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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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Flux compactifications and brane cosmology

Part II

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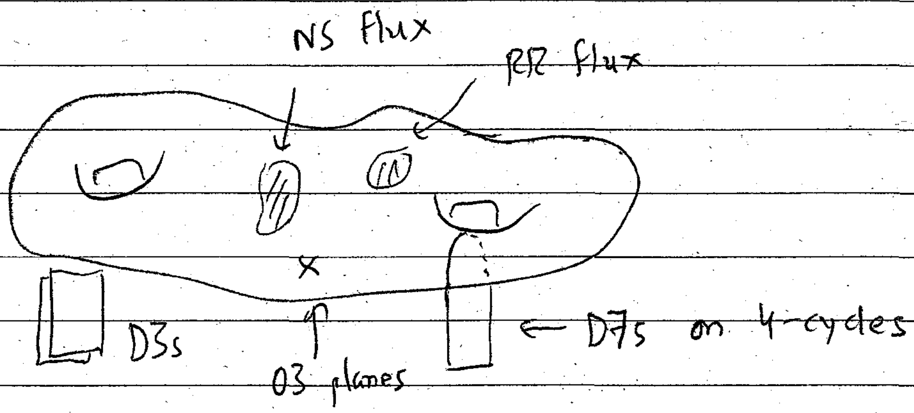
Last time, we developed a 4d description of a class of IIB vacua:



Today we'll:

- Describe how one can compute the vacuum structure in simple examples
- Discuss the more "global" picture of {vacua} that is emerging

I. Examples

The easiest examples to study are orientifolds of reduced holonomy CYs, eg $T^6/\mathbb{Z}_2$ or $K3 \times T^2/\mathbb{Z}_2$.

You can find extensive discussion of vacua in those cases in hep-th/0201028 and hep-th/0301139.

We'll focus on a more "generic" compact CY_3 example. Consider for M_6 the hypersurface in $W\mathbb{P}^4_{1,1,1,1,4}$, w/ defining equation

$$M_6 : z_1^8 + z_2^8 + z_3^8 + z_4^8 + 4 z_5^2 - 8\psi (z_1 \cdots z_4)^2 = 0$$

It has $h^{1,1} = 1$, $h^{2,1} = 149$. [So in addition to "ψ" $\exists$ 148 other monomials we could use to non-redundantly deform defining eqn.]

$\mathbb{Z}_2$ symmetry : $(z_5 \to -z_5) \circ$ (WS orient reversal)

"How much" flux are we allowed to turn on?

M_6 arises as a limit of F-theory compactification on hypersurface $\in W\mathbb{P}^5_{1,1,1,1,8,12}$ [as in Sen's prescription hep-th/9702165] with $\boxed{\frac{\chi}{24} = 972}$.

③

Now, we will wish to compute

$$W = \int (F - \tau H) \wedge \Omega$$

and find vacua. Do we <u>have</u> to consider turning on all of 300 possible NS & RR fluxes, & compute W as a function of 150 variables?

<u>No</u>. M_6 is invariant under a discrete symmetry group

$$G = \mathbb{Z}_8 \times \mathbb{Z}_8 \times \mathbb{Z}_2 \qquad \begin{aligned} g_1 &= (1\,0\,0\,7\,0) \\ g_2 &= (1\,0\,7\,0\,0) \\ g_3 &= (1\,7\,0\,0\,0) \end{aligned} \qquad g_1^2 g_2^2 g_3^2 \text{ trivial in the } WP^4 \ldots$$

$$z_1 \to e^{2\pi i/8} z_1$$
$$z_2 \to e^{2\pi i\, 7/8} z_2 \quad \text{etc.}$$

<u>All</u> moduli of the complex structure <u>except</u> ψ transform nontrivially. [Quotienting $M_6/G \to$ the mirror manifold!]

- So M_6 has <u>4</u> <u>periods</u> which (up to $|\text{ord } G|$) coincide w/ those of mirror on this locus, and depend <u>only</u> on <u>G invariant</u> combos of other moduli!

- So, if we turn on only fluxes consistent with the G symmetry [4 each of RR + NS], low order (eg linear) terms in G charged moduli ψ^i <u>cannot</u> appear in W. So if we find a vacuum for φ + τ on this locus, can set $\psi^i = 0$ + get a full solution (where ψ_i fields generically fixed at 0 by higher order W).

Note that the $T^6/\mathbb{Z}_2$ + $K3 \times T^2/\mathbb{Z}_2$ cases have been studied with W(all moduli); calculations are much easier there ...

Now, focus on the 4d subspace of $H_3(M_6)$ of interest.

- Basis of 3-cycles A_a, B^a + integral cohomology α_a, β^a $(a=1,2)$ satisfies:

⑤

$$\int_{A_a} \alpha_b = \delta^a_b \qquad \int_{B^b} \beta^a = -\delta^a{}_b$$
$$\int_{M_6} \alpha_a \wedge \beta^b = \delta^b_a$$
symplectic basis

In this basis the holomorphic 3-form Ω is:

$$\Omega = z^a \alpha_a - G_a \beta^a$$

$$\int_{A_a} \Omega = z^a \quad , \quad \int_{B^a} \Omega = G_a$$

And: $\displaystyle\int_{M_6} \Omega \wedge \bar{\Omega} = \bar{z}^a G_a - z^a \bar{G}_a = -\Pi^\dagger \Sigma \Pi$

$\Pi \equiv$ period vector $\qquad \Sigma = \begin{pmatrix} 0_{2\times2} & \mathbb{1}_{2\times2} \\ -\mathbb{1}_{2\times2} & 0_{2\times2} \end{pmatrix}$

$\Downarrow$

$\Pi^T = (G_1, G_2, z^1, z^2)$

Choose fluxes:

$F_3 = f_1\beta_1 + f_2\beta_2 + f_3\alpha_1 + f_4\alpha_2$

$H_3 = h_1\beta_1 + h_2\beta_2 + h_3\alpha_3 + h_4\alpha_4$

Then the amount of D3 charge $\in$ the fluxes is:

⑥

$$N_{flux} = \int H_3 \wedge F_3 = \vec{f}^T \Sigma \vec{h}$$

The <u>superpotential</u> is given by

$$W = \int_M (F_3 - \tau H_3) \wedge \Omega = \vec{f} \cdot \vec{\Pi} - \tau \vec{h} \cdot \vec{\Pi}$$

And the Kähler potential is

$$K = K_\tau + K_{cs} = -\ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(i \int \Omega \wedge \bar{\Omega}\right)$$

$$= -\ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i \vec{\Pi}^\dagger \Sigma \vec{\Pi}\right)$$

What is $\vec{\Pi}(\psi)$? Following Klemm + Theisen (hep-th/9205041), define the "fundamental period"

$$\omega_0(\psi) = \frac{(2\pi i)^3 \pi}{8} \sum_{n=1}^{\infty} \frac{1}{\Gamma(n) \prod_{i=0}^{4} \Gamma\left(1 - \frac{n}{8}\nu_i\right)} \frac{e^{i\frac{7n\pi}{8}}}{\sin\left(\frac{n\pi}{8}\right)} (4\psi)^n$$

Here $\nu_i = 1,1,1,1,4$; and this is valid for $|\psi| < 1$.

In terms of ω_0, a basis for all 4 periods is given by

$$\vec{\omega}^T = \frac{1}{\psi}\left[\omega_0(\alpha^2\psi), \omega_0(\alpha\psi), \omega_0(\psi), \omega_0(\alpha^7\psi)\right]$$

where $\alpha \equiv \exp\left(+\frac{i\pi}{4}\right)$. The "$\frac{1}{\psi}$" just changes gauge $\Omega \to \frac{1}{\psi}\Omega$ to avoid $\Omega \to 0$ as $\psi \to 0$.

We actually <u>want</u> the periods in a symplectic basis. In the $\vec{\Pi}$ variables, one finds:

$$\vec{\Pi} = m \cdot \vec{\omega}$$

$$\vec{m} \equiv \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

So in principle the problem (on this 1d slice of the 149d cplx moduli space, w/ only 4 of 300 fluxes turned on!) is simple.

- Choose $\vec{f}, \vec{h}$
- Then $W = \vec{f} \cdot \vec{\Pi} - \tau \vec{h} \cdot \vec{\Pi}$
- Solve $D_\tau W = D_\psi W = 0$.

Although this sounds easy (and is in eg $T^6/\mathbb{Z}_2$), it is tough here. In hep-th/0312104, we found:

- Explicit nonsusy sol'ns at $\psi = 0$, some with small $e^{K}|W|^2 \sim 10^{-5}$
- Explicit SUSY soln's in another CY_3 (a bit more complicated).

<u>Natural Questions</u>:

1 - How many solutions?

2 - Where on $\mathcal{M}_\psi$? (moduli space)

3 - How small can $\cancel{SUSY}$ scale $e^{K/2}|W|$ be?

...

<u>1 - How many solutions?</u>

A natural model (along lines of Bousso-Polchinski)

Note that on a <u>solution</u> of flux eqns:

$$N_{flux} \sim \int H \wedge F \underset{\substack{\text{ISD} \\ \text{condition}}}{\sim} \geq 0$$

$$= \underbrace{\frac{\chi(X_4)}{24}}_{\text{in F-Thy}}$$

And (assuming no $\overline{D3}$s) $N_{flux} \lesssim$ "N_{D3}"

Replacing N_{flux} by a simple quadratic positive form in fluxes $\Rightarrow$

$$\sum f_i^2 + h_i^2 \leq N_{D3}$$

A sphere of radius $\sqrt{N_{D3}}$ in $2b_3$ dimensions

$\Rightarrow$ assuming $\exists$ 1 vacuum per integral flux choice (+ approximating # integral pts by vol. of sphere):

$$\# \text{ vacua} \sim (N_{D3})^{b_3} \sim \left(\frac{\chi(X_4)}{24}\right)^{b_3}$$

Typical values of χ, $b_3 \Rightarrow$ very large #.

Much larger than eg # of known CY 3-folds ($\sim 10^4$).

We argued last time that generically after no scale breaking, these can $\to$ AdS vacua.

2 - Where on $\mathcal{M}_\psi$?

This does not follow from a simple B-P type of argument. However, Douglas has conjectured that

$$\#\ \text{vacua} \ \propto \int_{\mathcal{F}} \det(-R - \omega).$$

i.e. density in fundamental domain $\mathcal{F}$ of moduli space is given by det of Kähler form on $\mathcal{M}$ + curvature tensor.

$$K = -\ln\left[-i\,\vec{\Pi}^{\dagger}\,\Sigma\,\vec{\Pi}\right] \implies$$

we can compute prediction.

How to check? In the model described earlier

We have started running computer code:

- Randomly select f_i, h_i subject to

$$\vec{f}^T \Sigma \vec{h} \leq 972$$

- Solve flux eqns (in $|\psi|<1$ region)
- Plot $N_{vacua}(|\psi| \leq |\psi_0|)$ vs $|\psi_0|$ + compare to conjecture!

In fact, for $|\psi|<1$ we find the prediction is

$$\int_{\mathcal{F}} \det(-R-\omega) \quad \propto \quad \int d\psi \, d\bar{\psi} \; |\psi|^2 \quad \Rightarrow$$

predict $\qquad N_{vacua}(|\psi| \leq |\psi_0|) \sim |\psi_0|^4$

The predictions are matched <u>very well</u> by our random selection of $\sim 10^6$ vacua!

But clearly, this "taxonomy" of vacua is in it's infancy.

3 - ~~SUSY~~ scales?

Recall the generic expectation is

$$W_{total} = W_{Flux}(z_\alpha, \tau) + W_{np}(\rho) + \cdots$$

$\Rightarrow$ below scale α'/R^3

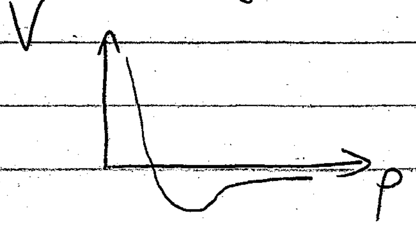
$$W = W_0 + A e^{ia\rho} + \cdots$$

<u>Suppose</u> one can achieve very small values of W_0 by tuning $\in$ large space of flux choices. Then: the soln to

$D_\rho W = 0$ can be at large ρ $\Rightarrow$ trust "sugra"! (i.e., trust that soln exists)

Note small $\underline{\underline{a}}$ also helps ($\rightarrow$ SU(N) gaugino cond. for large N).

Can we find a way to add a bit of energy to



to find dS space?

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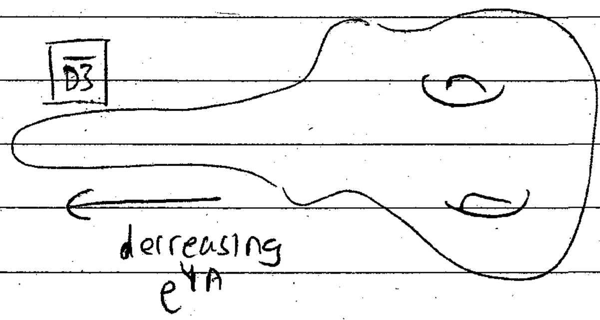
Several very plausible suggestions:

i) ~~SUSY~~ by anti-branes

Consider a model w/ significant warping

e.g. Klebanov-Strassler throat $\left\{ e^{4A} \ll 1 \right.$

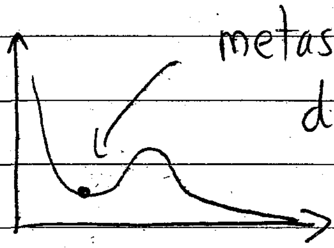


$e^{4A} \sim O(1)$

Satisfy tadpole w/ an extra $\overline{D3}$ + go through same analysis $\Rightarrow$

$$\delta V \sim T_{D3}\, e^{4A}$$

Can hope to arrange so



The arguments that this requires only plausible tuning of W_0, e^{A} are $\in$ KKLT.

We'll describe how to make such significantly warped models (one special way) next time.

ii) Just "start" with vacuum of no-scale V with

$\partial_{z_\alpha} V = \partial_\tau V = 0$ $\quad V > 0$ of course

$\partial_\rho V \neq 0$ (tadpole)

Then the $W_{np}(\rho)$ can play off against ρ tadpole $\Rightarrow$ ρ stabilization, plausibly in dS. (See Saltman/Silverstein)

iii) Multiple gaugino condensates (complicated $W(\rho_a)$ as a function of ≥ 2 fields).

So it is believed that the flux discretuum has a huge # of approximate vacua; by tuning:
- can get dS vacua
- Can get them into regime of control
- Can get (tunneling) lifetime >> string time

(15)

References : Same as last time, plus

Giryavets, Kachru, Tripathy, Trivedi hep-th/0312104

Saltman, Silverstein hep-th/0402135

Ashok, Douglas hep-th/0307049