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IN SUPERSYMMETRIC GAUGE THEORIES**

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# NON-RENORMALIZATION THEOREMS IN SUSY GAUGE THEORIES (Part I)

**PhilipArgyres**
**University of Cincinnati, U.S.A.**

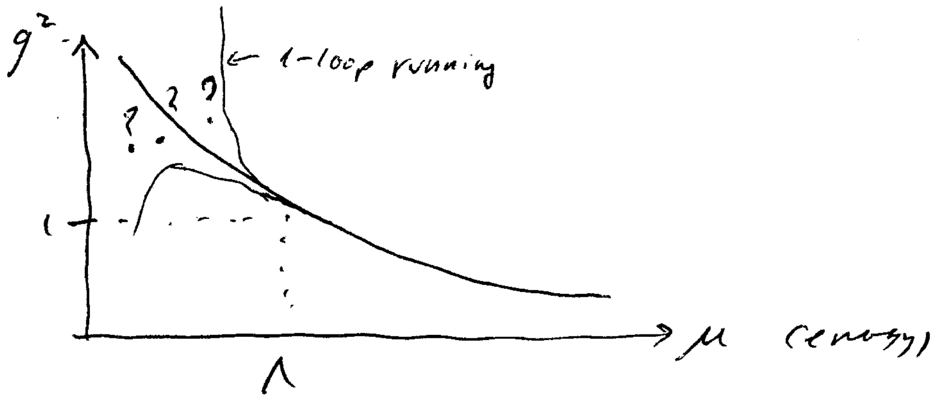
strada costiera, 11 - 34014 trieste italy - tel. +39 0402240111 fax +39 040224163 - sci_info@ictp.trieste.it - www.ictp.trieste.it

# Non-renormalization Theorems in SUSY Gauge Theories

- The problem:

  Determine the long wave-length physics that results from a given AF gauge theory.

  It is hard since, though weakly coupled at short distances, (UV) it is strongly coupled in IR:




  Quite different physics can describe low energies than UV:

  e.g. massless QCD: { in UV is free theory of quarks & gluons
  in IR is free theory of pions & (massive) glueballs
  & for $E \sim \Lambda$ (100's MeV), is complicated.

- The strategy
  - make educated guess for long-distance fields & symmetries
  - write down all possible effective actions built w/ these fields & consistent w/ symmetries, constrained by selection rules & weak coupling limits.
  - check your original guess somehow. using predictions from the effective action.

  (Ancient strategy in CM physics.)

  Works very well in SUSY theories because of powerful selection rules, which is what I want to explain:

- "non-renormalization theorem" has come to mean (somewhat imprecisely) these selection rule constraints on low-energy effective actions.
  - will use superspace technology of S. R-D.'s lectures.
  - the use of these NR theorems to solve for the exact (non-perturbative) low-energy action will be the subject of next week's lectures.
  - Will stick mostly to $d=4$, $N=1$ SUSY; but applies equally to many $d$'s, $N$'s & to sugra.

- Outline:
  1. Effective actions
  2. Symmetries & selection rules
  3. NR thm for chiral super-fields
  4. Anomalies
  5. NR thm for vector superfields

## 1. Effective Actions (Wilsonian) $S_\mu$

> $S_\mu = \int d^4x\, \mathcal{L}_\mu(\phi)$ describes physics at energies $E < \mu$. Obtained by averaging over ("integrating out") the short distance fluctuations of the fields on length scales $x < \frac{1}{\mu}$.

- Local, on scales $x \gtrsim \frac{1}{\mu}$.
- Unitary for processes involving energies $E < \mu$.
- For $E \sim \mu$, classical couplings (tree level) in $S_\mu$ describe effective couplings & masses. (Not renormalized by loops since higher-energy d.o.f. already integrated out)

- Physical processes at $E \ll \mu$ will get quantum corrections due to fluctuations of modes in effective action w/ energies between $E$ & $\mu$.
- These corrections can be absorbed in couplings to define new effective action at lower scale $E$

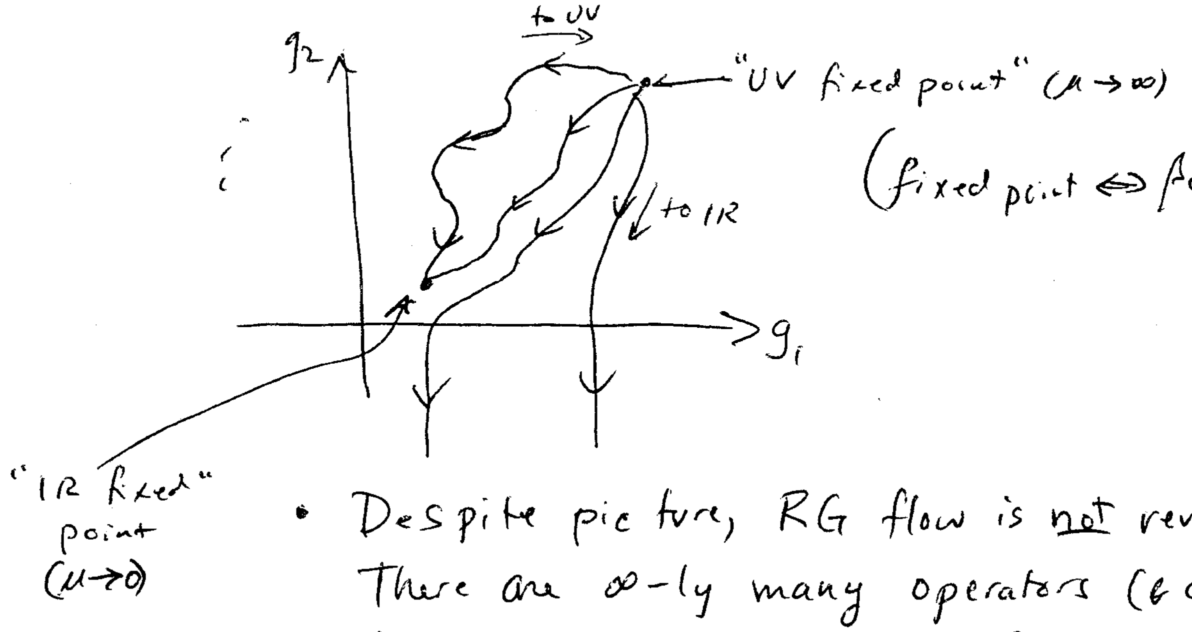
$\Longrightarrow$ renormalization group: change in $S_\mu$ as $\mu$ decreases. E.g.

$$S_\mu = \int d^4x \sum_i g_i(\mu)\, \mathcal{O}_i$$

eff. couplings — operators

β-functions

RG eqns: $\mu \dfrac{\partial g_i}{\partial \mu} = \beta_i(g_k, \mu)$ } Flow in coupling space




(fixed point $\Leftrightarrow \beta_i = 0$.)

- Despite picture, RG flow is <u>not</u> reversible! There are ∞-ly many operators (& couplings) "integrated out" along each flow!

- IR effective actions are $S_\mu$ near an IR fixed point ($\mu$ <u>finite</u>, but small compared to all other scales).
(NB: not 1PI eff. act. = $S_{\mu=0}$ ).

- If fixed point theory is free, e.g.

$$S_{free} = \int_{1/\mu_0} d^4x_0 \left[ (\partial_0 \phi_0)^2 + \bar{\psi}_0 \gamma_0 \psi_0 + F_0^{\mu\nu} F_{0\mu\nu} \right]$$

then fields should scale to keep $S_{free}$ independent of scale $\mu_0$:

I.e. replace scale: $\mu_0 \to \mu = \left(\frac{\mu}{\mu_0}\right) \cdot \mu_0 \quad \therefore \underset{\text{old scale}}{\mu_0} = \left(\frac{\mu_0}{\mu}\right) \underset{\text{new scale}}{\mu}$

$\therefore$ lower all energies by replacing $E \to \frac{\mu_0}{\mu} E$

$$\Rightarrow \partial_0 \to \frac{\mu_0}{\mu} \partial \ , \ dx_0 \to \frac{\mu}{\mu_0} dx$$

So replace fields by

$$\begin{cases} \phi_0 \to \left(\frac{\mu_0}{\mu}\right) \phi & \psi_0 \to \left(\frac{\mu_0}{\mu}\right)^{3/2} \psi \\ F_0^{\mu\nu} \to \left(\frac{\mu_0}{\mu}\right)^2 F^{\mu\nu} \end{cases}$$

to keep $S_{free}$ the same in terms of new variables.

- In general, if operator $\mathcal{O}_i$ scales as

$$\mathcal{O}_i \to \left(\frac{\mu_0}{\mu}\right)^{\Delta_i} \mathcal{O}_i \ , \quad \text{say } \mathcal{O}_i \text{ has dimension } \Delta_i.$$

$$\Rightarrow \int_{1/\mu_0} d^4x_0 (\mathcal{O}_i)_0 \Rightarrow \left(\frac{\mu_0}{\mu}\right)^{\Delta_i - 4} \int_{1/\mu} d^4x \, \mathcal{O}_i \qquad \text{(classical)}$$

So
$\Delta_i > 4$ : $\mathcal{O}_i$ irrelevant – less important in IR
$\Delta_i = 4$ : $\mathcal{O}_i$ marginal
$\Delta_i < 4$ : $\mathcal{O}_i$ relevant – more important in IR

- For given local field content, only a finite number of relevant + marginal local operators. This is why finding the effective action is not a hopeless task.

- Quantum corrections modify classical scaling.

Write:

dimles. coupling "constant"

$$S_{\mu_0} = S_{free} + \int d^4x \, \mu_0^{4-\Delta_i} \lambda_i(\mu_0) \, \mathcal{O}_i$$

$$\Rightarrow S_\mu = S_{free} + \int d^4x \, \underbrace{\mu^{4-\Delta_i} \lambda_i(\mu)}_{} \, \mathcal{O}_i$$

$$\text{w/} \quad \mu \frac{d\tilde{\lambda}_i}{d\mu} = \underbrace{(4-\Delta_i)\tilde{\lambda}_i}_{\text{classical scaling}} + \underbrace{\beta_i(\tilde{\lambda}_j)}_{}$$

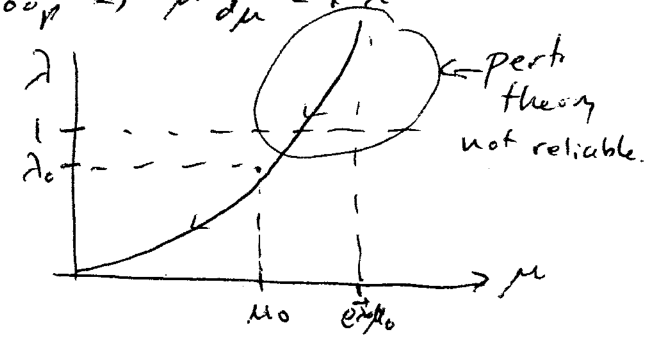
quant. correct. from fluctuations w/ $\mu + d\mu > E > \mu$

$\Rightarrow$ finite (No UV or IR divergences).

- E.g. $\mathcal{L} = (\partial\phi)^2 + \lambda\phi^4$

$\Rightarrow \mathcal{O} = \phi^4$, $\Delta = 4$, 1-loop $\Rightarrow$ $\mu \frac{d\lambda}{d\mu} = +\lambda^2$

$$\Rightarrow \lambda(\mu) = \frac{\lambda(\mu_0)}{1 + \ln\left(\frac{\mu_0}{\mu}\right)\lambda(\mu_0)}$$




IR free theory $\Longleftarrow$

- Quantum corrections to kinetic terms = wave function renormalization: e.g. write

$$S_\mu = \int d^4x \, \{ Z(\mu)(\partial\phi)^2 + \mu^2 m^2(\mu)\phi^2 + \lambda(\mu)\phi^4 + \cdots \}$$

wv. fnc. renorm. dimensionless mass param.

Can absorb in redefinition of fields:

$$\phi \to \phi_{CN} \equiv \sqrt{Z}\,\phi$$

"canonically normalized"

$$\Rightarrow S_\mu = \int d^4x \, \{ (\partial\phi_{CN})^2 + \underbrace{\mu^2 m_{CN}^2(\mu)}_{} \phi_{CN}^2 + \underbrace{\lambda_{CN}(\mu)}_{} \phi_{CN}^4 + \cdots \}$$

(canonical) effective couplings

w/ $\quad m_{cN}(\mu) = \dfrac{m(\mu)}{\sqrt{Z(\mu)}} \qquad \lambda_{cN}(\mu) = \dfrac{\lambda(\mu)}{Z^2(\mu)}$ etc.

Thus, physical mass:

$$m_{phys} = \mu\, m_{cN}(\mu) = \frac{\mu\, m(\mu)}{\sqrt{Z(\mu)}} \quad (\text{limit } \mu \to 0 \text{ really})$$

- As flow to IR, eventually reach a point where $m_{eff} = \mu\, m_{cN}(\mu) > \mu$

  $\Rightarrow$ Mass term dominates kinetic term, & fixes $\phi$ to be a constant (for $x > \frac{1}{m_{eff}}$).

  $\therefore$ For $\mu < m_{eff}$, $\phi \simeq$ constant, & drop kinetic terms
  = "integrating out" $\phi$.

  Massless fields, though, have d.o.f. for all $\mu$.

- Generally, in calculating quantum corrections, can get arbitrary (non-singular) field <u>redefinitions</u>. These are changes of variables & do not affect the physics. The particular choice of field redefinitions that occur depend on details of the <u>renormalization scheme</u> (e.g. regularization procedure, subtraction procedure, matching conditions ...)

  - I don't want to get into details of a specific renormalization scheme. So, I will simply <u>assume</u> that such a scheme has been picked (eg $\overline{DR}$ ...) which preserves the symmetries of the theory (cf. Anomalies)

  - Since scalars can shift by constants & still preserve Lorentz invariance, $\phi' = \phi + \langle\phi_0\rangle$, etc it is important to keep <u>all</u> powers of $\phi$ in effective actions:

$$S_{eff} = \int d^4x \, \{ V(\phi) + g(\phi)(\partial\phi)^2 + h(\phi)\bar{\psi}\gamma\psi + y(\phi)\psi\psi + \tau(\phi) F^2 + \underset{\text{(irrelevant)}}{\text{higher derivatives}} \}$$

- This is important in determining the vacuum (i.e. $\langle\phi\rangle$). Once $\langle\phi\rangle$ is fixed, then, in expanding around $\phi = \langle\phi\rangle$, can use kinetic term scaling.

## Types of IR fixed points

- Trivial: all fields massive, so for $\mu <$ masses all integrated out & no propagating d.o.f.
- Free: all massless fields are non-interacting in far IR (e.g. $\lambda\phi^4$ theory, ~~massless~~ QED).
- Interacting: in 4 dim's, we have no good description of interacting theories of massless particles. (i.e. non-lagrangian).

$$\boxed{\text{So: for lack of anything better, we will guess that our IR effective theory is free (as } \mu \to 0).}$$

Coleman-Gross (74) $\Rightarrow$ any theory of scalars, spinors & U(1) gauge fields is IR-free. This gives a large class of IR-free theories. (Will see later that only other is non-Abelian gauge theory w/ many massless flavors.)

$\therefore$ Take field content of IR eff. theory to be scalars $\phi^n$ (complex) Weyl spinors $\psi^a_\alpha$, U(1) vectors $A^A_\mu$.

$\{n, a, A\} \to$ just label different fields.

$\therefore$ General form of leading (relevant) IR action

$$S_{eff} = \int d^4x \Big\{ V(\phi^n) + \tfrac{1}{2} g_{mn}(\phi)\, \overline{D_\mu \phi^n}\, D^\mu \phi^m$$
$$+ h_{ab}(\phi)\ \bar{\psi}^a \not{D} \psi^b$$
$$- \frac{1}{4 g^2_{AB}(\phi)} F^A_{\mu\nu} F^{B\mu\nu} + \frac{\theta_{AB}(\phi)}{\cancel{32}\pi^2} F^A_{\mu\nu} \tilde{F}^{B\mu\nu}$$
$$+ y_{ab}(\phi)\, \psi^a \psi^b + \text{c.c.} \Big\}$$

"gauged sigma model"

- where: $D_\mu \chi = (\partial_\mu + q_A A^A_\mu)\chi$

  $q_A$ ← charges of $\chi$

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu$$

$$\tilde{F}^A_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{A\rho\sigma}$$ "dual field strength"

- Constraints:
  - $g_{mn}$ real, symmetric, pos. definite
  - $V$ bounded below
  - $g^2_{AB}$ real, symm, pos. definite
  - ...

- Convenient to define generalize complex coupling

$$\tau_{AB}(\phi) = \frac{\theta_{AB}}{2\pi} + i \frac{4\pi}{g^2_{AB}}$$

Then gauge kinetic terms written

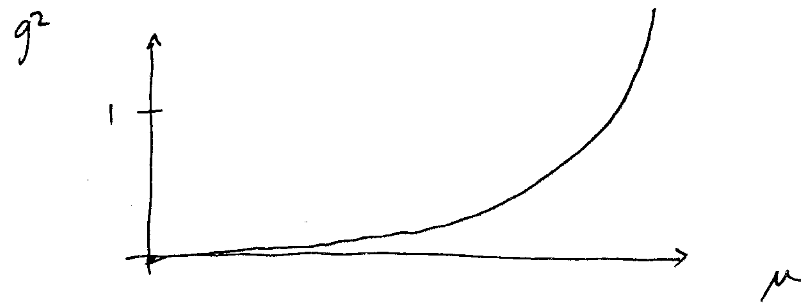
$$\frac{i\tau}{16\pi} (\mathcal{F})^2 + \text{c.c.} \quad \text{w/}\ \mathcal{F}^{\mu\nu} \equiv \frac{1}{2} F^{\mu\nu} - \frac{i}{2} \tilde{F}^{\mu\nu}$$ "self-dual"

- Will drop flavor labels where possible.

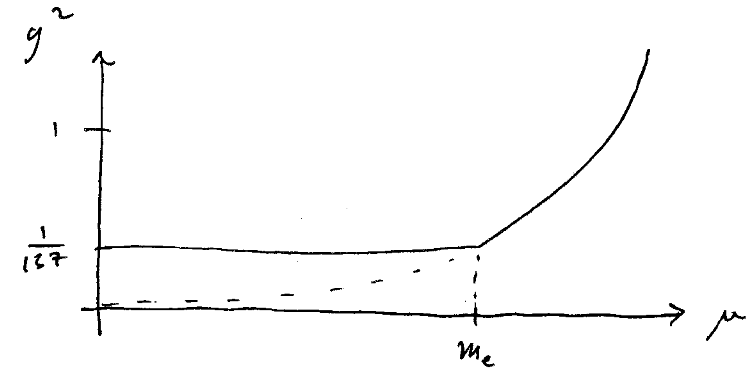
- Meaning of $\tau_{AB}$: since fields are free in IR, what is the meaning of the gauge couplings $\tau$?

2 cases:

(1) massless charged $\phi$ or $\psi$: 1-loop running of $U(1)$ coupling $\Rightarrow$ free in IR




(2) all charged fields massive: $U(1)$ coupling stops running for $\mu$ < lightest charged particle (since ~~nothing~~ integrated out)




- In this case $\tau$ measures strength of coupling only to massive (classical) sources.
- $\theta$-angles, sensitive to instanton # of field configurations. In presence of electric & magnetic massive charges, can have non-trivial $\theta$-effects.
- Electric-Magnetic duality: field redefinitions in $U(1)$ theories $\Rightarrow$ $\tau \simeq \frac{A\tau + B}{C\tau + D}$

w/ $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2r, \mathbb{Z})$ ... see L. A-G. lectures.

# Supersymmetric Effective Actions

- Scalar, spinor, & vector fields appear in

  chiral superfield: $\Phi(x,\theta) \sim \phi(x) + \theta\cdot\psi(x) + \cdots$

  $(\bar{D}_{\dot\alpha}\Phi = 0)$

  And in

  vector superfield: $V(x,\theta,\bar\theta) \sim \bar\theta\sigma^\mu\theta\cdot A_\mu(x) + \bar\theta^2\bar\theta\cdot\lambda(x) + \cdots$

  or chiral field-strength: $W_\alpha(x,\theta) \sim \lambda_\alpha(x) + (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}(x) + \cdots$

  $$\left(\begin{array}{l} W_\alpha \sim \bar{D}^2 e^{-V} D_\alpha e^{V} \quad \Rightarrow \quad D_\alpha W^\alpha = \bar{D}^{\dot\alpha}\bar{W}_{\dot\alpha} \text{ : Bianchi} \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \& \ \bar{D}_{\dot\alpha} W_\alpha = 0 \\ \text{Gauge inv: } \begin{cases} e^{-V} \to e^{-i\bar\Lambda} e^{-V} e^{+i\Lambda} \\ W_\alpha \to e^{-i\Lambda} W_\alpha e^{+i\Lambda} \end{cases} \\ \Lambda = \text{chiral superfield.} \end{array}\right)$$

- Supersymmetric actions given by

  $$\mathcal{L} = \int d^2\theta\, d^2\bar\theta\, \tilde{\mathcal{K}} + \int d^2\theta\, \tilde{\mathcal{W}} + \text{c.c.}$$

  where $\tilde{\mathcal{K}}$ is general superfield
  $\tilde{\mathcal{W}}$ is chiral superfield

- Kinetic terms:

  (minimal coupling)

  $$\int d^2\theta\, d^2\bar\theta\ \bar\Phi\Phi \xrightarrow{\text{gauge}} \int d^2\theta\, d^2\bar\theta\ \bar\Phi e^V \Phi$$

  $$\int d^2\theta\ \mathrm{tr}(W_\alpha W^\alpha)$$

- General superfield:

  arbitrary derivatives

  $$\tilde{\mathcal{K}} = \mathcal{K}(\bar\Phi e^V, \Phi) + \mathcal{K}'(\bar\Phi e^V, \Phi, D, \bar{D}, \partial_\mu)$$

  "Kahler potential"

- Chiral superfield:

  "superpotential" "generalized coupling"

  $$\tilde{\mathcal{W}} = \mathcal{W}(\Phi) + \tau(\Phi)\,\mathrm{tr}(W_\alpha W^\alpha) + \mathcal{O}(W^4) + \mathcal{W}'(\Phi, \partial_\mu) + \cdots$$

  higher-derivative

- To see which terms we need to keep in IR action, scale coordinates

$$x_0 \to \left(\frac{\mu}{\mu_0}\right) x \qquad \Rightarrow \left\{ \begin{array}{l} \theta_0 \to \left(\frac{\mu}{\mu_0}\right)^{1/2} \theta \\ \partial_{\theta_0} \to \left(\frac{\mu_0}{\mu}\right)^{1/2} \partial_\theta \end{array} \right\} \text{ since } D^2 \sim \partial_\mu$$

$$\partial_{\mu 0} \to \left(\frac{\mu_0}{\mu}\right) \partial_\mu$$

$$dx_0 \to \left(\frac{\mu}{\mu_0}\right) dx \qquad d\theta_0 \to \left(\frac{\mu_0}{\mu}\right)^{1/2} d\theta \quad \Big\} \text{ since } \int d\theta \sim \frac{\partial}{\partial \theta}$$

Then to preserve form of kinetic terms:

$$\Phi_0 \to \left(\frac{\mu_0}{\mu}\right) \Phi$$

$$W_{0\alpha} \to \left(\frac{\mu_0}{\mu}\right)^{3/2} W_\alpha \qquad \left( V_0 \to \left(\frac{\mu_0}{\mu}\right)^0 V \right)$$

i.e. scale in same way as lowest component.

- Chiral superfield: $\mathcal{O}_{i0} \to \left(\frac{\mu_0}{\mu}\right)^{\Delta_i} \mathcal{O}_i$

$$\Rightarrow \int d^4x_0 \int d^2\theta_0\, \mathcal{O}_{i0} \to \left(\frac{\mu_0}{\mu}\right)^{\Delta_i - 3} \int d^4x\, d^2\theta\, \mathcal{O}_i$$

$\therefore$ $\underline{\Delta_i = 3 \text{ is marginal } \chi\text{sf}}$

- ~~Vector~~ General superfield: $\mathcal{N}_{i0} \to \left(\frac{\mu_0}{\mu}\right)^{\Delta_i} \mathcal{N}_i$

$$\Rightarrow \int d^4x_0 \int d^2\theta_0\, d^2\bar\theta_0\, \mathcal{N}_{i0} \to \left(\frac{\mu_0}{\mu}\right)^{\Delta_i - 2} \int d^4x\, d^2\theta\, d^2\bar\theta\, \mathcal{N}_i$$

$\therefore$ $\underline{\Delta_i = 2 \text{ is marginal gen. superfield}}$

These funny dimensions are just a result of our scaling superfields by their lowest component.

- So write relevant terms:

$$S_\mu = \int d^4x \Big[ \int d^2\theta \left\{ \tau(\mu)\, W^2 \right\} + \text{c.c.} + \int d^4\theta \left\{ Z(\mu)\, \bar\Phi e^V \Phi \right\}$$
$$+ \int d^2\theta \left\{ \mu^{3-\Delta_i} \lambda_i(\mu)\, \mathcal{O}_i(\Phi) \right\} + \text{c.c.}$$
$$+ \underbrace{\int d^4\theta \left\{ \mu^{2-\Delta_i} v_i(\mu)\, \mathcal{N}_i(\Phi, \bar\Phi e^V) \right\}}_{\text{all irrelevant}} + \text{higher derivatives} \Big].$$

But keeping arbitrary powers of scalars (in $\Phi$) to account for field redefinitions, look at SUSY $\sigma$-models:

$$\boxed{S_{eff} = \int d^4x \left[ \int d^4\theta\, \mathcal{K}(\bar{\Phi} e^V, \Phi) + \int d^2\theta \left[ \tau(\Phi) W^2 + \mathcal{W}(\Phi) \right] + c.c. \right]}$$

($\underline{NB}$: should also put in Fayet-Iliopoulos term:

$$\int d^4x\, \xi \int d\theta_\alpha W^\alpha .$$

Will leave out for simplicity.)