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**Workshop on
Nuclear Reaction Data and Nuclear Reactors:
Physics, Design and Safety**

16 February - 12 March 2004

RIPL Database +
Statistical Model of Nuclear Reactions

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These are preliminary lecture notes, intended only for distribution to participants

EMPIRE-II statistical model

(version: 2.18 Mondovi)
(version: 2.19beta12 Lodi)

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- Huser-Feshbach
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Statistcal theory of nucl. reactions

Compound Nucleus formation

- multiple scattering of a projectile with target nucleons and among target nucleons =>
- many nucleons excited around the Fermi level =>
- relatively low excitation energy per particle =>
- low probability of particle emission =>
- enough time for mixing of configurations

Statistcal theory of nucl. reactions

Main features

- long life-time ($\sim 10^{-18}$ sec.)
- initial and final channel factorization (lost memory)
- symmetric angular distributions
- time independence

Formal development

"At the beginning there was chaos"

$$S_{ab}^{fl} = S_{ab} - < S_{ab} >$$

$$< \sigma_{ab} > = |\delta_{ab} - < S_{ab} > |^2 + < |S_{ab}^{fl}|^2 > = \sigma_{ab}^{dir} + \sigma_{ab}^{fl}$$

$< S_{ab} >$known from Optical Model

Goal of statistical theory

express: $< |S_{ab}^{fl}|^2 >$in terms of $< S_{ab} >$

Elimination of dir. reactions

$$P = 1 - < S >< S^{+} >$$

if U diagonalizes P it does it also for S

$$\widetilde{S} = USU^{T}$$

$$< |S_{ab}^{fl}|^2 > = \sum_{efgh} U_{ea}^{*} U_{fb}^{*} U_{ga} U_{hb} < \widetilde{S}_{ef}^{fl} \widetilde{S}_{gh}^{fl*} >$$

This elimination is **NOT** performed in EMPIRE

Huaser-Feshbach formula

$$T_a = 1 - | < S_{aa} > |^2$$

unitarity of S-matrix

$$T_a = \sum_{b=1}^{\Lambda} \sigma_{ab}^{fl}$$

Bohr's hypothesis

$$\sigma_{ab}^{fl} = \frac{T_a T_b}{\sum_c^{\Lambda} T_c}$$

Width fluctuation correction

elastic channel is correlated

$$\sigma_{ab}^{fl} = \frac{V_a V_b W_{ab}}{\sum_{c=1}^{\Lambda} V_c}$$

pole expansion of S-matrix

$$S_{ab}(E) = S_{ab}^0 - i \sum_{\mu=1}^{N} \frac{g_{\mu a} g_{\mu b}}{E - \epsilon_\mu + i\Gamma_\mu/2}$$

Width fluctuation (cont.)

transmission coefficient

$$T_c \simeq 2\pi < |g_{\mu c}|^2 >_\mu /D$$

Compound Nucleus cross section

$$\sigma_{ab}^{fl} \simeq \frac{T_a T_b W_{ab}}{\sum_c T_c}$$

Width fluctuation (cont.)

the width fluctuation factor is

$$W_{ab} = \left\langle \frac{|g_{\mu a}|^2 \, |g_{\mu b}|^2}{\Gamma_\mu} \right\rangle_\mu \left(\frac{\left\langle |g_{\mu a}|^2 \right\rangle_\mu \left\langle |g_{\mu b}|^2 \right\rangle_\mu}{\langle \Gamma_\mu \rangle_\mu} \right)^{-1}$$

Width fluctuation (cont.)

if fluctuations of Γ_μ are neglected elastic enhancement reads

$$W_a = \frac{\left\langle |g_{\mu a}|^4 \right\rangle}{\left\langle |g_{\mu a}|^2 \right\rangle \left\langle |g_{\mu a}|^2 \right\rangle}$$

and

$$\begin{aligned} W_a &= 3 \;\; for \;\; T_a \ll 1 \\ W_a &= 2 \;\; for \;\; T_a \simeq 1 \end{aligned}$$

WF - Moldauer approach

$$W_{ab} = (1 + 2\delta_{ab}/\nu_a) \times$$
$$\int_0^\infty dt \prod_{c=1}^{N} \left(1 + \frac{2tT_c}{\nu_c \sum_d T_d}\right)^{-\frac{1}{2}-\delta_{ca}-\delta_{cb}}$$

Moldauer had found

$$\nu_c = 1.78 + \left(T_c^{1.212} - 0.78\right) e^{-0.228 \sum_a T_a}$$

Moldauer approach is **NOT** used in EMPIRE-II

WF - the HRTW approach

In the case of no direct reactions

$$< S >_{ab} = \delta_{ab} e^{i\varsigma_{ab}} (1 - T_a)^{1/2}$$

$$T_a = 1 - | < S >_{aa} |^2$$

HRTW model assumes cross sections **factorization**

$$< \sigma_{ab}^{fl} > = \xi_a \xi_b \quad a \neq b \qquad and \qquad < \sigma_a^{fl} > = W_a \xi_a^2$$

WF - the HRTW approach

Setting

$$\xi_a = \frac{V_a}{\sqrt{\sum_c V_c}}$$

we get for the CN cross section

$$\sigma_{ab}^{CN} = [1 + \delta_{ab}(W_a - 1)] \frac{V_a V_b}{\sum_c V_c}$$

WF - the HRTW approach

Taking into account flux conservation (unitarity condition)

$$V_a = T_a \left[1 + \frac{V_a}{(\sum_c V_c)} (W_a - 1) \right]^{-1}$$

Can be solved for V_a by iteration once all W_a are known

WF - the HRTW approach

K-matrix representation

$$S = \frac{(1+iK)}{(1-iK)}$$

$$K_{ab} = K^0_{ab} + \sum_{\mu=1}^{N} \frac{\gamma_{\mu a}\gamma_{\mu b}}{E_\mu - E}$$

$\gamma_{\mu a}$ and $E_\mu$are real and uncorrelated, K_0 - direct reactions

WF - the HRTW approach

Numerical experiment:
$\gamma_{\mu a}$- drawn randomly with normal distribution
E_μ- spacing ($x = E_\mu - E_{\mu+1}$) according to the Wigner distribution

$$Q_W(x)dx = \frac{\pi}{2} exp(-\frac{\pi x^2}{4})dx$$

$$T = \frac{4\pi < \gamma^2 > /D}{(1 + \pi < \gamma^2 > /D)^2}$$

Generate ensemble of K-matrices => S-matrices => <S> => σ_{ab}

HRTW (cont.)

V_a can be found by iteration with W_a given by:

$$W_a = 1{+}2\left[1+T_a^F\right]^{-1}{+}87\left(\frac{T_a - T_{ave}}{\sum_c T_c}\right)^2\left(\frac{T_a}{\sum_c T_c}\right)^5$$

with

$$F = 4\frac{T_{ave}}{\sum_c T_c}\left(1+\frac{T_a}{\sum_c T_c}\right)\left(1+3\frac{T_{ave}}{\sum_c T_c}\right)^{-1}$$

By default, the HRTW model is applied below 5 MeV incident energy. Users have the option to apply it at all energies or to turn it off.

Triple integral

$$< S_{ab}(E_1) S^*_{cd}(E_2) > \quad =$$
$$< S_{ab}(E_1) >< S^*_{cd}(E_2) > \quad +$$
$$\frac{1}{8} \int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda \quad \times$$
$$\frac{(1-\lambda)\lambda|\lambda_1-\lambda_2|}{[(1+\lambda_1)\lambda_1(1+\lambda_2)\lambda_2]^{1/2}(\lambda+\lambda_2)^2(\lambda+\lambda_2)^2} \quad \times$$
$$exp\left[-i\pi(E_2^* - E_1)(\lambda_1 + \lambda_2 + 2\lambda)/d\right] \quad \times$$
$$\prod_{e=1}^{\Lambda} \frac{(1-T_e\lambda)}{[(1+T_e\lambda_1)(1+T_e\lambda_2)]^{1/2}} \quad \times$$

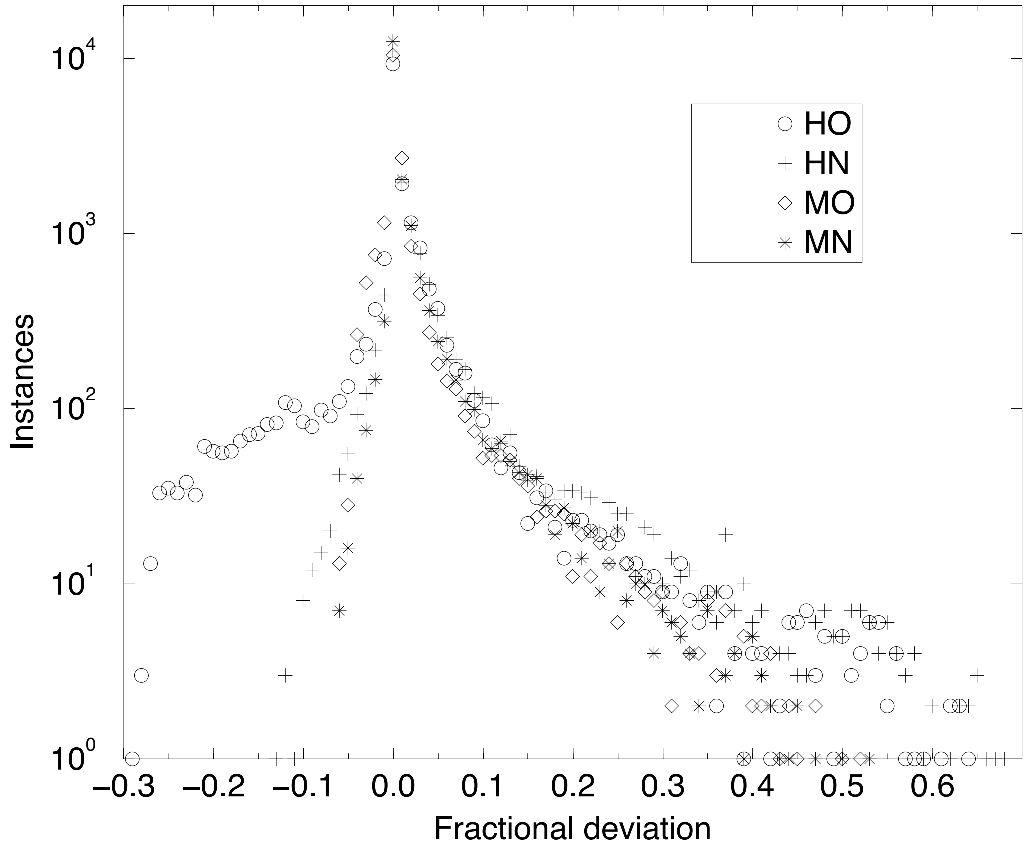
Triple integral (cont.)

$$
\begin{aligned}
&\{\delta_{ab}\delta_{cd} < S_{aa} >< S_{cc} > T_a T_c && \times \\
&\left(\frac{\lambda_1}{1+T_a\lambda_1} + \frac{\lambda_2}{1+T_a\lambda_2} + \frac{2\lambda}{1+T_a\lambda}\right) && \times \\
&\left(\frac{\lambda_1}{1+T_c\lambda_1} + \frac{\lambda_2}{1+T_c\lambda_2} + \frac{2\lambda}{1+T_c\lambda}\right) + (\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})\, T_a T_c && \times \\
&\left[\frac{\lambda_1(1+\lambda_1)}{(1+T_a\lambda_1)(1+T_b\lambda_1)} + \frac{\lambda_2(1+\lambda_2)}{(1+T_a\lambda_2)(1+T_b\lambda_2)}\right. && + \\
&\left.\frac{2\lambda(1+\lambda)}{(1+T_a\lambda)(1+T_b\lambda)}\right]\}
\end{aligned}
$$

NOT used in EMPIRE

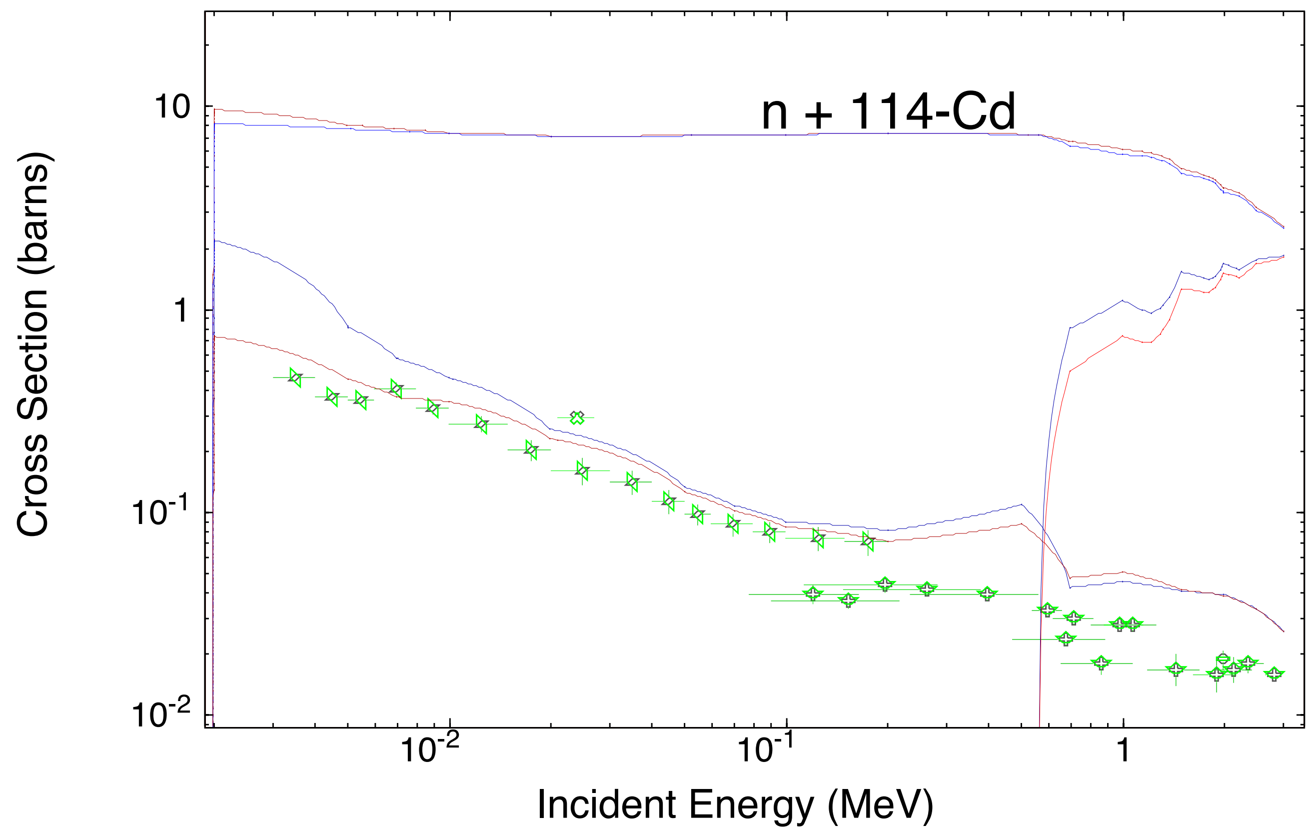
Validation of approx. approaches

Deviations of HRTW & Moldauer from the Triple Integral (≃8000 cases)



Effect of HRTW

elastic, inelastic, capture



CN and other nuclear models

- **Optical Model** - provides averaged S-matrix (transmission coefficients)
- **Direct Reaction Models** - provides averaged S-matrix including off-diagonal elements
- **Preequilibrium emission** - CN formation mechanism, reduction of the CN absorption cross section

CN and other nuclear models

- **Shell Model** - basis for the CN development
- **Fermi Gas and BCS** - determination of level densities
- **Nuclear Structure** - discrete level schemes, γ-transition probabilities
- **Liquid Drop + Shell Model** - binding energies, shell corrections, fission barriers, nuclear shape

Hauser-Feshbach in EMPIRE

- Each Compound Nucleus state contributes with a cross section

$$\sigma_b(E,J,\pi) = \sigma_a(E,J,\pi)\frac{\Gamma_b(E,J,\pi)}{\sum_c \Gamma_c(E,J,\pi)}$$

These have to be summed over spin J and parity π, and integrated over excitation energy E (in case of daughter CN)

HF in EMPIRE (cont.)

- Particle decay width

$$\Gamma_c(E,J,\pi) = \frac{1}{2\pi\rho_{CN}(E,J,\pi)}\sum_{J'=0}^{\infty}\sum_{\pi'}\sum_{j=J'-J}^{J+J'} \int_0^{E-B_c}\rho_c(E',J',\pi')T_c^{l,j}(E-B_c-E')dE',$$

B_c - binding energy of particle c,
ρ - level density (for discrete levels
$\rho = \delta(E-E_i)\delta_{(J',J_i)}\delta_{(\pi',\pi_i)}$)
$T_c^{l,j}(\epsilon)$ - transmission coefficient,

HF in EMPIRE (cont.)

- Gamma decay width assumed to be a mixture of single-particle estimate and Giant Resonance contributions

$$T_{Xl} = (1-t)T_{Xl}^{Weiss} + tT_{Xl}^{GMR}$$

Single-particle estimates (Weisskopf):

$$T_{E1}^{Weiss} = C_{E1} 4.599^{-7} A^{2/3} E_\gamma^3 [MeV^{-3}]$$

$$T_{M1}^{Weiss} = C_{M1} 1.3^{-7} E_\gamma^3 [MeV^{-3}]$$

HRTW implementation

HRTW is applied to each J^π state in the highest energy bin in the first CN

- fusion cross section to J^π has to be decomposed into its l components
- first sweep: T_l for all channels are stored and elastic channels are recorded
- second sweep: V_l and HF-type denominator are calculated, elastic channels enhanced
- third sweep: partial widths are normalized

Conclusions

EMPIRE contains a fairly complete implementation of statistical model

- angular momentum coupling (although T_l scheme is used rather than T_{lj})
- Width fluctuation correction
- multiple emission of n, p, α, γ, and optionally one light ion (d, ^{3}He, t, etc.)
- full γ-cascade
- treatment of discrete levels (isomeric cross sections)