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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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Strings in AdS 3 and the SL (2,R) WZW model

Hirosi OOGURI
CALTECH
Department of Physics
Mail Code 452-48
Pasadena, CA 91125
U.S.A.

Please note: These are preliminary notes intended for internal distribution only.

STRINGS IN AdS_3

AND THE $SL(2,\mathbb{R})$ WZW MODEL

HIROSI OOGURI

(CALTECH)

BASED ON WORK WITH JUAN MALDACENA

HEP-TH / 0001053

0005183 (WITH J. SON)

0111180

SHOULD THE CONCEPT OF TIME

BE MODIFIED IN STRING THEORY ?

C.F. THE CONCEPT OF SPACE HAS BEEN REVISED.

- MIRROR SYMMETRY. T-DUALITY
- SMOOTH TOPOLOGY CHANGES

  PERTURBATIVE (FLOP, ⋯) & NONPERTURBATIVE (CONIFOLD, ⋯)
- DIMENSIONALITY IS NOT INVARIANT.

  10^d IIA STRING $\rightarrow$ 11^d SUPERGRAVITY

AdS_3 IS AN INTERESTING CASE TO STUDY

SINCE $g_{00}(x)$ IS NON-TRIVIAL.

$\Rightarrow$ THE TIME VARIABLE DOES NOT DECOUPLE

ON THE WORLDSHEET.

WITH A BACKGROUND NS-NS 2-FORM,

THE WORLDSHEET THEORY IS

THE $SL(2,\mathbb{R})$ WZW MODEL.

- THE PROOF OF NO-GHOST THEOREM WAS NON-TRIVIAL.

  ($\sim$ 10 YEARS)
- TODAY I WILL DISCUSS HOW THE EUCLIDEAN ROTATION WORKS FOR CORRELATION FUNCTIONS.

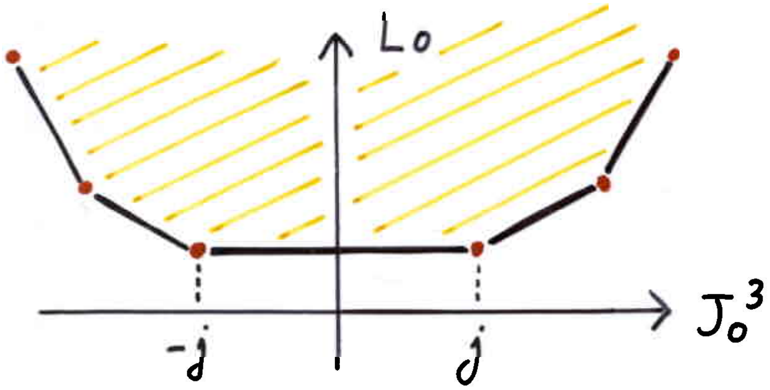
1. Spectrum

- SU(2) WZW Model ··· Compact Target Space

$$S = \frac{k}{8\pi}\int d^2z \,\mathrm{Tr}\left[g^{-1}\partial g \, g^{-1}\bar{\partial} g \right] + k\,\Gamma_{WZ}$$

$k = 1, 2, \cdots$: LEVEL OF $\widehat{SU(2)}$

$$\boxed{\text{Hilbert Space} = \bigoplus_{j=0,\frac{1}{2},\cdots,\frac{k}{2}} \left[\mathcal{H}_j \otimes \mathcal{H}_j \right]}$$



$\mathcal{H}_j$: IRREDUCIBLE REPRESENTATION OF $\widehat{SU(2)}$

(GROUND STATES = SPIN j)

✔ Modular Invariance

✔ Operator Product Expansion

The spectral flow

$$J_m^3 \to J_m^3 + \frac{k}{2}\,\omega\,\delta_{m,0}$$

$$J_m^{\pm} \to J_{m\pm\omega}^{\pm} \qquad \omega = 0, \pm 1, \pm 2, \cdots$$

MAPS $\mathcal{H}_j \longleftrightarrow \mathcal{H}_{\frac{k}{2}-j}$

• SL(2,R) WZW MODEL ··· NONCOMPACT TARGET SPACE

LEVEL k : REAL , >2

IN HEP-TH/0001053, WE PROPOSED

HILBERT SPACE

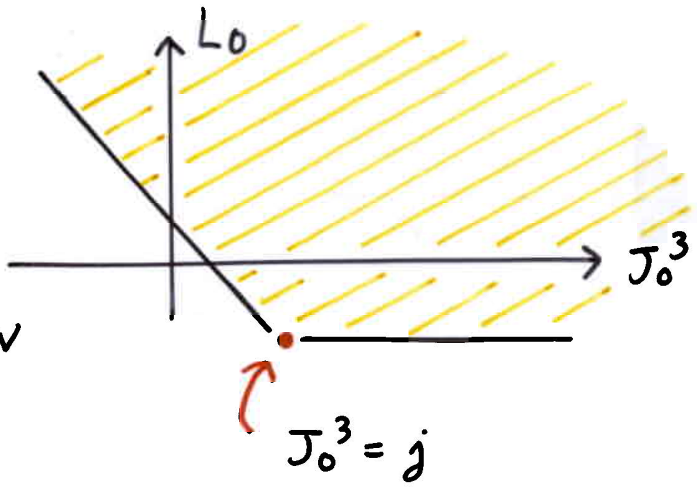
$$= \bigoplus_{\omega=-\infty}^{+\infty} \left[\int_{\frac{1}{2}}^{\frac{k-1}{2}} dj \, D_j^{\omega} \otimes D_j^{\omega} \oplus \int_{\frac{1}{2}+i\mathbb{R}} dj \int_0^1 d\alpha \, C_{j,\alpha}^{\omega} \otimes C_{j,\alpha}^{\omega} \right]$$

D_j^{ω} , $C_{j,\alpha}^{\omega}$: IRREDUCIBLE REPRESENTATIONS OF $\widehat{SL(2,\mathbb{R})}$

$D_j^{\omega=0}$

GROUND STATES

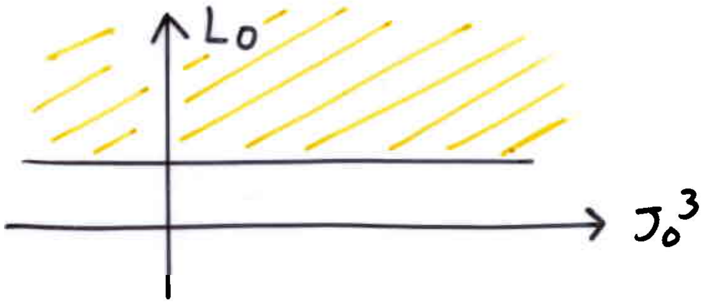
= DISCRETE REPRESENTATION



$C_{j,\alpha}^{\omega=0}$

GROUND STATES

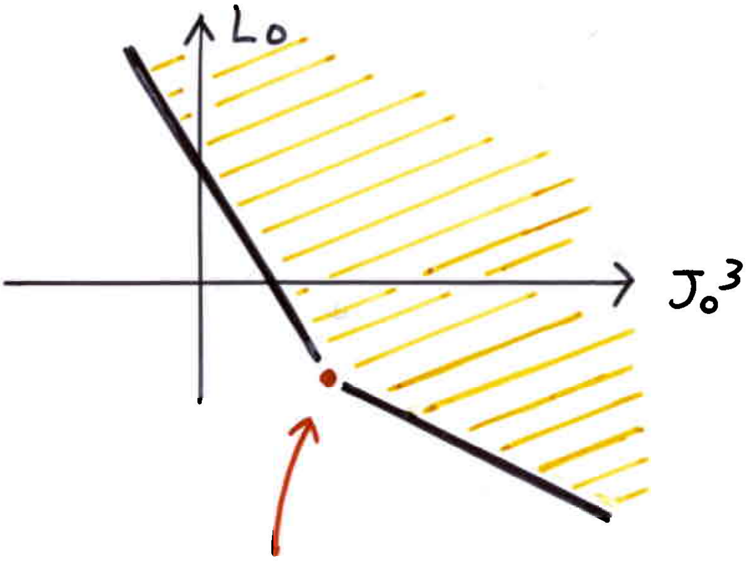
= CONTINUOUS REPRESENTATION



THEY ARE NOT INVARIANT UNDER THE SPECTRAL FLOW.

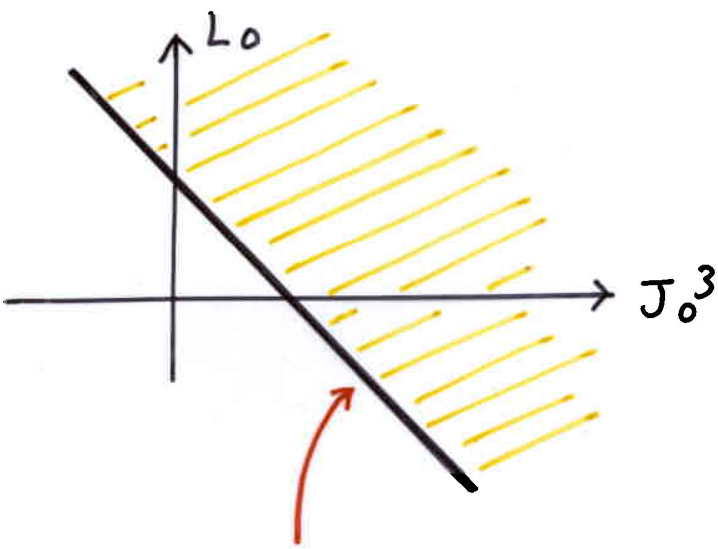
D_j^{ω}, $C_{j,\alpha}^{\omega}$: SPECTRAL FLOW OF $D_j^{\omega=0}$, $C_{j,\alpha}^{\omega=0}$

D_j^{ω}



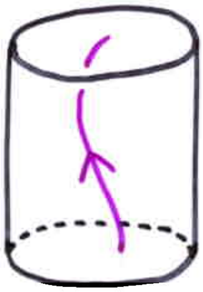
ANNIHILATED BY $J^-_{-\omega}$, $J^-_{1-\omega}$, $J^-_{2-\omega}$, $\cdots$
$J^+_{1+\omega}$, $J^+_{2+\omega}$, $\cdots$

$C_{j,\alpha}^{\omega}$



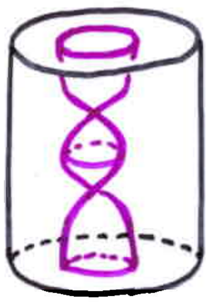
ANNIHILATED BY $J^-_{1-\omega}$, $J^-_{2-\omega}$, $\cdots$
$J^+_{1+\omega}$, $J^+_{2+\omega}$, $\cdots$

- THE PROPOSED HILBERT SPACE FITS WELL WITH THE SEMI-CLASSICAL DESCRIPTION OF STRINGS IN AdS_3



MASSIVE POINT PARTICLE

$\in \mathcal{D}_j^{\omega=0} \otimes \mathcal{D}_j^{\omega=0}$

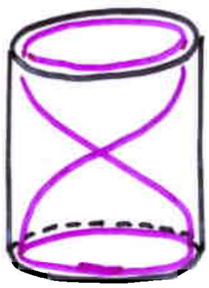


SHORT STRING WITH WINDING NUMBER ω

$\in \mathcal{D}_j^{\omega} \otimes \mathcal{D}_j^{\omega}$

TACHYON

$\in \mathcal{C}_{j,\alpha}^{\omega=0} \otimes \mathcal{C}_{j,\alpha}^{\omega=0}$



LONG STRING WITH WINDING NUMBER ω

$\in \mathcal{C}_{j,\alpha}^{\omega} \otimes \mathcal{C}_{j,\alpha}^{\omega}$

- NO GHOST THEOREM HOLDS FOR ALL ω.

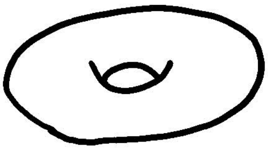
2. ONE-LOOP TEST

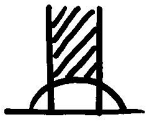
FREE ENERGY AT TEMPERATURE T.

$$\sum_i \log\left(1 - e^{-E_i/T}\right),$$

E_i : SINGLE STRING SPECTRUM IN LORENTZIAN AdS_3

$$\|$$

$$-\int d^2\tau$$ 



COMPUTED ON EUCLIDEAN AdS_3 WITH PERIODIC IMAGINARY TIME

$$t \sim t + \frac{1}{T}$$

- EUCLIDEAN PATH INTEGRAL IS ITERATIVELY GAUSSIAN.
- $\int d^2\tau$ EVALUATED A LA POLCHINSKI.

$\{E_i\}$ AGREES WITH THE SPECTRUM DERIVED FROM OUR PROPOSAL.

✓ CONTAINS ALL THE SECTORS, $\omega = 0, \pm 1, \pm 2, \cdots$

✓ CONSTRAINT $\frac{1}{2} < j < \frac{k-1}{2}$ ON D_j^{ω}

✓ CORRECT DENSITY OF STATES FOR $C_{j,\alpha}^{\omega}$

$$\rho(E) \sim \Lambda_{IR} + \frac{d}{dE}\delta(E)$$

$\delta(E)$: PHASE SHIFT (COMPUTED INDEPENDENTLY)

ONE-LOOP FREE ENERGY

THERMAL AdS_3 : CONSIDER THE EUCLIDEAN AdS_3, AND PERIODICALLY IDENTIFY THE IMAGINARY TIME,

$$t \to t + \beta$$

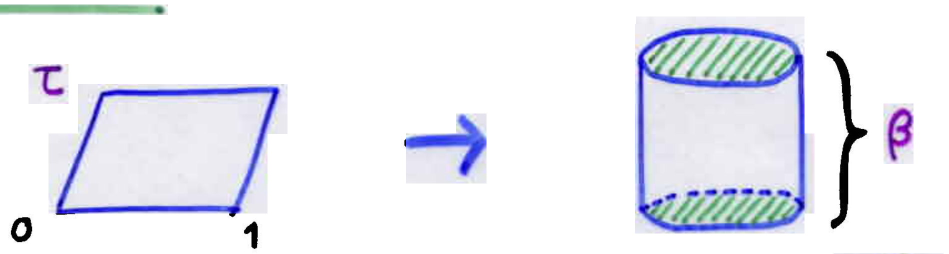
THE WORLDSHEET THEORY FOR THE EUCLIDEAN AdS_3,

$$S = \frac{k}{\pi} \int d^2 z \left(\partial\phi \bar{\partial}\phi + (\partial + \partial\phi)\bar{v} \cdot (\bar{\partial} + \bar{\partial}\phi) v \right),$$

IS EXACTLY SOLVABLE BY THE ITERATIVE GAUSSIAN INTEGRAL.

(GAWEDZKI, 1989)

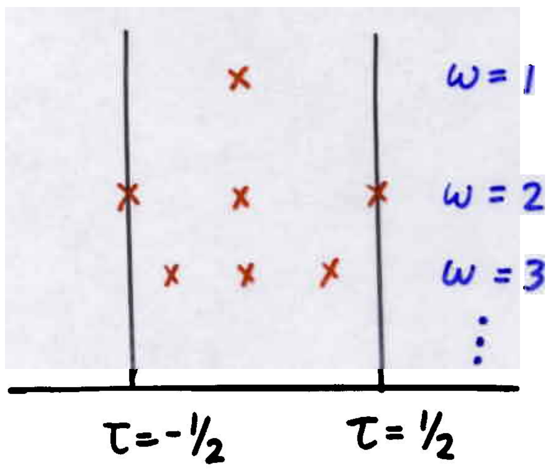
PATH INTEGRAL



WORLDSHEET TORUS → THERMAL AdS_3

THE AMPLITUDE $Z(\tau, \beta)$ HAS SINGULARITIES

$$\text{AT} \quad \tau = \frac{i\beta}{2\pi\omega}, \quad \frac{i\beta \pm 1}{2\pi\omega}, \quad \frac{i\beta \pm 2}{2\pi\omega}, \quad \cdots$$



$\Updownarrow$

$\exists$ HOLOMORPHIC MAP

THE WORLDSHEET CAN GROW INDEFINITELY LARGE.

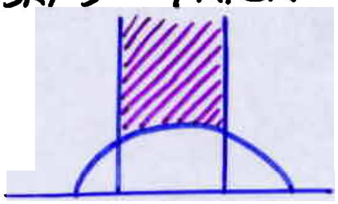
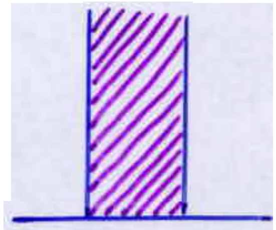
$$\int d^2\tau \underbrace{Z(\tau,\beta)}_{} = \sum_i \log\left(\frac{1}{1-e^{-\beta E_i}}\right)$$

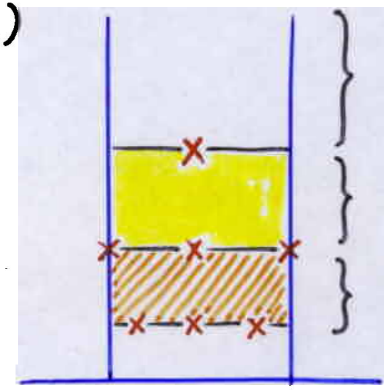
GAWEDZKI, 1991

E_i : SINGLE STRING SPECTRUM GIVEN IN HEP-TH/0001053.

TO VERIFY THIS,

(1) POLCHINSKI'S TRICK (1986)

$\sum_{(n,m)}$  $=$ $\sum_{(0,m)}$ 

(2) 

→ SHORT STRING WITH $w=0$

→ SHORT STRING WITH $w=1$

→ 〃 〃 WITH $w=2$

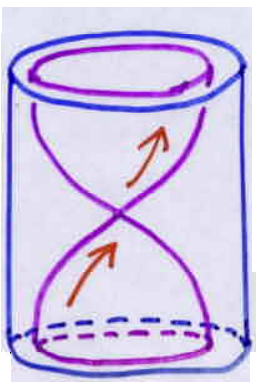
WITH THE CONSTRAINT

$$\frac{1}{2} < j < \frac{k-1}{2}$$

THE POLES AT ×

⇒ LONG STRING WITH CONTINUOUS SPECTRUM

THE DENSITY OF STATES : $\rho(E) \sim L_{IR} + \frac{d}{dE}\delta(E)$



$\delta(E)$ = PHASE SHIFT

AGREES WITH THE 2-POINT FUNCTION COMPUTED BY TESCHNER, (ZAMOLODCHIKOV)^2.

SL(2,R) W2W MODEL, LEVEL $k>2$

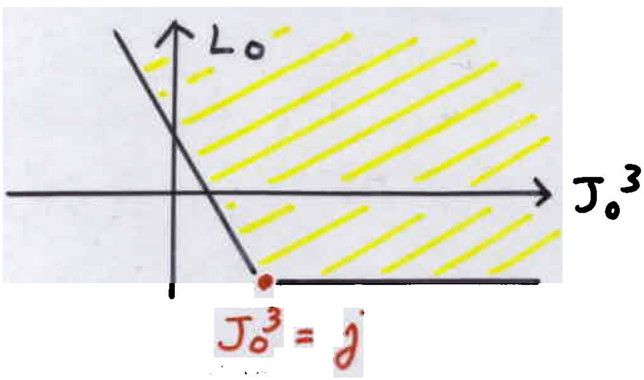
HILBERT SPACE

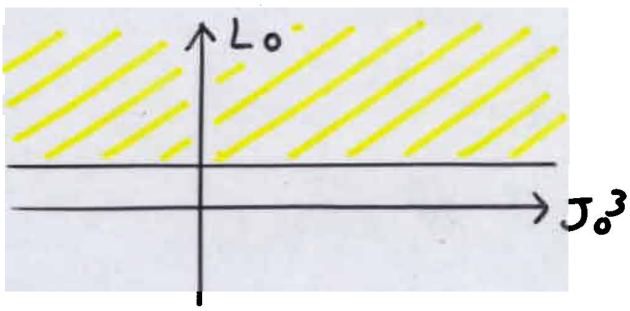
$$= \bigoplus_{\omega=-\infty}^{+\infty} \left[\int_{\frac{1}{2}}^{\frac{k-1}{2}} dj \; D_j^{\omega} \otimes D_j^{\omega} \oplus \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj \int_0^1 d\alpha \; C_{j,\alpha}^{\omega} \otimes C_{j,\alpha}^{\omega} \right]$$

D_j^{ω}, $C_{j,\alpha}^{\omega}$: IRREDUCIBLE REPRESENTATION OF $\widehat{SL}(2,\mathbb{R})$

$D_j^{\omega=0}$: DISCRETE REPRESENTATION

$C_{j,\alpha}^{\omega=0}$: CONTINUOUS REPRESENTATION





D_j^{ω} : SHORT STRING, $C_{j,\alpha}^{\omega}$: LONG STRING $\omega \neq 0$

ARE THEIR SPECTRAL FLOW IMAGES.

- THIS SPECTRUM WAS PROPOSED AND THE NO-GHOST THEOREM WAS PROVEN IN HEP-TH/0001053.
- THE SPECTRUM WAS CONFIRMED BY EXACT ONE-LOOP COMPUTATION IN HEP-TH/0005183.

SL(2, C) / SU(2) COSET MODEL

EUCLIDEAN ROTATION OF AdS_3

$$S = \frac{k}{\pi} \int d^2z \left(\partial\phi \bar{\partial}\phi + e^{2\phi} \partial\bar{\gamma} \bar{\partial}\gamma \right)$$

VERTEX OPERATOR

$$\Phi_j(z, x) = -\frac{2j-1}{\pi} \left(e^{-\phi(z)} + |\gamma(z) - x|^2 e^{\phi(z)} \right)^{-2j}$$

z : POINT ON THE WORLDSHEET

x : POINT ON THE BOUNDARY OF AdS_3

NORMALIZABLE $\longleftrightarrow$ $j = \frac{1}{2} + is$, $s \in \mathbb{R}$ • GAWEDZKI

CORRELATION FUNCTIONS OF THESE OPERATORS HAVE BEEN COMPUTED.

• TESCHNER

• FATEEV + (ZAMOLODCHIKOV)^2

THE SPECTRUM IS DIFFERENT FROM THAT OF THE SL(2, R) WZW MODEL.

- ONLY $\omega = 0$ SECTOR EXISTS
- NO DISCRETE REPRESENTATIONS

EXCEPT FOR THE TACHYON, ALL THE STATES IN THE SL(2, R) WZW MODEL CORRESPOND TO NON-NORMALIZABLE STATES IN THE SL(2, C) / SU(2) COSET MODEL.

FOR PHYSICAL OBSERVABLES IN STRING THEORY, CORRELATION FUNCTIONS IN EUCLIDEAN AdS_3 AND LORENTZIAN AdS_3 SHOULD BE RELATED BY ANALYTIC CONTINUATION.

WORLDSHEET 2-POINT FUNCTION

$$\langle \Phi_j(z,x)\,\Phi_{j'}(z',x')\rangle = B(j)\,\delta(j-j')\,|z-z'|^{-4\Delta(j)}\,|x-x'|^{-4j}$$

$$B(j) = \frac{k-2}{\pi}\,\nu^{1-2j}\,\frac{\Gamma\left(\frac{k-1-2j}{k-2}\right)}{\Gamma\left(\frac{2j-1}{k-2}\right)}$$

ν : CONST $= \pi\,\dfrac{\Gamma\left(1-\frac{1}{k-2}\right)}{\Gamma\left(1+\frac{1}{k-2}\right)}$

EUCLIDEAN MODEL : $j = \frac{1}{2} + is$

$\Downarrow$

LORENTZIAN MODEL : $j \in \mathbb{R}$ FOR $D_j^{\omega=0}$.

$B(j)$ IS REGULAR AND POSITIVE FOR $\frac{1}{2} < j < \frac{k-1}{2}$, IN AGREEMENT WITH THE SPECTRUM OF THE $SL(2,\mathbb{R})$ W2W MODEL.

$B(j)$ HAS A POLE AT $j = \frac{k-1}{2}$.

FROM THE POINT OF VIEW OF WORLDSHEET, THIS IS DUE TO LARGE WORLDSHEET INSTANTONS.

TARGET SPACE 2-POINT FUNCTION

$$\langle \Phi_j(x)\, \Phi_j(x') \rangle_{\text{TARGET}} = (2j-1)\, B(j)\, |x-x'|^{-4j}$$

FOR SHORT STRING WITH $\omega = 0$.

- THE EXTRA FACTOR $(2j-1)$ COMES FROM REGULARIZING

$$\frac{\infty}{\infty} \quad \begin{matrix} \leftarrow \delta(j-j')|_{j=j'} \\ \nwarrow \text{VOL (CONFORMAL GROUP)} \end{matrix}$$

- THIS FACTOR CAN ALSO BE DERIVED FROM THE WARD IDENTITY IN TARGET SPACE.

$$\langle J(x)\, \Phi_j(x_1)\, \Phi_j(x_2) \rangle \leftarrow \text{UNAMBIGUOUS}$$

$$= \sum_{a=1,2} \frac{Q_a}{x - x_a} \langle \Phi_j(x_1)\, \Phi_j(x_2) \rangle$$

THE VERTEX OPERATOR FOR $J(x)$ WAS DEFINED BY KUTASOV AND SEIBERG.

THIS IS A STRING THEORY VERSION OF THE RESULT BY FREEDMAN, MATHUR, MATUSIS AND RASTELLI.

$$\boxed{\begin{array}{l}\text{THE FACTOR } (2j-1) \text{ IS NECESSARY} \\ \text{FOR THE FACTORIZATION OF 4-POINT FUNCTION} \\ \text{TO WORK PROPERLY.}\end{array}}$$

TARGET SPACE 2-POINT FUNCTION FOR $\omega \neq 0$

$(J, \bar{J})$: CONFORMAL WEIGHTS IN THE BOUNDARY CFT_2

- SHORT STRING $\quad D_j^{\omega} \otimes D_j^{\omega}, \quad \frac{1}{2} < j < \frac{k-1}{2}$

$$\langle \Phi^{\omega j}_{J\bar{J}}(x)\, \Phi^{\omega j}_{J\bar{J}}(x') \rangle$$

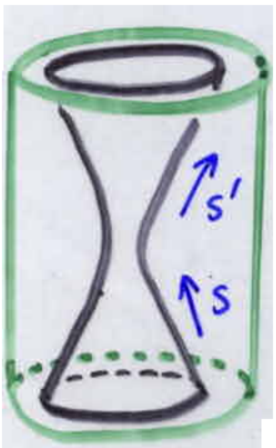
$$= |2j-1+(k-2)\omega| \cdot \frac{\Gamma(J+j-\frac{k}{2}\omega)\,\Gamma(\bar{J}+j-\frac{k}{2}\omega)}{(J-j-\frac{k}{2}\omega)!\,(\bar{J}-j-\frac{k}{2}\omega)!\,\Gamma(2j)^2} \cdot \frac{B(j)}{(x-x')^{2J}(\bar{x}-\bar{x}')^{2\bar{J}}}$$

- LONG STRING $\quad C_{j,\alpha}^{\omega} \otimes C_{j,\alpha}^{\omega}, \quad j = \frac{1}{2} + is$

$$\langle \Phi^{\omega j}_{J\bar{J}}(x)\, \Phi^{\omega j}_{J\bar{J}}(x') \rangle$$

$$= \left[\delta(s+s') + e^{i\delta(s)}\, \delta(s-s') \right] \cdot \frac{1}{(x-x')^{2J}(\bar{x}-\bar{x}')^{2\bar{J}}}$$

$$e^{i\delta(s)} = - \frac{\Gamma(-\frac{2is}{k-2})\,\Gamma(-2is)\,\Gamma(\frac{1}{2}+is+J-\frac{k}{2}\omega)\,\Gamma(\frac{1}{2}+is-\bar{J}+\frac{k}{2}\omega)}{\Gamma(+\frac{2is}{k-2})\,\Gamma(+2is)\,\Gamma(\frac{1}{2}-is+J-\frac{k}{2}\omega)\,\Gamma(\frac{1}{2}-is-\bar{J}+\frac{k}{2}\omega)}\, \nu^{-2is}$$



PHASE SHIFT

WE WILL USE THESE EXPRESSIONS TO STUDY THE FACTORIZATION OF 4-POINT FUNCTIONS.

3 POINT FUNCTION

$$- \frac{G(1-j_1-j_2-j_3)\,G(j_3-j_1-j_2)\,G(j_2-j_3-j_1)\,G(j_1-j_2-j_3)}{2\pi^2\,\nu^{j_1+j_2+j_3+1}\,\gamma\left(\frac{k-1}{k-2}\right)G(-1)\,G(1-2j_1)\,G(1-2j_2)\,G(1-2j_3)}$$

WHERE

$$G(j) \equiv (k-2)^{\frac{j(k-j-1)}{k-2}}\,\Gamma_2(-j\,|\,1,k-2)\,\Gamma_2(k-1+j\,|\,1,k-2)$$

BARNES DOUBLE GAMMA FUNCTION

$$\Gamma_2(x\,|\,1,\omega) \sim \text{“}\, x^{-1} \prod_{n,m=0}^{\infty}{}' \left(1+\frac{x}{n+m\omega}\right)^{-1} \text{”}$$

(C.F. $\Gamma(x) \sim \text{“}x^{-1}\prod_{n=1}^{\infty}\left(1+\frac{x}{n}\right)^{-1}\text{”}$ WEIERSTRASS)

THIS EXPRESSION IS FOR $\omega = 0$.

WE HAVE GENERALIZED THIS FOR $\omega \neq 0$.

$G(j)$ HAS POLES AT $j = \begin{cases} n + m(k-2) \\ -(n+1) - (m+1)(k-2) \end{cases}$

$n, m = 0, 1, 2, \cdots$

① SINGULARITIES AT $j_1 + j_2 = j_3 + n$, $n = 0, 1, 2, \cdots$

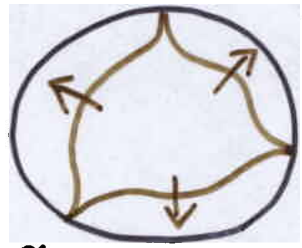
INTERPRETED AS DUE TO MIXING OF 1 PARTICLE STATE WITH 2 PARTICLE STATE.

$j_1 + j_2 > j_3$

$j_1 + j_2 = j_3$

② SINGULARITY AT $j_1 + j_2 + j_3 = k$

- FROM THE WORLDSHEET POINT OF VIEW, IT IS DUE TO LARGE WORLDSHEET INSTANTONS.



VERTEX OPERATOR $\sim e^{2j\phi}$

$e^{-(\text{INSTANTON ACTION})} \sim e^{-2k\phi}$

- FROM THE TARGET SPACE POINT OF VIEW, THE DIVERGENCE IS NON-LOCAL IN THE BOUNDARY CFT_2.

  ... SHOULD WE ADD NON-LOCAL COUNTER TERMS?

  — NO, WE DO NOT RENORMALIZE THIS DIVERGENCE.

- THE BOUNDARY CFT_2 IS A LOCAL QFT.
- THE DIVERGENCE IS DUE TO THE NON-COMPACTNESS OF THE TARGET SPACE OF CFT_2.

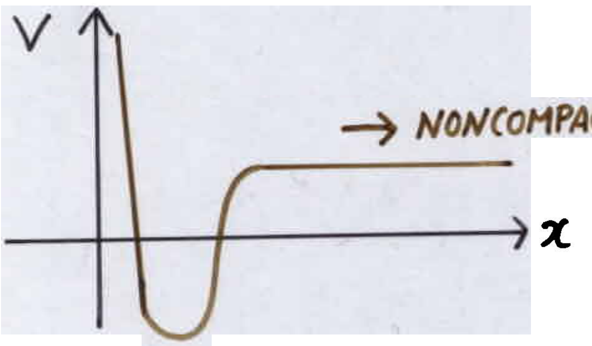
  TARGET SPACE = MODULI SPACE OF YM INSTANTONS

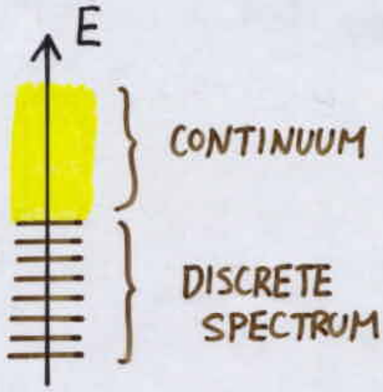
  NON-COMPACTNESS ↑ SMALL YM INSTANTONS

THIS HAPPENS IN A SIMPLER QM MODEL ALSO.

A SIMPLE MODEL WITH SIMILAR BEHAVIOR

$$\left[-\frac{1}{2}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\,\psi(x)$$





THE VACUUM WAVE FUNCTION $\psi_0(x) \sim e^{-\frac{K}{2}x}$

CORRELATION FUNCTIONS OF OPERATORS $e^{\lambda x}$

$$\langle \psi_0 | e^{\lambda_1 x(t_1)} e^{\lambda_2 x(t_2)} \cdots e^{\lambda_m x(t_m)} | \psi_0 \rangle$$

IS WELL-DEFINED ONLY IF $\lambda_1 + \lambda_2 + \cdots + \lambda_m < K$.

OTHERWISE THE AMPLITUDE IS DIVERGENT.

- THE DIVERGENCE IS NON-LOCAL IN t
- WE DO NOT TRY TO RENORMALIZE THE DIVERGENCE.

OPERATORS OF THE FORM $e^{\lambda_1 x(t_1)} e^{\lambda_2 x(t_2)} \cdots$

CAN TAKE ψ_0 OUT OF THE HILBERT SPACE.

4-POINT FUNCTION

$$\mathcal{F}_{TARGET}(x,\bar{x}) = \int d^2z \underbrace{\mathcal{F}_{WORLDSHEET}(z,\bar{z};x,\bar{x})}$$

$$\parallel$$

$$\mathcal{F}_{SL(2)}(z,\bar{z};x,\bar{x})\,\mathcal{F}_{INTERNAL}(z,\bar{z})$$

x: CROSS RATIO OF 4-POINTS ON THE BOUNDARY OF AdS_3

z: CROSS RATIO OF 4-POINTS ON THE WORLDSHEET.

FOR NORMALIZABLE OPERATORS IN $SL(2,\mathbb{C})/SU(2)$,

i.e. $j_1 = \frac{1}{2} + is_1$, ...

$$\mathcal{F}_{SL(2)}(z,\bar{z};x,\bar{x}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\, C(j)\, |\mathcal{F}_j(z,x)|^2$$

FACTORIZATION ON THE WORLD SHEET

- $C(j) = C(j_1,j_2,j)\cdot \dfrac{1}{B(j)}\cdot C(j,j_3,j_4)$

- $\mathcal{F}_j(z,x)$: CONFORMAL BLOCK.

  UNIQUELY DETERMINED BY

  - THE KNIZHNIK-ZAMOLODCHIKOV EQUATION
  - THE BOUNDARY CONDITION FOR $z, x \to 0$

$$\mathcal{F}_j(z,x) \sim z^{\Delta(j)-\Delta(j_1)-\Delta(j_2)}\, x^{j-j_1-j_2}$$

TO DO :

① PERFORM THE j - INTEGRAL

TO OBTAIN $\mathcal{F}_{SL(2)}(z,\bar{z};x,\bar{x})$

② ANALYTICALLY CONTINUE THE RESULT,

$$j_1 = \tfrac{1}{2} + i s_1 \quad \rightarrow \quad \tfrac{1}{2} < j_1 < \tfrac{k-1}{2}$$

$$j_2 = \tfrac{1}{2} + i s_2 \quad \rightarrow \quad \tfrac{1}{2} < j_2 < \tfrac{k-1}{2}$$

$$\cdots$$

SOME POLES IN $C(j)$ CROSS THE CONTOUR OF THE j- INTEGRAL IN ①.

$\Rightarrow$ WE NEED TO TAKE INTO ACCOUNT THE POLE RESIDUES.

(NO POLES IN $\mathcal{F}_j$ CROSS THE CONTOUR.)

③ PERFORM THE z - INTEGRAL.

z - INTEGRAL

A NOVEL FEATURE : SINGULARITY AT $z = x$

AS WELL AS AT $z = 0, 1, \infty$.

THE KNIZHNIK-ZAMOLODCHIKOV EQUATION IMPLIES

$$\boxed{\mathcal{F}_j(z, x) \sim (z - x)^{k - j_1 - j_2 - j_3 - j_4}}$$

WHEN $z = x$, THERE IS A HOLOMORPHIC MAP

FROM THE WORLDSHEET TO THE BOUNDARY OF AdS_3

$\Rightarrow$ LARGE WORLDSHEET INSTANTONS

A SEMI-CLASSICAL ANALYSIS $\sim |z - x|^{2(k - j_1 - j_2 - j_3 - j_4)}$

- $\mathcal{F}_{SL(2)}(z, x)$ IS MONODROMY INVARIANT AROUND $z = x$,

  AFTER THE j-INTEGRAL.

- THE z-INTEGRAL CAN BE EVALUATED EXPLICITLY,

  USING FORMULAE SUCH AS,

$$\int d^2z \, z^{d-1} \bar{z}^{\bar{d}-1} \left[|F(a, b, c : z)|^2 - \frac{\gamma(c)^2 \gamma(a-c+1) \gamma(b-c+1)}{(1-c)^2 \gamma(a) \gamma(b)} \times |F(1+b-c, 1+a-c, 2-c : z)|^2 \right]$$

$$= \pi \frac{\Gamma(d) \Gamma(a - \bar{d}) \Gamma(b - \bar{d}) \Gamma(1 - c + d) \, \gamma(c)}{\Gamma(1 - \bar{d}) \Gamma(1 - a + d) \Gamma(1 - b + d) \Gamma(c - \bar{d}) \gamma(a) \gamma(b)}$$

FINAL RESULT

4-POINT FUNCTION OF SHORT STRINGS WITH $\omega = 0$

$$\frac{1}{2} < j_1, j_2, j_3, j_4 < \frac{k-1}{2}$$

IF $j_1 + j_2 < \frac{k+1}{2}$, $j_3 + j_4 < \frac{k+1}{2}$ ARE SATISFIED.

$$\mathcal{F}_{TARGET}(x, \bar{x}) = \int d^2z\, \mathcal{F}_{WORLDSHEET}(z, \bar{z}; x, \bar{x})$$

$$= \sum_{\substack{\text{SHORT STRINGS} \\ \text{WITH } \omega = 0}} \frac{(3\,\text{POINT})(3\,\text{POINT})}{(2\,\text{POINT})} x^{J - j_1 - j_2} \bar{x}^{\bar{J} - j_1 - j_2}$$

$$+ \int_{\substack{\text{LONG STRINGS} \\ \text{WITH } \omega = 1}} \frac{(3\,\text{POINT})(3\,\text{POINT})}{(2\,\text{POINT})} x^{J - j_1 - j_2} \bar{x}^{\bar{J} - j_1 - j_2}$$

$$+ \left[\begin{array}{l} \text{CONTRIBUTION} \\ \text{FROM 2-PARTICLE STATES} \end{array} \right]$$

THE COEFFICIENTS OF THE x-EXPANSION OF $\mathcal{F}_{TARGET}(x, \bar{x})$ EXACTLY AGREE WITH THE COMPUTATION OF TARGET SPACE 2 AND 3 POINT FUNCTIONS, INCLUDING THE FACTOR $(2j-1)$ FOR SHORT STRING AND THE PHASE SHIFT $e^{i\delta(s)}$ FOR LONG STRING.

COMMENTS

(1) SELECTION RULES

- CONSTRAINTS DUE TO AFFINE $SL(2,\mathbb{R})$ SYMMETRY

$$\boxed{\begin{array}{l} (\text{SHORT STRING}, \omega=0) \otimes (\text{SHORT STRING}, \omega=0) \\ = (\text{SHORT STRING}, \omega=0,1) \oplus (\text{LONG STRING}, \omega=1) \end{array}}$$

- NORMALIZING TARGET SPACE 2 POINT FUNCTION FINITE, THE 3 POINT FUNCTION OF SHORT STRINGS WITH $\omega_1=0$, $\omega_2=0$, $\omega_3=1$ VANISHES.

(2) WHAT HAPPENS WHEN $j_1+j_2 > \frac{k+1}{2}$?

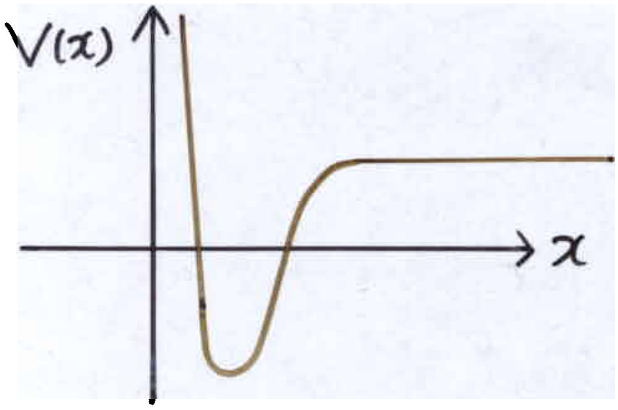
THE ANALYTIC CONTINUATION,

$$j_{1,2} = \frac{1}{2} + i s_{1,2} \Rightarrow \frac{1}{2} < j_1, j_2 < \frac{k-1}{2},$$

PICKS UP ADDITIONAL POLE RESIDUES FROM THE j-INTEGRAL AND GENERATES TERMS THAT CANNOT BE INTERPRETED AS DUE TO EXCHANGES OF PHYSICAL STATES OF STRING.

<u>FACTORIZATION FAILS.</u>

THIS ALSO HAPPENS IN THE SIMPLE QM MODEL



VACUUM WAVE FUNCTION

$$\psi_0(x) \sim e^{-\frac{K}{2}x}$$

$$\lambda_1 + \lambda_2 > \frac{K}{2}$$

$$\Rightarrow \; e^{\lambda_1 x(t_1)} e^{\lambda_2 x(t_2)} |\psi_0\rangle \notin \text{HILBERT SPACE}$$

THEREFORE

$$\langle \psi_0 | e^{\lambda_3 x(t_3)} e^{\lambda_4 x(t_4)} \uparrow e^{\lambda_1 x(t_1)} e^{\lambda_2 x(t_2)} |\psi_0\rangle$$

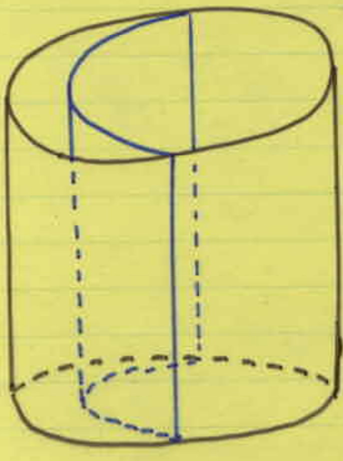
WE <u>CANNOT</u> INSERT $\mathbb{1} = \sum_a |a\rangle\langle a|$

> THE FAILURE OF FACTORIZATION
> OF THE 4-POINT FUNCTION FOR $j_1 + j_2 > \frac{k+1}{2}$
> IS CORRECT PHYSICS FOR THIS STRING THEORY.

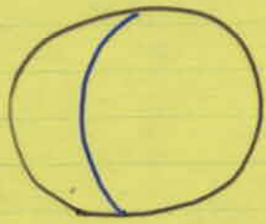
AdS_2 BRANES IN AdS_3

WITH P. LEE AND J.-W. PARK, HEP-TH / 0106129
0112188

c.f. PONSOT, SCHOMERUS, AND TESCHNER, HEP-TH /0112198.



SPACELIKE SECTION



$\psi = \psi_0$

$$ds^2 = d\psi^2 + \cosh^2\psi \, d\theta^2$$

THE BOUNDARY STATE IS DETERMINED BY

THE ONE-POINT FUNCTION.

$$\langle \Phi_j(x, z) \rangle = \begin{cases} \dfrac{U^+(j)}{|x-\bar{x}|^{2j}\,|z-\bar{z}|^{2\Delta_j}} & \mathrm{Im}\, x > 0 \\[2ex] \dfrac{U^-(j)}{|x-\bar{x}|^{2j}\,|z-\bar{z}|^{2\Delta_j}} & \mathrm{Im}\, x < 0 \end{cases}$$

$$U^{\pm}_{\psi_0}(j) = \Gamma\left(1 - \frac{2j-1}{k-2}\right) \nu^{\frac{1}{2}-j} e^{\pm \psi_0 (2j-1)}$$

CONCLUSION

WE HAVE REACHED COMPLETE UNDERSTANDING OF THE TREE-LEVEL STRING THEORY IN AdS_3 WITH A BACKGROUND NS-NS 2-FORM.

- SINGLE STRING SPECTRUM IS DERIVED EXACTLY.
  NO-GHOST THEOREM HOLDS.
- PRESCRIPTION FOR ANALYTIC CONTINUATION
  $SL(2,\mathbb{C})/SU(2) \to SL(2,\mathbb{R})$
- CORRELATION FUNCTIONS HAVE SINGULARITIES, AND WE HAVE FOUND PHYSICAL INTERPRETATIONS FOR ALL OF THEM, BOTH ON THE WORLDSHEET AND IN THE TARGET SPACE.
- 4-POINT FUNCTIONS FACTORIZES, WHEN THEY SHOULD.

$$\mathcal{F}_{\text{TARGET}} = \sum \frac{(\text{3 POINT})(\text{3 POINT})}{(\text{2 POINT})}$$

  - THE COEFFICIENTS MATCH COMPLETELY.
  - WE UNDERSTAND WHEN FACTORIZATION FAILS AND WHY.