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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

15 - 23 March 2004

Challenges for string cosmology

Part III

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Challenges for String Cosmology

1. Motivation
2. Challenges for String Cosmology
3. Essential tools of String Cosmology
4. Brane Gas Cosmology
5. Progress & Problems

Conceptual Problems of Scalar Field-Driven Inflationary Cosmology

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a) Fluctuation Problem
b) Trans-Planckian Problem
c) Singularity Problem
d) Cosmological Constant Problem

Origin of Inflation

Phase of Inflation in very early Universe

↓

almost scale invariant spectrum of adiabatic fluctuations

↓

agreement with data

but

In the context of

- space-time – classical GR
- matter – quantum fields

there is no convincing realization of inflation

Q: Does string theory provide a convincing realization of inflation?

(or an alternative?)

a) Fluctuation Problem

inflation $\longrightarrow$ density fluctuations on all scales
~ scale invariant spectrum
Mukhanov & Chibisov 81

quantum theory of cosmological perturbations
Mukhanov ; Sasaki
Mukhanov, Feldman & R.B.

classical fluctuations emerge via squeezing of initial vacuum state

$\downarrow$

amplitude $\frac{\delta M}{M}(k, t_f(k)) \underset{\uparrow}{\sim} 10^2 \lambda^{1/2}$

$V(\varphi) = \lambda \varphi^4$

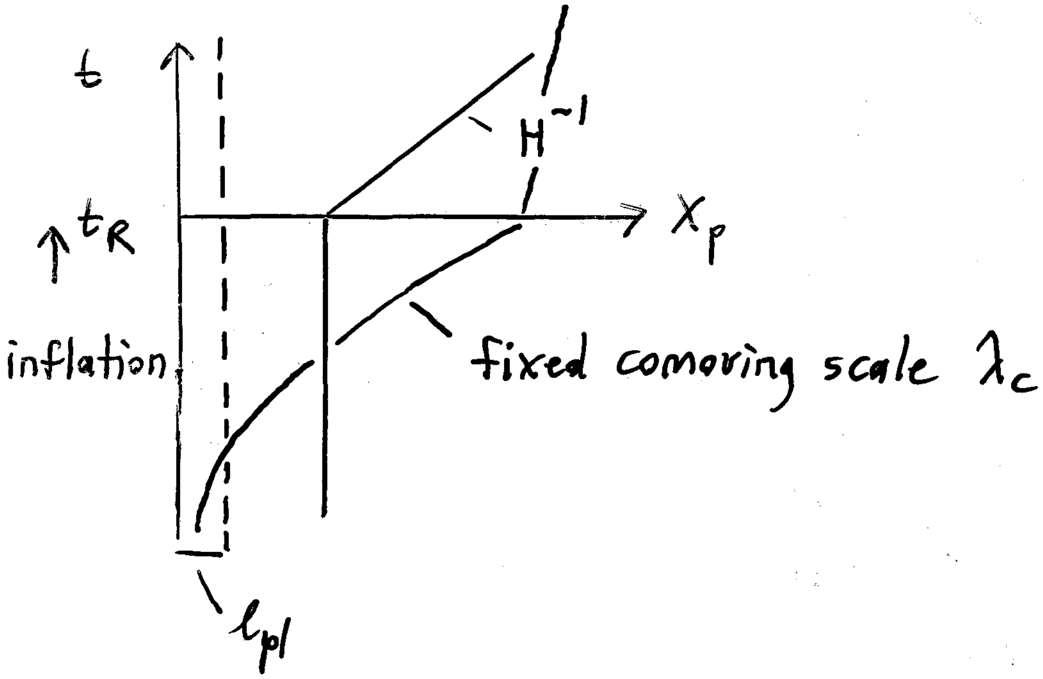
$\Rightarrow \lambda \leq 10^{-12}$ hierarchy problem

N.B. problem is generic
Adams, Freese & Guth

Window of Opportunity

b) Trans-Planckian ~~Problem~~

R.B 1999



Success of inflation : at early times $\lambda_c(t) < H^{-1}(t)$

$\Downarrow$

causal generation mechanism for fluctuations

$\downarrow$

Problem : if $\Delta t_{\text{inflation}} > 70 H^{-1}$

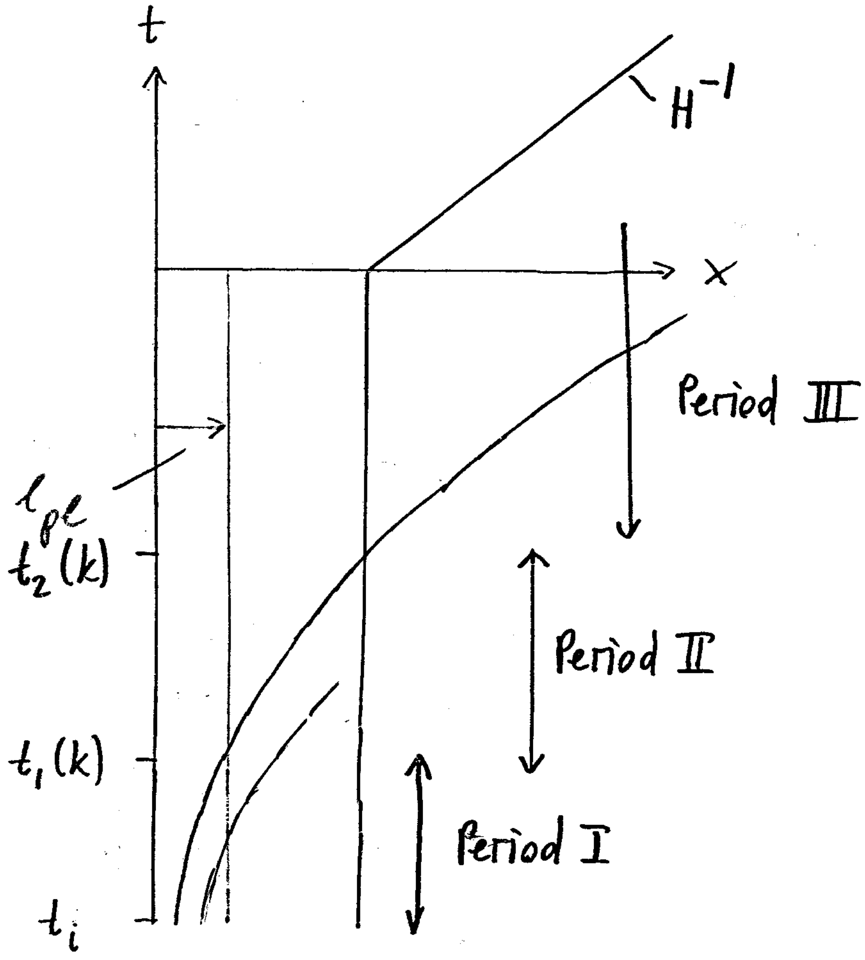
$\Downarrow$

$\lambda_c(t_i) < \ell_{pl}$

t_i: beginning of inflation

$\Downarrow$

new physics MUST enter into calculation of fluctuations



If evolution in Period I non-adiabatic then $P(k)$ likely not scale invariant.

R.B. & J. Martin (2000)

$\Downarrow$

Planck scale physics testable in observations

c) Singularity Problem

standard cosmology : Penrose - Hawking theorems

$\downarrow$

initial singularity ("Big Bang")

$\downarrow$

incompleteness of theory

Penrose - Hawking theorems

Ass : Einstein action

(*) weak energy condition

$\rho > 0 \, , \, \rho + 3p \geq 0$

$\Big\} \Rightarrow$ space-time is geodesically incomplete

inflationary cosmology : weak energy condition violated ($p = -\rho$)

$\downarrow$

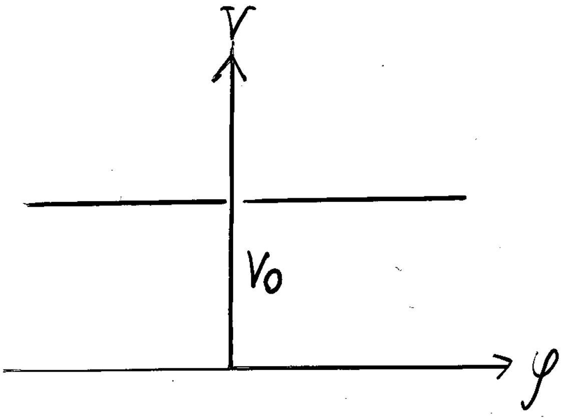
Penrose - Hawking theorems do not apply

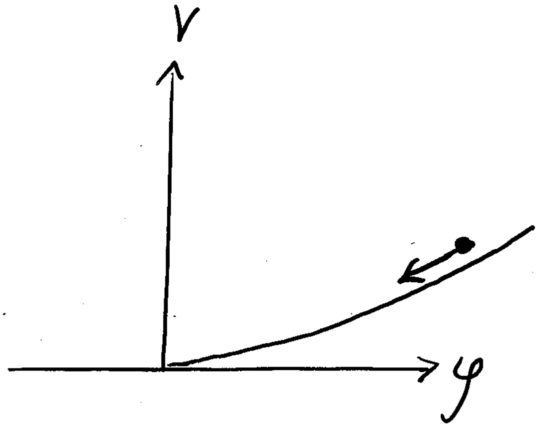
but : Theorem : In a chaotic inflation model initial singularity persists [Borde & Vilenkin]

$\downarrow$

incompleteness of theory

d) Cosmological Constant Problem





Quantum vacuum energy of matter fields do not gravitate

$$\frac{V_0}{\Lambda_{obs}} > 10^{122} \quad !!$$

Cosmological Constant Problem

Why does the unknown mechanism which renders V_0 gravitationally inert also render $V(\varphi)$ gravitationally inert ?

driving inflation

Why String Theory can Help (?)

a) Fluctuation Problem

$$\text{string theory moduli} \xrightarrow{?} \text{small parameters required for } \frac{\delta\rho}{\rho} \sim 10^{-5}$$

$$\downarrow ?$$

inflation

b) Trans-Planckian Problem

string theory determines physics when

$\lambda_{ph} \sim l_{pl}$

→ determines initial conditions for fluctuations in classical regime

c) Singularity Problem

string theory → nonsingular cosmology

↑

goal

R.B. & C. Vafa Nucl. Phys. B 1989

S. Alexander, R.B. & D. Easson , 2000

d) Cosmological constant problem ??

2. Challenges for string cosmology

Dimensionality Problem

Critical, perturbative Superstring Theory
is mathematically consistent only in
$d = 9 + 1$ space-time dimensions

fatal problem ?

Traditional approach :

extra dimensions compactified

ad hoc ?

stabilization ?

New approach :

brane world scenarios :

we live on a $d = 3+1$ brane

ad hoc

why $3+1$ brane ?

.....

⇓ my conclusion

key challenge for string cosmology

Some Challenges for String Cosmology

- * Resolve Cosmological Singularities
- * Predict Dimensionality
- * Make contact with observations

either : provide convincing theory of inflation •

or : yield alternative cosmology • which maintains successes of inflation

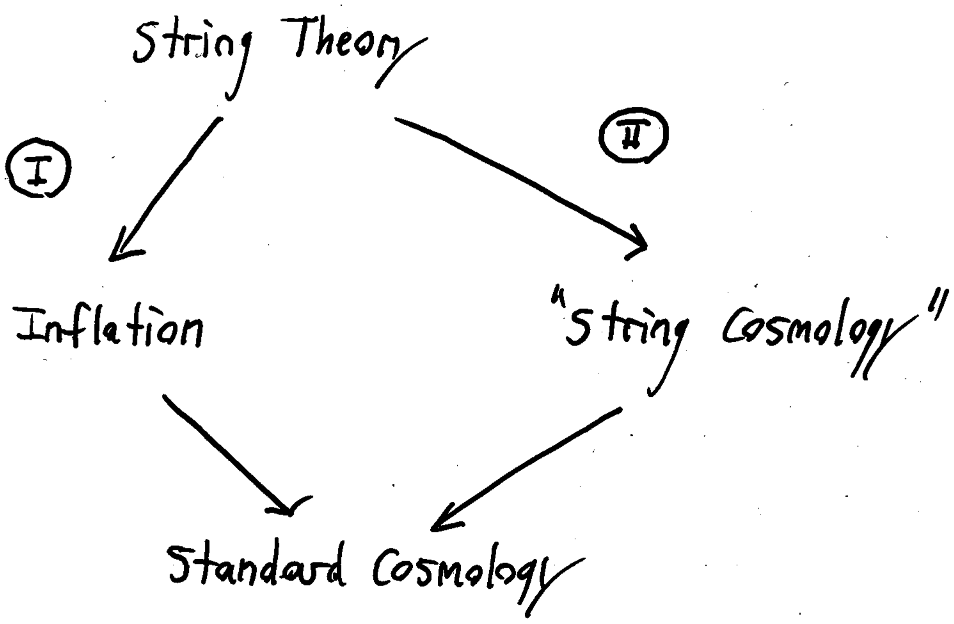
— — — — — —

- * Address cosmological constant problem

- • Fluctuations mathematically consistent

Evolution equations physically consistent

Possibilities



String Theory

Ⓘ

Ⓘⓘ

Inflation

"String cosmology"

Standard Cosmology

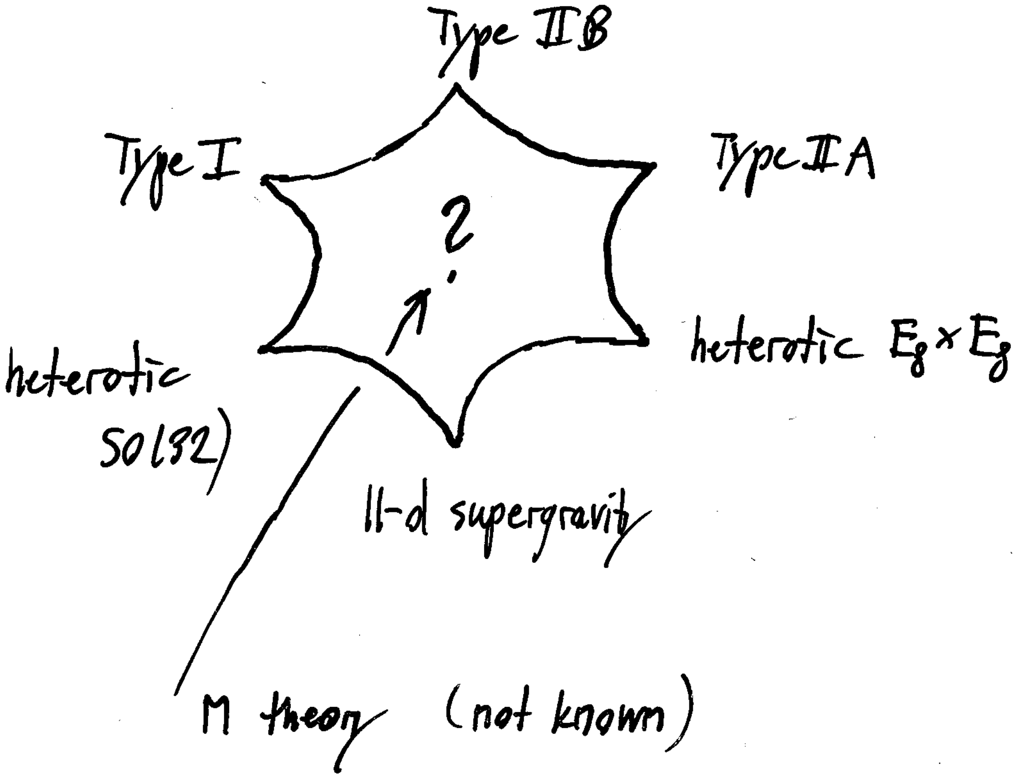
I : maintains successes of infl. cosmology

II : must provide new solutions to the problems which inflation solves

3. Essential tools of string cosmology

The Problem

String cosmology does not exist because nonperturbative string theory is not yet known.



Most work on string cosmology starts in one corner of string theory moduli space.

A Comparison

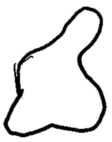
	space-time	matter
standard cosmology	classical GR	perfect fluid
inflationary cosmology	classical GR	quantum field theory
	fundamental inconsistency	
string cosmology	quantum string theory	

N.B. Quantum string theory is "under construction".

Building Blocks of String Cosmology vs. Inflationary Cosmology

Inflation		String
point particle	basic classical building block	$\boxed{\text{string}}$
	↓	
point particle scattering rates	quantization	string scattering rates
	↓	
quantum field theory		string field theory
scalar field = inflaton		

$\boxed{\text{string}}$:



① different excitation modes ⇔ different particles (including graviton)

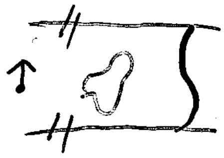
→ unification

② New degrees of freedom

Consider toroidal space, radius R

particle: momentum modes

$$E_m = \frac{m}{R}$$



string: momentum modes $E_m = \frac{m}{R}$

oscillatory modes* E indep. of R

$\boxed{\text{winding modes}}$ $E_n = nR$

* density of these states ↗ exponentially

↓

maximum temperature

(Hagedorn temp.)

③ New symmetries

$\boxed{\text{t-duality}}$ $R \rightarrow \frac{1}{R}$

$(n, m) \rightarrow (m, n)$

spectrum of closed strings invariant

is symmetry of perturbative string theory

The "Rules" for String Cosmology

A Hot Beginning

Expect gas of strings & branes
of all dimensionalities

see Brane gas cosmology

B Cold Beginning

Expect special initial state determined
by symmetry

see PBB
" Ekpyrotic

Some Approaches

Brane gas cosmology	type A
Pre-big-bang scenario	type B
Ekpyrotic	type B

Brane world scenario ?

- Could come from type A
  M. Majumdar & A.-C. Davis
- Moving branes → variations of fund. constants

↑

Mirage cosmology

4. Brane Gas Cosmology

Brane Gas Cosmology

Theory, Background & Initial Conditions

Theory : Type II superstring theory

Background : $\underset{\text{time}}{\underset{\uparrow}{\mathbb{R}}} \times \underset{\text{space}}{\underset{\uparrow}{T^9}}$

"democratic"

Initial Conditions : * hot gas

all degrees of freedom excited

* $R_i = R = 1$

"conservative"

R.B. & C. Vafa (89)

T Duality

for fundamental strings

oscillatory modes : E ind. of R

momentum modes : $E = \frac{n}{R}$

winding modes : $E = mR$

n, m integer

$$M^2 = \frac{n^2}{R^2} + m^2 R^2 + 2(N + \tilde{N} - 2)$$

$$\boxed{\begin{array}{ll} \text{t-duality :} & R \to \frac{1}{R} \\ & (n, m) \to (m, n) \end{array}}$$

spectrum of states invariant under t-duality

vertex operators 〃 〃

$\Downarrow$

t-duality is symmetry of perturbative string theory

K. Kikkawa & M. Yamasaki (84)

...

for branes
T. Boehm & R.B. (02)

Note: t-duality is used to argue for the existence of branes

Polchinski (95)

T-duality in direction // to p-brane:

p-brane → p-1 brane

T-duality in direction ⊥ to p brane:

p-brane → p+1 brane

T-dualizing in all spatial dimensions

p-brane → 9-p brane

R large		R small
Type II A brane gas $\mathcal{B}$	$\xleftrightarrow{T}$ duality	Type II B brane gas $\mathcal{B}^*$
0, 2, 4, 6, 8 branes		1, 3, 5, 7, 9 branes
string coupling g		string coupling g'

$$g' = \left(\frac{\alpha'^{1/2}}{R}\right)^9 g$$

effects tensions

$$\tau_p = e^{-\phi} T_p = \frac{1}{g} T_p = (2\pi)^{-p} g^{-1} \alpha'^{-\frac{p+1}{2}}$$

Ex : $$M_4 = (2\pi)^4 \tau_4 R^4 = g^{-1} \alpha'^{-5/2} R^4$$

$$M'_5 = (2\pi)^5 \tau'_5 R'^5 = M_4$$

using $$R' = \frac{l_s}{R}$$

$$g' = \left(\frac{l_s}{R}\right)^9 g \qquad \left[l_s = \alpha'^{1/2}\right]$$

$$M'_{9-p} = M_p$$

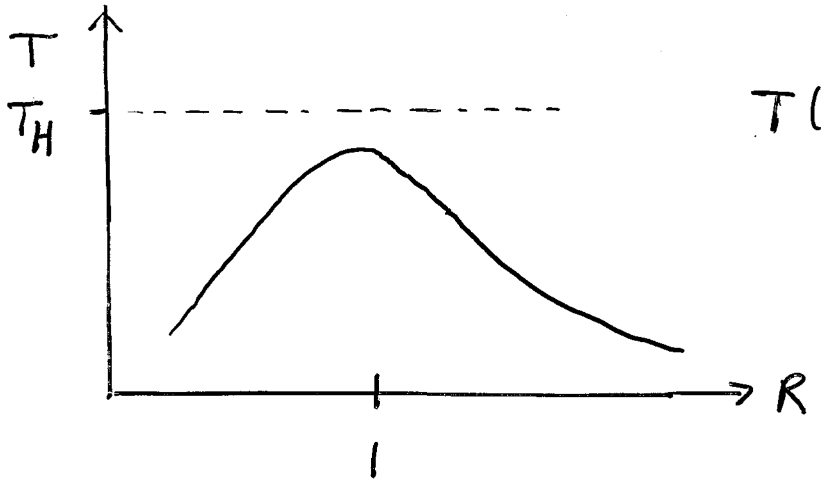
$$\Downarrow$$

$$\boxed{\text{mass spectrum of stable brane states invariant under T-duality}}$$

T-duality → Nonsingular Cosmology

Consider adiabatic change in R

A. Thermodynamics



$T(R) = T(\frac{1}{R})$

⇒ temperature nonsingular as $R \searrow 0$

B. "Physical" Length

$R > 1$ ℓ measured in terms of x

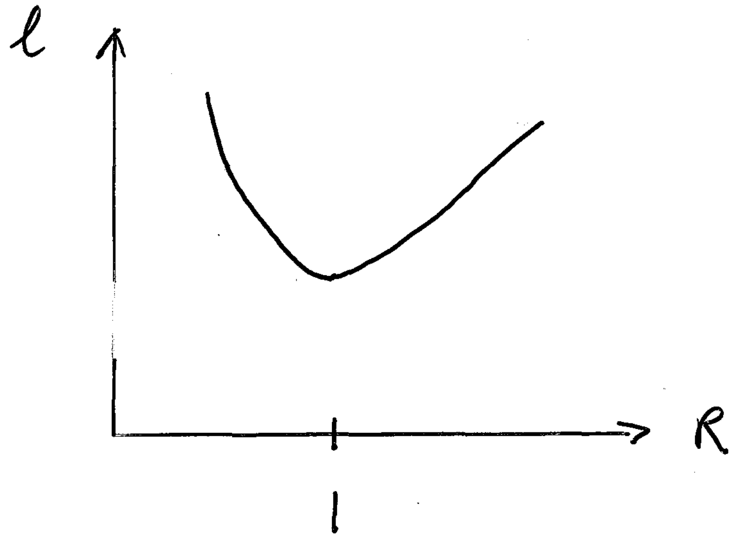
$$|x\rangle = \sum_p e^{ixp} |p\rangle$$

↖ momentum eigenstates

$R < 1$ ℓ measured in terms of $\tilde{x}$

$$|\tilde{x}\rangle = \sum_{p_w} e^{i\tilde{x}p_w} |p_w\rangle$$

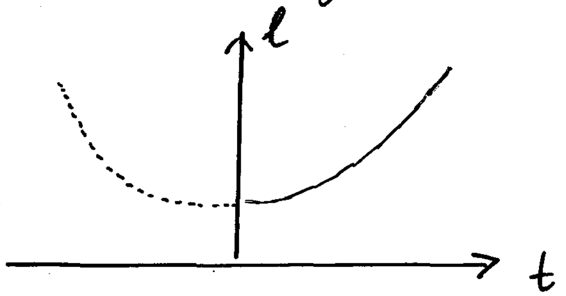
↑ winding eigenstates



$\Rightarrow$ physical length nonsingular as $R \to 0$

R.B. & C. Vafa (89)

N.B. Obtain bouncing Universe!



Cosmic Loitering

R.B., D. Easson & D. Kimberly (01)

To avoid overclosure : need all winding modes in 3 large dims. to annihilate

Kibble mechanism : $\gtrsim 1$ winding mode per Hubble volume persists

$$\Downarrow$$

need loitering $(H=0)$

Consider R large

$$\rho_w(t) = \mu \tilde{\nu}(t) t^{-2} \quad \text{winding modes}$$
$$\rho_\ell(t) = \gamma(t) e^{-3(\lambda(t)-\lambda(t_0))}$$

$$\boxed{\begin{aligned} \frac{d\tilde{\nu}(t)}{dt} &= 2\tilde{\nu}\left(t^{-1}-\ell\right) - cc'\tilde{\nu}^2 t^{-1} \\ \frac{d\gamma(t)}{dt} &= cc'\mu t^{-3}\tilde{\nu}^2 e^{3(\lambda(t)-\lambda(t_0))} \end{aligned}}$$

A. Vilenkin (85)

$$\ell \equiv \frac{\dot{a}}{a}$$

energy transfer from winding modes to loops

see also : A. Campos (03)

Equations of Motion

Dilaton gravity !
(else no T-duality)

$$S = \int d^{10}x \sqrt{-g}\, e^{-2\phi}\left[R + 4\partial^\mu\phi\,\partial_\mu\phi\right]$$

$$ds^2 = dt^2 - a(t)^2 d\underline{x}^2$$

$$\lambda = \log a(t)$$

$$\varphi = 2\phi - d\lambda \qquad [d = 9]$$

$\Downarrow$ variational EOM

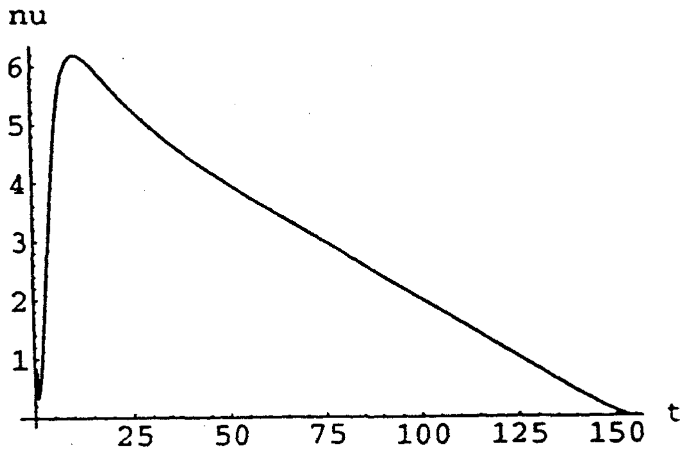
$$\boxed{\begin{aligned} -d\dot{\lambda}^2 + \dot{\varphi}^2 &= e^{\varphi} E \\ \ddot{\lambda} - \dot{\varphi}\dot{\lambda} &= \frac{1}{2} e^{\varphi} P \\ \ddot{\varphi} - d\dot{\lambda}^2 &= \frac{1}{2} e^{\varphi} E \end{aligned}}$$

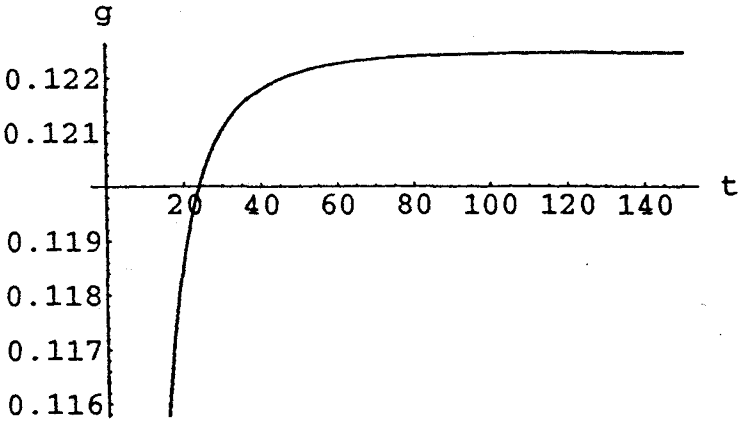
winding modes → $P < 0$

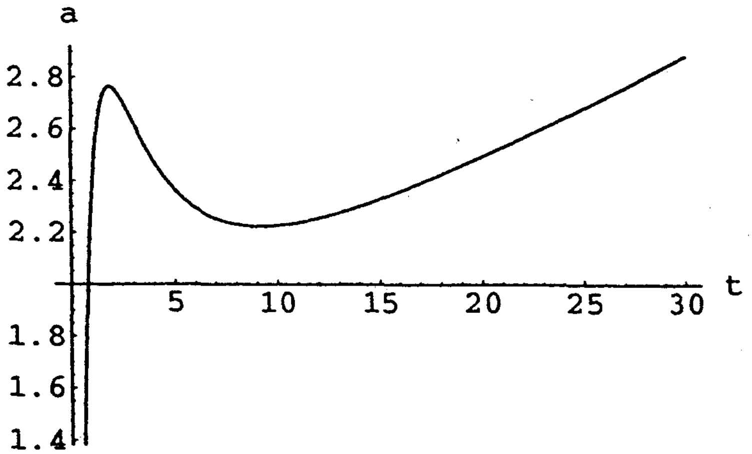
→ confining potential for λ

Veneziano (91)
Tseytlin & Vafa (92)

duality: $\left.\begin{aligned} \phi &\to \phi - d\lambda \\ \lambda &\to -\lambda \end{aligned}\right\} \Rightarrow \varphi$ invariant







R.B., D. Easson & D. Kimberly (01)

Scenario

- * Brane winding modes prevent expansion
- * p branes : annihilate in $\leq$ d_c spatial dims.

$$2(p+1) = d_c + 1$$

Consider : $R(t_0) = 1$
$\dot{R}(t_0) > 0$

- * Heaviest branes fall out of equilibrium first
  - └ largest p

no obstructions to $p=4$, ... brane annihilation

- * $p = 2$ branes
  - → $d_c = 5$ expand

S. Alexander, R.B. & D. Easson (00)

- * within these 5 dimensions :

fundamental string winding modes

→ only 3 spatial dims. grow large

R.B. & C. Vafa (89)

→ Solution of Dimensionality Problem

see also : M. Sakellariadou (96)

5. Progress and Problems in Brane Gas Cosmology

S. Watson & R.B. (03)

Radion Stabilization

Q: How does radius of "small" dimensions evolve once the 3 "large" dimensions are expanding?

Ansatz: $$ds^2 = dt^2 - \underset{\underset{\mathbb{R}^3 \text{ large}}{\uparrow}}{e^{2\lambda} d\underline{x}^2} - \underset{\mathbb{R}^6 \text{ small}}{e^{2\nu} d\underline{y}^2}$$

Dilaton gravity action + perfect fluid matter

$$-3\ddot{\lambda} - 3\dot{\lambda}^2 - 6\ddot{\nu} - 6\dot{\nu}^2 + 2\ddot{\phi} = \frac{1}{2} e^{2\phi} \rho$$

$$\ddot{\lambda} + 3\dot{\lambda}^2 + 6\dot{\lambda}\dot{\nu} - 2\dot{\lambda}\dot{\phi} = \frac{1}{2} e^{2\phi} p_\lambda$$

$$\ddot{\nu} + 6\dot{\nu}^2 + 3\dot{\lambda}\dot{\nu} - 2\dot{\nu}\dot{\phi} = \frac{1}{2} e^{2\phi} p_\nu$$

$$4\ddot{\phi} - 4\dot{\phi}^2 - 12\dot{\lambda}\dot{\phi} - 24\dot{\nu}\dot{\phi} + 3\ddot{\lambda} + 6\dot{\lambda}^2 + 6\ddot{\nu} + 21\dot{\nu}^2 + 18\dot{\lambda}\dot{\nu} = 0$$

Matter (string gas) sources obeying T duality

$\downarrow$

include equal number of winding and momentum modes

$$E = 3\mu N^{(3)} e^{\lambda} + 3\mu M^{(3)} e^{-\lambda} + 6\mu N^{(6)} e^{\nu} + 6\mu M^{(6)} e^{-\nu}$$
$$P_\lambda = -\mu N^{(3)} e^{\lambda} + \mu M^{(3)} e^{-\lambda}$$
$$P_\nu = -\mu N^{(6)} e^{\nu} + \mu M^{(6)} e^{-\nu}$$

1st application : $N^{(3)} = M^{(3)} = N^{(6)} = M^{(6)}$.

$\lambda(t_i) =, \nu(t_i) \neq 0$

$\Downarrow$

damped oscillations of radius about self-dual point

2nd application : $N^{(3)} = 0$; $N^{(6)} = M^{(6)}$

[after decompactification]

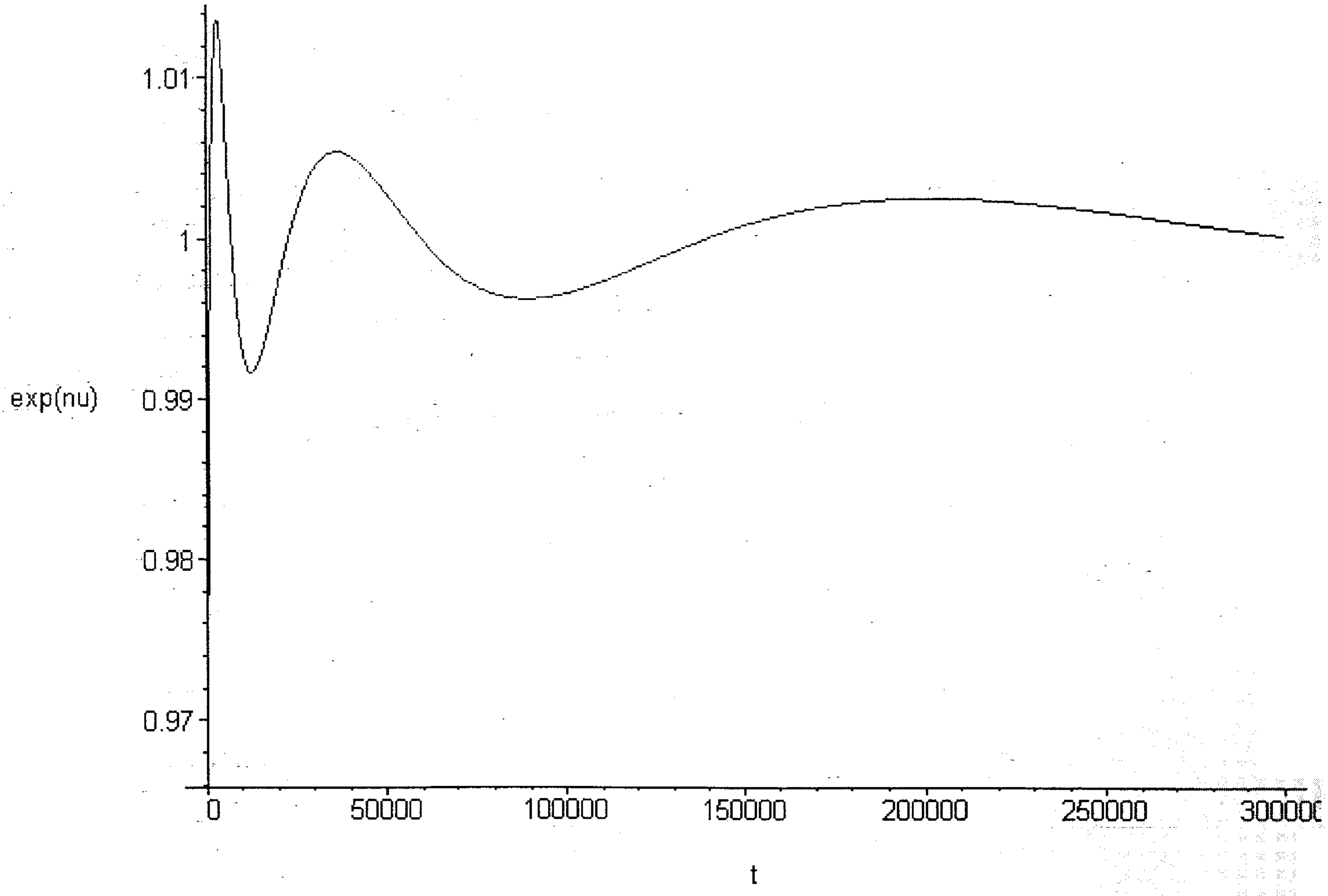
$\Downarrow$

λ increases

ν oscillates about $\nu = 0$

expansion of $\lambda \Rightarrow$ damping of ν oscillations radion stabilization

S. Watson & R.B. (03)



S. Patil & R.B. (04)

Late Time Radion Stabilization

Q: Does the radius of the extra dimensions remain stabilized after the dilaton is fixed ?

Model: GR + string gas

5d

winding & momentum modes about 5th dim.

initial conditions:

our 4d space-time in expanding phase

Einstein eqs. (5d)

$$ds_5^2 = dt^2 - a^2(t)\,d\underline{x}^2 - b^2(t)\,dy^2$$

$$\ddot{b} + 3H\dot{b} + \frac{b}{M_{pl_5}^3}\underbrace{\left(p - \frac{2r}{3} - \frac{\rho}{3}\right)}_{S} = 0$$

$$H = \frac{\dot{a}}{a}$$

$$r = -T^5_{\ 5} \qquad p = -T^i_{\ i} \qquad \rho = T^0_{\ 0}$$

winding strings: winding number w

momentum number n

$$v = \frac{1}{2\pi}\frac{\mu_0}{a^3 b}\frac{\frac{n^2}{b^2} - \frac{w^2 b^2}{\alpha'^2}}{\sqrt{p^i p_i + \frac{4}{\alpha'}(N-1) + \left(\frac{n}{b} + \frac{wb}{\alpha'}\right)^2}}$$

$$\underbrace{\frac{b}{M_{pl_5}^3}\left(\rho - \frac{2r}{3} - \frac{f}{3}\right)}_{S} = \frac{\mu_0}{2\pi a^3 M_{pl_5}^3} \times \frac{\frac{w^2 b^2}{3\alpha'^2} - \frac{2nw}{3\alpha'} - \frac{n^2}{b^2} - \frac{4(N-1)}{3\alpha'}}{\text{Denominator}}$$

key: S changes sign as $b \nearrow$

$S < 0$ for small b ⇒ expansion

$S > 0$ for large b ⇒ contraction

$\Downarrow$

$$\boxed{\text{stabilization at self-dual radius}}$$

for $N=1$

Note: string loops, gravitons & photons do not contribute to S

Challenges for Brane Gas Cosmology

Isotropy Problem

Are the 3 large dimensions isotropic ?

A : yes !

S. Watson & R. B. (02)

Isotropy is scaling fixed pt.

$R_i < R_j$

→ more efficient annihilation of i winding modes

→ $\frac{R_i}{R_j} \nearrow 1$

Dependence on Background

Existence of cycles not crucial

R. Easther et al. (01)

extension to orbifolds

dynamical obstruction to efficient annihilation

D. Easson (01)

Radion Stabilization

Interplay of Winding & Momentum Modes
about internal tori

$\Downarrow$

$R_i \rightarrow 1$

S. Watson & R.B. (03)

Homogeneity of Internal Dimensions

??

S. Watson, R. Easther, B. Greene
in prep.

Extensions to other Corners of M-Theory moduli space

M theory (11-d supergravity)

a) 2 branes

R. Easther et al. (02)

$\Downarrow$

hierarchy of dimensions

b) 2 brane / 5 brane intersections

$\Downarrow$

3 d get very large fastest

S. Alexander (02)

Connection with Observations

Need: i) lage Universe

ii) mechanism to generate fluctuations

i) flatness problem

↓

require inflation

ii) "only" viable mechanism now is inflation

Q: Can we get inflation from brane gas cosmology?

Conclusions

1. String theory can provide a new paradigm of early Universe cosmology
    - non singular
    - explains why only 3 large spatial dimensions
2. New degrees of freedom & symmetries play a crucial role
3. Brane gas cosmology

    One approach to string cosmology
    hot small beginning
    "explains" why 3 large dims.
    nonsingular
    radion stabilization
    but : too heuristic
    successful connection with t_0 ?