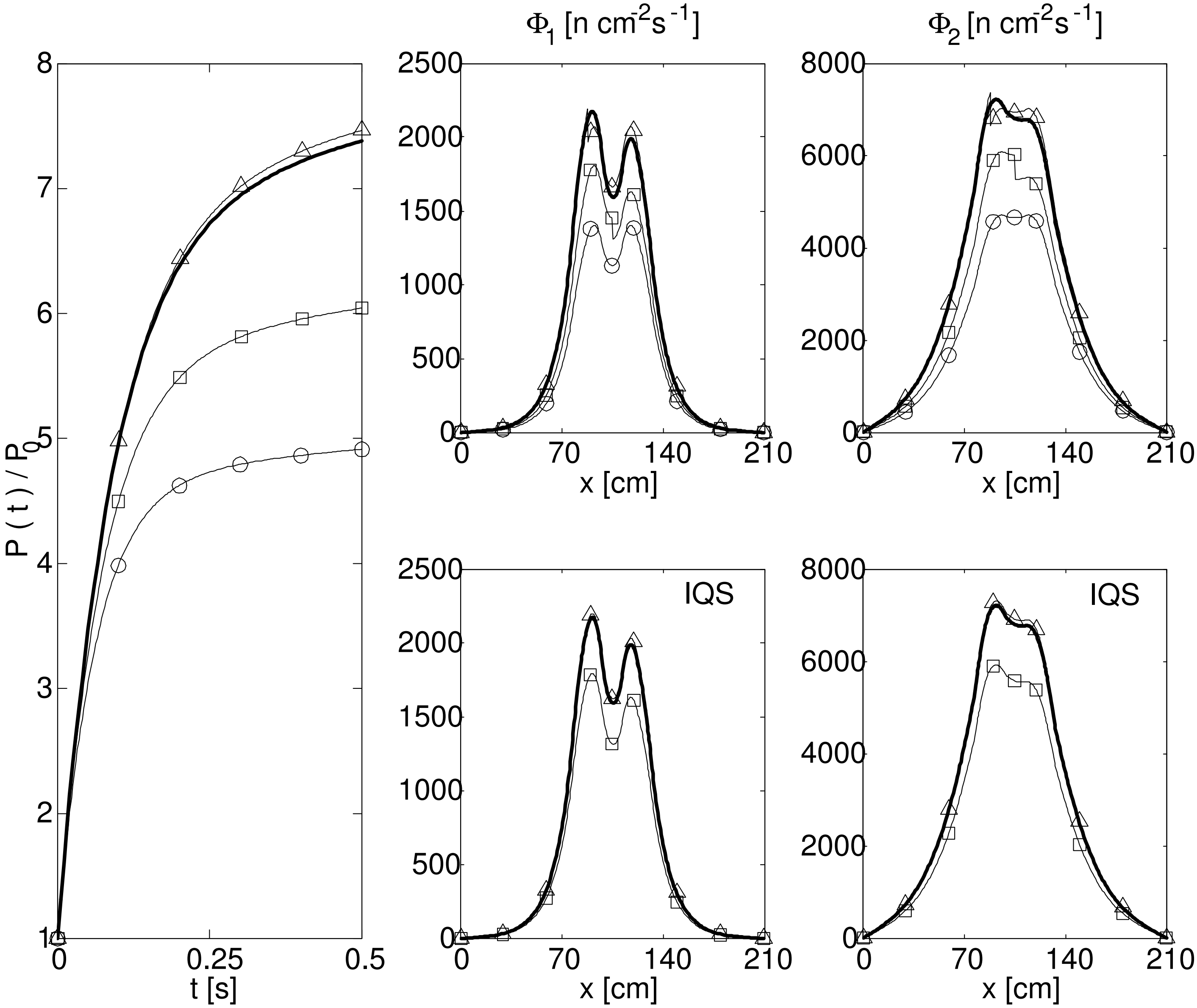
3.Transient caused by a $\delta\Sigma_{a1} = -0.1\delta\Sigma_1$ in the r_2 zone of system A. The multiplication constant becomes $k_{eff} = 0.998447.$ On the right, the fluxes at the last time considered in the transient ($t = 1$ s) are plotted.



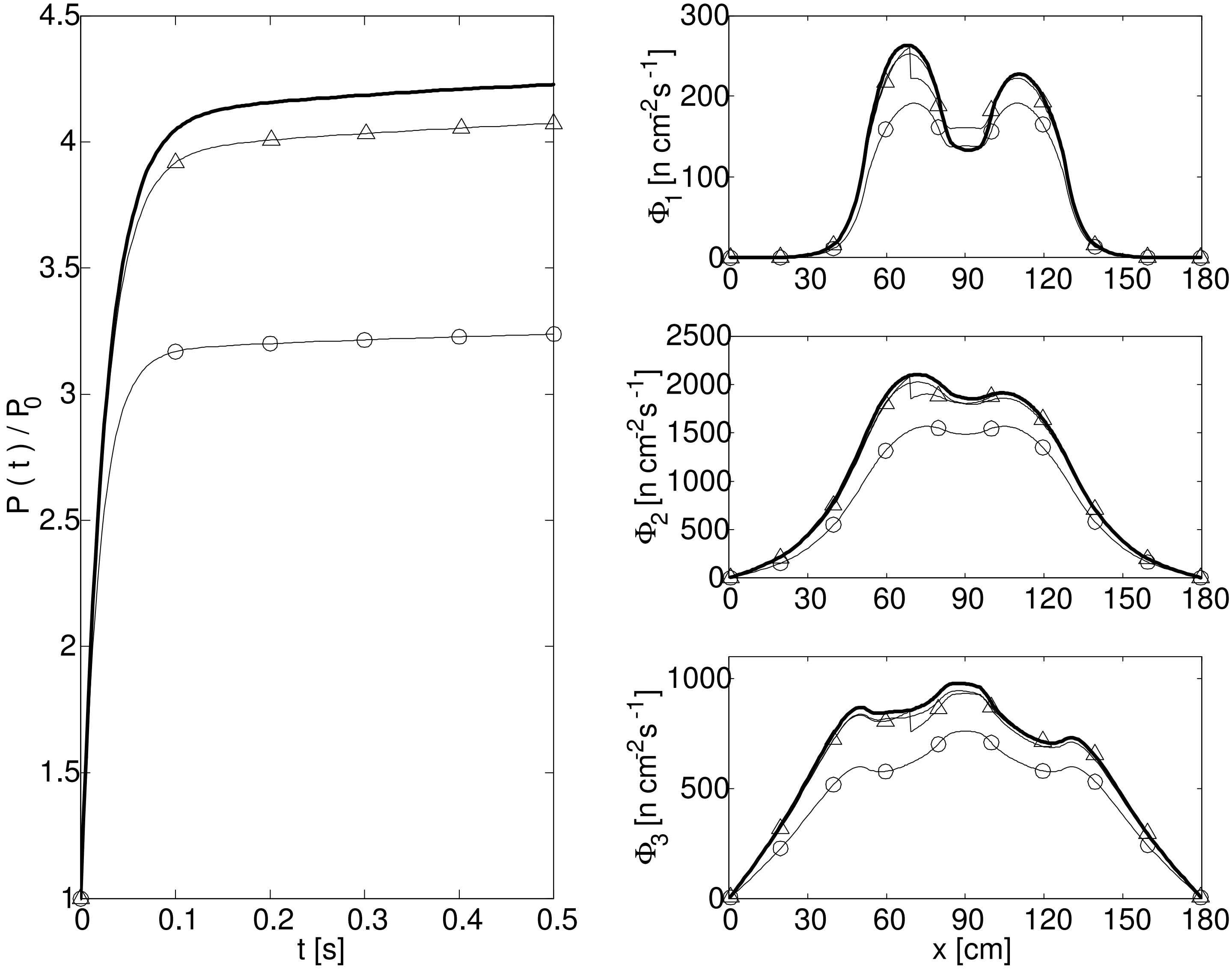
4.Transient caused by a $\delta\Sigma_{a1} = +0.05\delta\Sigma_1$ in the r_1 and r_2 zones of system A. The multiplication constant becomes $k_{eff} = 0.961767$. Final fluxes on the right.



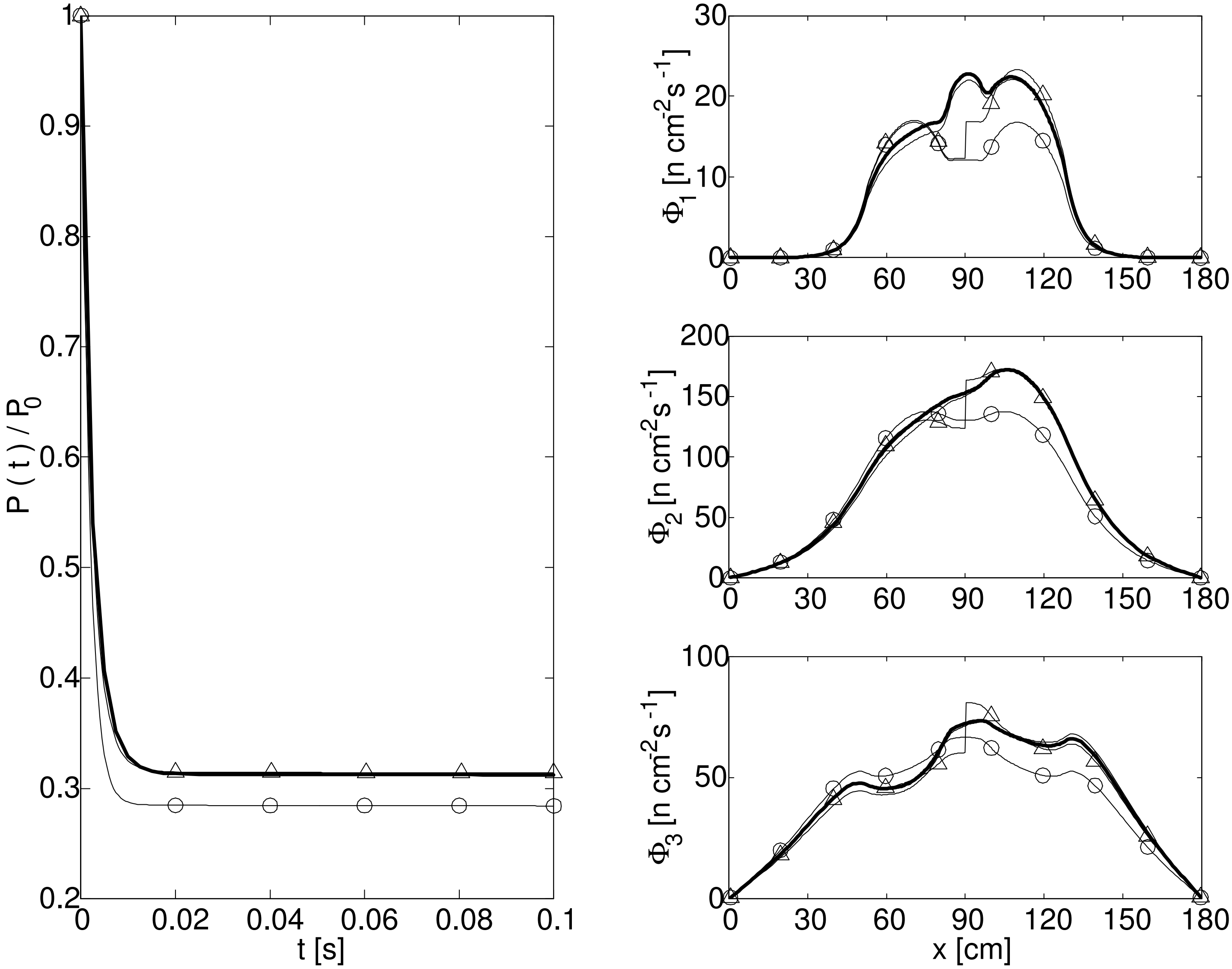
5.Transient caused by a $\delta\Sigma_{a1} = +0.1\delta\Sigma_1$ in the r_1 zone of system A. The multiplication constant becomes $k_{eff} = 1.00489$.



6.Transient caused by a $\delta\Sigma_{a1} = -0.05\delta\Sigma_1$ and $\delta\Sigma_{a2} = -0.1\delta\Sigma_2$ in the r_2 zone of system A. The multiplication constant becomes $k_{eff} = 0.998850$. In the right on the top, comparison of reference, point and two-point kinetics can be seen. At the bottom, IQS results are reported.



7.Transient caused by a $\delta\Sigma_{ag} = -0.2\delta\Sigma_g$ for all groups in the p_1 zone of system B. The multiplication constant becomes $k_{eff} = 0.994264$.



8.Transient caused by a $\delta\Sigma_{ag} = +0.2\delta\Sigma_g$ for all groups in the p_2 zone of system B. The final multiplication constant is $k_{eff} = 0.912024$.

Fluid-fuel systems

Summary of lecture

► General model

► Discussion of dynamic effects of fuel motion

► Derivation of consistent point kinetics

► Quasi-static scheme

The neutron kinetics of circulating-fuel reactors

Balance equations for neutrons:

$$\begin{cases} \dfrac{\partial n(\mathbf{r}, E, \Omega, t)}{\partial t} = \left[\hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)\right] n(\mathbf{r}, E, \Omega, t) + \\ \qquad \displaystyle\sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r}, E, t) + S(\mathbf{r}, E, \Omega, t) \\ \\ \dfrac{1}{\lambda_i}\dfrac{\partial \mathcal{E}_i(\mathbf{r}, E, t)}{\partial t} + \dfrac{1}{\lambda_i}\boldsymbol{\nabla} \cdot (\mathbf{u}\mathcal{E}_i(\mathbf{r}, E, t)) = \\ \qquad \hat{\mathrm{M}}_i(t) n(\mathbf{r}, E, \Omega, t) - \mathcal{E}_i(\mathbf{r}, E, t), \\ \\ \qquad\qquad i = 1, 2, ..., R, \end{cases}$$

$$\mathcal{E}_i(\mathbf{r}, E, t) = \lambda_i C_i(\mathbf{r}, t) \frac{\chi_i(E)}{4\pi}.$$

In the equations for precursors a convective term appears due to fuel motion, and appropriate boundary conditions must then be introduced:

$$\mathcal{E}_i(\mathbf{r}, E, t)\mathbf{u}(\mathbf{r}) \cdot (-\mathbf{n}) =$$

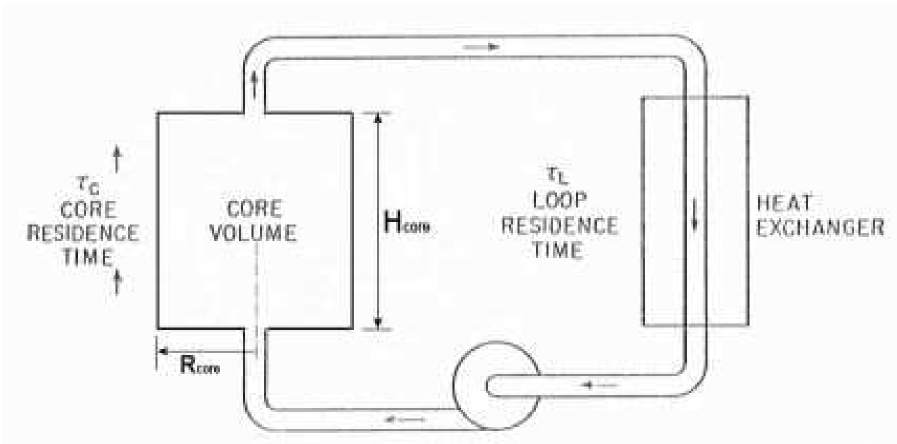
$$\int\limits_{\mathcal{A}_{out}} \mathcal{E}_i(\mathbf{r}', E, t - \tau(\mathbf{r}' \rightarrow \mathbf{r}))e^{-\lambda_i \tau(\mathbf{r}' \rightarrow \mathbf{r})} \times$$

$$\mathbf{u}\left(\mathbf{r}'\right) \cdot \mathbf{n}' \mathfrak{F}\left(\mathbf{r}' \rightarrow \mathbf{r}\right) d\mathcal{A}',$$

$$\mathbf{r} \in \mathcal{A}_{in}.$$

Fission products are moved through and outside the core by the motion of the fissile material.

Geometrical structure of a circulating-fuel reactor



Multigroup diffusion model in cilindrical gometry + slug flow

In slug-flow conditions the velocity field is maintained by externally-driven devices and is one-dimensional (axial):

$$\nabla \cdot (C_i \mathbf{u}) = \frac{\partial}{\partial z}(u C_i)$$

$$\tau_c = \frac{H_{core}}{\bar{u}_{core}} \qquad \tau_L = \frac{L_{loop}}{\bar{u}_{loop}}$$

$$\frac{\bar{u}_{core}}{\bar{u}_{loop}} = \frac{\mathcal{A}_{loop}}{\mathcal{A}_{core}} \qquad \mathcal{A}_{core} = \pi R_{core}^2$$

$$\frac{1}{v_g}\frac{\partial \Phi_g}{\partial t} = \bigtriangledown \cdot D_g \bigtriangledown \Phi_g - \Sigma_g \Phi_g + \sum_{g'} \left[\chi_{p,g} \nu \Sigma_{fg'} (1 - \beta) + \Sigma_{g' \to g}\right] \Phi_{g'} + S_g + \sum_i \lambda_i \chi_{i,g} C_i$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \sum_g \nu \Sigma_{fg} \Phi_g - \frac{\partial}{\partial z}(u C_i)$$

$\star$ boundary conditions:

$$u(0) C_i(z = 0, r, t) = u(H) \frac{e^{-\lambda_i \tau_L}}{\mathcal{A}_{core}} \int_{\mathcal{A}_{core}} C_i(z = H, r, t - \tau_L) d\mathcal{A}$$

Discussion of dynamic effects of motion

i) the delayed-precursor equations cannot be eliminated in steady-state configuration.

solid fuel: the concentrations of precursors can be expressed as functions of the fission term and substituted into the neutron balance equation

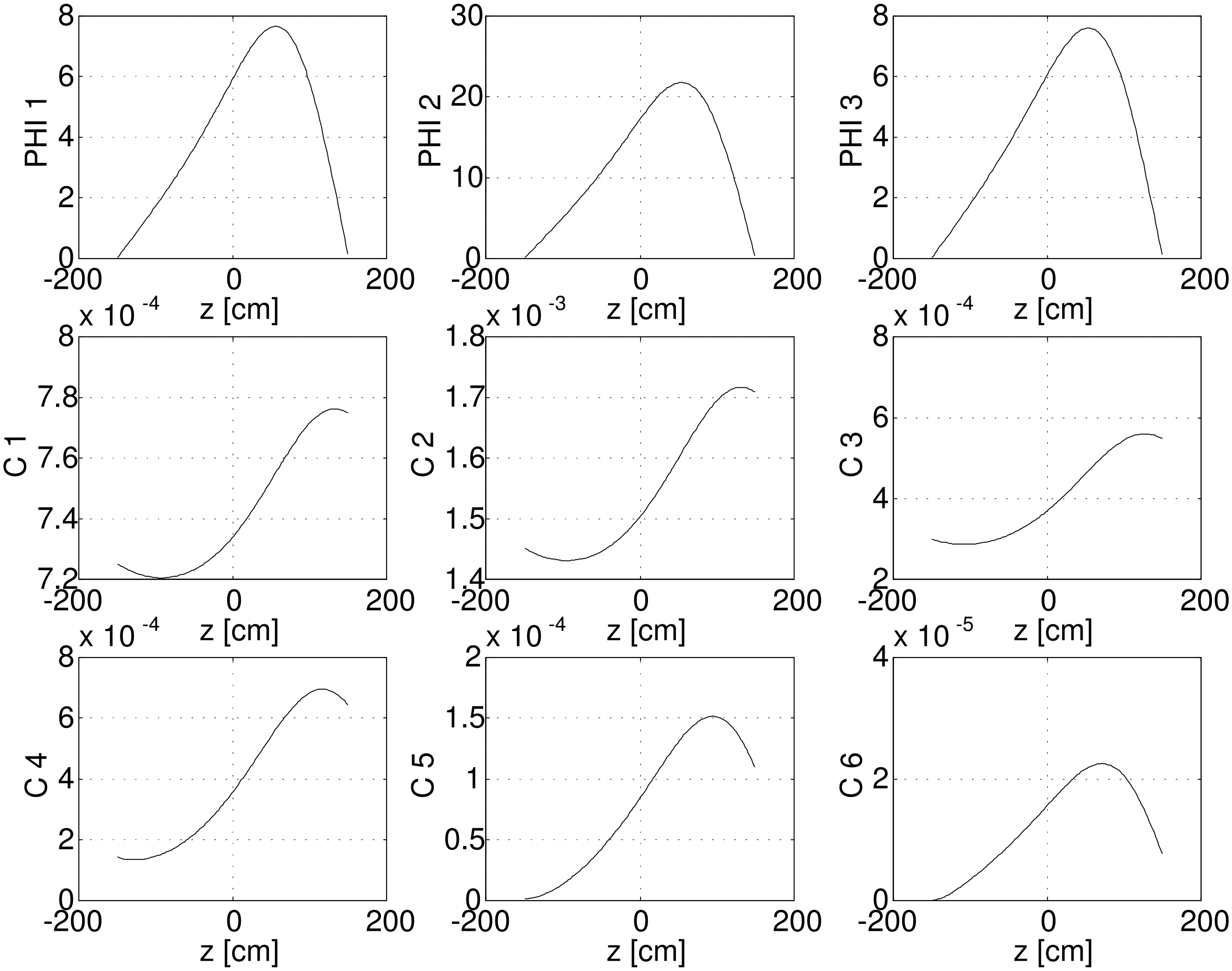
$$\mathcal{E}_i(\mathbf{r}, E) = \hat{\mathrm{M}}_i n(\mathbf{r}, E, \Omega)$$

circulating fuel: the equations for precursors are still differential for the space variable and their concentrations can not be made explicit and substituted

$$\boldsymbol{\nabla} \cdot (\mathbf{u}\mathcal{E}_i(\mathbf{r}, E)) = \lambda_i \hat{\mathrm{M}}_i n(\mathbf{r}, E, \Omega) - \lambda_i \mathcal{E}_i(\mathbf{r}, E)$$

ii) the multiplication eigenvalue depends on delayed neutron and flow characteristics;

iii) the space distribution of delayed-precursors is not following the neutron distribution and is completely different from standard solid-fuel systems


PHI 1
PHI 2
PHI 3
C 1
C 2
C 3
C 4
C 5
C 6
z [cm]
x 10 -4
x 10 -3
x 10 -5
-200
0
200
0
2
4
6
8
10
20
30
7.2
7.4
7.6
7.8
1.4
1.5
1.6
1.7
1.8
0.5
1
1.5

iv) the role of delayed emissions is reduced by space-redistribution and external recirculation (reduction of effective β)

$$\beta_{i,eff} = \frac{\lambda_i \left\langle \Phi^+ \middle| C_i \right\rangle}{\sum_j \lambda_j \left\langle \Phi^+ \middle| C_j \right\rangle + (1-\beta) \left\langle \Phi^+ \middle| \nu \Sigma_f \Phi \right\rangle}$$

Ratio $\tilde{\beta}/\beta$ as a function of τ_L and k_{eff}

τ_L [s] $\longrightarrow$	0	5	10	15	u [cm/s] $\downarrow$
$k_{eff} = 0.95009$	0.842	0.542	0.470	0.443	60
	0.835	0.422	0.363	0.332	100
$k_{eff} = 0.97048$	0.843	0.540	0.469	0.441	60
	0.834	0.420	0.353	0.330	100
$k_{eff} = 0.99028$	0.842	0.539	0.467	0.440	60
	0.833	0.419	0.352	0.329	100
$k_{eff} = 1.00001$	0.843	0.539	0.468	0.440	60
	0.834	0.420	0.352	0.329	100

.

v) factorization schemes should be applied to both neutrons and precursors

vi) point kinetics model needs a specific formulation

vii) the numerical solution can take advantage of the slower time-scale of the motion of the delayed precursors

Point kinetic model for circulating fuel systems (consistent with Henry factorization procedure)

$\Rightarrow$ the time-dependent balance equations for neutrons and delayed precursors are considered:

$$\begin{cases} \dfrac{\partial n(\mathbf{r},E,\Omega,t)}{\partial t} = \left[\hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)\right] n(\mathbf{r},E,\Omega,t) + \\ \qquad \displaystyle\sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r},E,t) + S(\mathbf{r},E,\Omega,t) \\ \\ \dfrac{1}{\lambda_i}\dfrac{\partial \mathcal{E}_i(\mathbf{r},E,t)}{\partial t} + \dfrac{1}{\lambda_i}\boldsymbol{\nabla}\cdot\left(\mathbf{u}\mathcal{E}_i(\mathbf{r},E,t)\right) = \\ \qquad \hat{\mathrm{M}}_i(t) n(\mathbf{r},E,\Omega,t) - \mathcal{E}_i(\mathbf{r},E,t), \\ \qquad\qquad i = 1,2,...,R, \end{cases}$$

+initial and boundary condition

$\Rightarrow$ a reference configuration is introduced

$$
\begin{cases}
0 = \left[\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_{p,0}\right] N_0(\mathbf{r}, E, \Omega) + \\
\qquad \sum_{i=1}^{R} \mathcal{E}_{i,0}(\mathbf{r}, E) + S_0(\mathbf{r}, E, \Omega) \\
\\
\frac{1}{\lambda_i} \boldsymbol{\nabla} \cdot \left(\mathbf{u}_0 \mathcal{E}_{i,0}(\mathbf{r}, E)\right) = \\
\qquad \hat{\mathrm{M}}_{i,0} N_0(\mathbf{r}, E, \Omega) - \mathcal{E}_{i,0}(\mathbf{r}, E), \\
\qquad \qquad i = 1, 2, ..., R,
\end{cases}
$$

+boundary conditions

$\Rightarrow$ a physically consistent definition of the neutron concentrations and delayed emissions importance $n^\dagger$ and $\mathcal{E}^\dagger$ is introduced;

$\Rightarrow$ the balance equations for the importance functions are shown to be the mathematical adjoint to the balance equations for neutrons and precursors, having defined the inner product as:

$$\begin{aligned}(\mathbf{w}^\dagger, \mathbf{w}) = & \sum_{n=1}^{R+1} \langle w_n^\dagger \mid w_n \rangle = \\ & \sum_{n=1}^{R+1} \int_V dV \int_E dE \int_{4\pi} d\Omega \cdot w_n^\dagger w_n\end{aligned}$$

where

$$\begin{aligned}\mathbf{w} &= (n, \mathcal{E}_1, ..., \mathcal{E}_R)^t \\ \mathbf{w}^\dagger &= (n^\dagger, \mathcal{E}_1^\dagger, ..., \mathcal{E}_R^\dagger)\end{aligned}$$

⇒ The system of equations for the importance takes the form:

$$\begin{cases} \left[\hat{\mathrm{L}}_0^+ + \hat{\mathrm{M}}_{p,0}^+\right] N_0^+(\mathbf{r},E,\Omega) + \sum_{i=1}^{R} \hat{\mathrm{M}}_{i,0}^+ \mathcal{E}_{i,0}^+(\mathbf{r},E) + \\ \qquad S_0^+(\mathbf{r},E,\Omega) = 0, \\ \\ N_0^+(\mathbf{r},E,\Omega) + \frac{1}{\lambda_i}\mathbf{u}_0 \cdot \boldsymbol{\nabla}\left(\mathcal{E}_{i,0}^+(\mathbf{r},E)\right) - \\ \qquad \mathcal{E}_{i,0}^+(\mathbf{r},E) = 0, \qquad i = 1,2,...,R, \end{cases}$$

with boundary conditions:

$$\mathcal{E}_i^+(\mathbf{r},E) = \int\limits_{\mathcal{A}_{in}} \mathcal{E}_i^+(\mathbf{r}',E) e^{-\lambda_i \tau(\mathbf{r}\to\mathbf{r}')} \mathfrak{F}\left(\mathbf{r}\to\mathbf{r}'\right) d\mathcal{A}',$$

$$\mathbf{r} \in \mathcal{A}_{out}.$$

$\Rightarrow$ Both flux and delayed emission distributions are factorized with an amplitude-shape formula:

$$n(\mathbf{r}, E, \Omega, t) = A(t)\varphi(\mathbf{r}, E, \Omega; t),$$
$$\mathcal{E}_i(\mathbf{r}, E, t) = G_i(t)e_i(\mathbf{r}, E; t) \qquad i = 1, 2, ..., R.$$

$*$ The factorized formula is introduced into balance equations;

$*$ A projection on the adjoint solution is taken;

$*$ A normalization conditions is imposed on the shape functions to make the factorization unique:

$$\frac{d}{dt}\left\langle N_0^+ \right| \varphi\rangle = 0,$$

$$\frac{d}{dt}\left\langle \mathcal{E}_{i,0}^+ \right| e_i\rangle = 0, \qquad i = 1, 2, ..., R,$$

$\Longrightarrow$ a point-like model is obtained

$$
\begin{cases}
\Lambda_P \dfrac{dA}{dt} = \left(\rho_S - \tilde{\beta}\right) A + \displaystyle\sum_{i=1}^{R} \lambda_i \Gamma_i + \tilde{S} \\
\\
\Lambda_i \dfrac{d\Gamma_i}{dt} = \left(\tilde{\beta}_i + \rho_i\right) A - (\lambda_i + \mu_{u,i} + \mu_{\xi,i})\Gamma_i + \sigma_i \\
\\
i = 1, ..., R
\end{cases}
$$

→ structure equivalent to the point kinetic model for solid-fuel systems;

→ kinetic parameters and effective delayed neutron functions Γ_i have different definitions;

→ unconventional terms:

ρ_i : perturbation of production;

$\mu_{u,i}$: perturbation of fluid velocity;

$\mu_{\xi,i}$: perturbation of recirculation time;

- normalization factor

$$\mathcal{F} = \sum_{i=1}^{R} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \chi_{i,n} \lambda_i C_{i,0} \rangle +$$

$$+ (1-\beta) \sum_{n=1}^{G} \sum_{g=1}^{G} \left\langle \Phi_n^\dagger \right| \chi_n \left(\nu \Sigma_f \right)_{g,0} \Phi_{g,0} \rangle$$

- effective delayed neutron precursors

$$\Gamma_i = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \chi_{i,n} C_{i,\,0} \rangle \, G_i(t)$$

- effective neutron source

$$\tilde{S} = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| S_n \rangle$$

- effective delayed neutron fractions

$$\tilde{\beta}_i = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \chi_{i,n} \lambda_i C_{i,0} \rangle \,, \qquad \tilde{\beta} = \sum_{i=1}^{R} \tilde{\beta}_i .$$

- reactivity $\rho_S = \rho_0 + \rho_{pert}$, where

$$\rho_0 = -\frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle S_n^\dagger \right| \left. \Phi_{n,0} \right\rangle$$

$$\rho_{pert} = \frac{1}{\mathcal{F}} \left\{ -\sum_{n=1}^{G} \left\langle \boldsymbol{\nabla} \Phi_n^\dagger \cdot \right| \left. \delta D_n \boldsymbol{\nabla} \Phi_{n,0} \right\rangle + \right.$$

$$+ \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \left. \sum_{g=1}^{G} (1-\beta)\, \chi_n \delta \left(\nu \Sigma_f \right)_g \Phi_{g,0} \right\rangle +$$

$$- \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \left. \delta \Sigma_{R,n} \Phi_{n,0} \right\rangle +$$

$$\left. + \sum_{n=1}^{G} \sum_{g=1}^{G} \left\langle \Phi_n^\dagger \right| \left. \delta \Sigma_{g \to n} \Phi_{g,0} \right\rangle \right\}$$

- prompt neutron lifetime

$$\Lambda_P = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \left. \frac{1}{v_n} \Phi_{n,0} \right\rangle$$

- generalized precursor lifetime

$$\Lambda_i = \frac{\left\langle C_i^\dagger \middle| C_{i,0} \right\rangle}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} C_{i,0} \right\rangle}; \qquad i = 1, 2, ..., R$$

- unconventional terms

$$\rho_i = \frac{1}{\mathcal{F}} \left\langle C_i^\dagger \middle| \beta_i \sum_{g=1}^{G} \delta(\nu\Sigma_f)_g \Phi_{g,0} \right\rangle$$

$$\mu_{u,i} = \frac{\left\langle C_i^\dagger \middle| \frac{\partial}{\partial z} \left(\delta u C_{i,0}\right) \right\rangle}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} C_{i,0} \right\rangle}$$

$$\mu_{\xi,i} = \frac{u_0(H) \int_{\mathcal{A}_{core}} C_i^\dagger(H) C_{i,0}(r, H) d\mathcal{A}}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} C_{i,\,0} \right\rangle}$$

- apparent precursor source

$$\Xi = \frac{1}{\mathcal{A}_{core}} u_0(H) \left(\xi_{i,0} + \delta\xi_i\right) \times \int_{\mathcal{A}_{core}} C_i^\dagger(r,0) d\mathcal{A} \int_{\mathcal{A}_{core}} C_{i,\,0}(r,H)$$

$$\sigma_i = \begin{cases} \dfrac{\Xi}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \chi_{i,n} C_{i,0} \rangle} \Gamma_{i,0}, & \textbf{if } t \le T_R, \\ \dfrac{\Xi}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \chi_{i,n} C_{i,0} \rangle} \Gamma_i(t - T_R) & \textbf{if } t > T_R\,. \end{cases}$$

Quasi-statics

- Point kinetic calculation on the time interval ΔT_ϕ;

- At time T_ϕ, the factorized forms of the neutron density and delayed emissions are introduced in the spatial equations:

$$
\begin{cases}
A(t)\dfrac{\partial \varphi}{\partial t} + \varphi \dot{A} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right] \varphi A + \displaystyle\sum_{i=1}^{R} G_i e_i + S \\[2ex]
\dfrac{1}{\lambda_i}\dfrac{\partial e_i}{\partial t} G_i + \dfrac{1}{\lambda_i} e_i \dot{G}_i + \dfrac{1}{\lambda_i} \boldsymbol{\nabla} \cdot (\mathbf{u} e_i)\, G_i = \\
\qquad \hat{\mathrm{M}}_i \varphi A - e_i G_i, \\[2ex]
\qquad\qquad i = 1, 2, ..., R,
\end{cases}
$$

After time discretization the equations take the form:

$$\begin{cases} \dfrac{\varphi^{n+1}-\varphi^{n}}{\Delta T_\phi} + \varphi^{n+1} \left.\dfrac{\dot{A}}{A}\right|_{T_\phi} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right] \varphi^{n+1} + \\ \qquad \displaystyle\sum_{i=1}^{R} e_i^{n+1} \left.\frac{G_i}{A}\right|_{T_\phi} + \frac{S}{A|_{T_\phi}} \\ \\ \dfrac{1}{\lambda_i}\dfrac{e_i^{n+1}-e_i^{n}}{\Delta T_\phi} + \dfrac{1}{\lambda_i} e_i^{n+1} \left.\dfrac{\dot{G}_i}{G_i}\right|_{T_\phi} + \dfrac{1}{\lambda_i}\boldsymbol{\nabla}\cdot\left(\mathbf{u} e_i^{n+1}\right) = \\ \qquad -\lambda_i C_i^{n+1} + \beta_i \displaystyle\sum_g \nu\Sigma_{fg}\phi_g^{n+1} \left.\frac{P}{G_i}\right|_{T_\phi} \\ \\ \qquad i = 1, ..., \ R \end{cases}$$

Iterations on the values of the derivatives of the amplitude, to fulfill the normalization conditions.

Typical Results (multigroup diffusion model, 2 groups)