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Introductory School on

RECENT DEVELOPMENTS IN SUPERSYMMETRIC GAUGE THEORIES

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INSTANTON PHYSICS (Part III)

Nick Dorey
University of Wales, Swansea, U.K.

zero modes. Then we derive an explicit formulae for the appropriate supersymmetric volume form on $\mathfrak{M}_k$.

4.1. Action, supersymmetry and equations-of-motion

We start by defining theories in four-dimensional Minkowski space with $\mathcal{N}=1,2$ and 4 supersymmetry. In the interests of brevity we will develop a unified notion that allows us to deal with all these cases together. To this end, we introduce the fermionic partners of the gauge field λ^A and $\bar{\lambda}_A$. Here, $A=1,\ldots,\mathcal{N}$ is an R-symmetry index of the supersymmetry. Since we are working—at least initially—in Minkowski space, these spinors are subject to the reality conditions

$$(\lambda_\alpha^A)^\dagger = \bar{\lambda}_{\dot{\alpha}A}, \quad (\bar{\lambda}_A^{\dot{\alpha}})^\dagger = \lambda^{\alpha A} \quad (\alpha = \dot{\alpha}) . \tag{4.12}$$

In addition, for the theories with extended supersymmetry there are real scalar fields ϕ_a, $a=1,\ldots,2(\mathcal{N}-1)$. The Minkowski space action is[21]

$$S^{\text{Mink}} = \int \mathrm{d}^4x\, \mathrm{tr}_N \left\{ \frac{1}{2} F_{mn}^2 + \frac{\mathrm{i}\theta g^2}{16\pi^2} F^*_{mn} F^{mn} + 2\mathrm{i}\mathscr{D}_n \bar{\lambda}_A \bar{\sigma}^n \lambda^A - \mathscr{D}^n \phi_a \mathscr{D}_n \phi_a \right.$$
$$\left. + g\bar{\lambda}_A \Sigma_a^{AB}[\phi_a, \bar{\lambda}_B] + g\lambda^A \bar{\Sigma}_{aAB}[\phi_a, \lambda^B] + \frac{1}{2} g^2 [\phi_a, \phi_b]^2 \right\} . \tag{4.13}$$

The terms involving the scalar fields are, of course, absent in the $\mathcal{N}=1$ theory. The Σ-matrices are associated to the SU(2) and SU(4) R-symmetry group of the $\mathcal{N}=2$ and 4 theories, respectively. For $\mathcal{N}=2$ we take

$$\Sigma_a^{AB} = \varepsilon^{AB}(\mathrm{i}, 1), \quad \bar{\Sigma}_{aAB} = \varepsilon_{AB}(-\mathrm{i}, 1) . \tag{4.14}$$

In this case the indices $A, B, \ldots = 1, 2$ are spinor indices of the SU(2) subgroup of the U(1) $\times$ SU(2) R-symmetry group. In this case, we can raise and lower the indices using the ε-tensor in the usual way following the conventions of [47]. For the $\mathcal{N}=4$ case

$$\Sigma_a = (\eta^3, \mathrm{i}\bar{\eta}^3, \eta^2, \mathrm{i}\bar{\eta}^2, \eta^1, \mathrm{i}\bar{\eta}^1) ,$$
$$\bar{\Sigma}_a = (-\eta^3, \mathrm{i}\bar{\eta}^3, -\eta^2, \mathrm{i}\bar{\eta}^2, -\eta^1, \mathrm{i}\bar{\eta}^1) , \tag{4.15}$$

where η^c, $\bar{\eta}^c$, $c=1$–3, are 't Hooft's η-symbols defined in Appendix A.

Theory (4.13) is invariant under the on-shell supersymmetry transformations

$$\delta A_n = -\xi^A \sigma_n \bar{\lambda}_A - \bar{\xi}_A \bar{\sigma}_n \lambda^A , \tag{4.16a}$$

$$\delta\lambda^A = -\mathrm{i}\sigma^{mn}\xi^A F_{mn} - \mathrm{i}g\Sigma_{ab}{}^A{}_B \xi^B [\phi_a, \phi_b] + \Sigma_a^{AB} \sigma^n \bar{\xi}_B \mathscr{D}_n \phi_a , \tag{4.16b}$$

[21] We remind the reader that our gauge field is anti-Hermitian rather than Hermitian, otherwise our conventions in Minkowski space are those of Wess and Bagger [47].

$$\delta\bar{\lambda}_A = -i\bar{\sigma}^{mn}\bar{\xi}_A F_{mn} - ig\bar{\Sigma}_{abA}{}^B\bar{\xi}_B[\phi_a,\phi_b] + \bar{\Sigma}_{aAB}\bar{\sigma}^n\xi^B\mathscr{D}_n\phi_a\ , \tag{4.16c}$$

$$\delta\phi_a = i\xi^A\bar{\Sigma}_{aAB}\lambda^B + i\bar{\xi}_A\Sigma_a^{AB}\bar{\lambda}_B\ . \tag{4.16d}$$

In the above,

$$\sigma^{mn} = \tfrac{1}{4}(\sigma^m\bar{\sigma}^n - \sigma^n\bar{\sigma}^m),\quad \bar{\sigma}^{mn} = \tfrac{1}{4}(\bar{\sigma}^m\sigma^n - \bar{\sigma}^n\sigma^m) \tag{4.17}$$

and

$$\Sigma_{ab} = \tfrac{1}{4}(\Sigma_a\bar{\Sigma}_b - \Sigma_b\bar{\Sigma}_a),\quad \bar{\Sigma}_{ab} = \tfrac{1}{4}(\bar{\Sigma}_a\Sigma_b - \bar{\Sigma}_b\Sigma_a)\ . \tag{4.18}$$

In order to construct instanton solutions, we now Wick rotate to Euclidean space. Vector quantities in Minkowski space $a^n = (a^0, \vec{a})$, with $n = 0$–3, become $a_n = (\vec{a}, ia^0)$, with $n = 1$–4, in Euclidean space. The Euclidean action is then $-i$ times the Minkowski space action. The exception to this is that we define the Euclidean σ-matrices as in (2.8) and (2.9). So in Minkowski space $\sigma^n = (-1, \vec{\tau})$ and $\bar{\sigma}^n = (-1, -\vec{\tau})$, whereas in Euclidean space $\sigma_n = (i\vec{\tau}, 1)$ and $\bar{\sigma}_n = (-i\vec{\tau}, 1)$. Operationally, this means that when Wick rotating from Minkowski space to Euclidean space we should actually replace the Minkowski space σ-matrices by $-i$ times the Euclidean space σ-matrices. As usual we treat λ_α and $\bar{\lambda}^{\dot{\alpha}}$ as independent spinors, i.e. independent integration variables in the functional integral. The Euclidean space action is

$$S = \int d^4x\,\mathrm{tr}_N\left\{-\tfrac{1}{2}F_{mn}^2 - \frac{i\theta g^2}{16\pi^2}F_{mn}^*F_{mn} - 2\mathscr{D}_n\bar{\lambda}_A\bar{\sigma}_n\lambda^A + \mathscr{D}_n\phi_a\mathscr{D}_n\phi_a\right.$$
$$\left. - g\bar{\lambda}_A\Sigma_a^{AB}[\phi_a,\bar{\lambda}_B] - g\lambda^A\bar{\Sigma}_{aAB}[\phi_a,\lambda^B] - \tfrac{1}{2}g^2[\phi_a,\phi_b]^2\right\}\ . \tag{4.19}$$

As discussed above, the fact that the fermionic terms in this action are not real will not concern us further. For the case with $\mathcal{N}=2$ supersymmetry, we can recover the more conventional presentation of the theory by defining a complex scalar field

$$\phi = \phi_1 - i\phi_2,\quad \phi^\dagger = \phi_1 + i\phi_2 \tag{4.20}$$

and spinors $\lambda \equiv \lambda^1$ and $\psi \equiv \lambda^2$. The fields $\Phi = \{\phi/\sqrt{2}, \psi\}$ form a chiral multiplet and $V = \{A_m, \lambda\}$ a vector multiplet of $\mathcal{N}=1$ supersymmetry. In terms of these variables, the Euclidean space action of the $\mathcal{N}=2$ theory (4.19) is

$$S_{\mathcal{N}=2} = \int d^4x\,\mathrm{tr}_N\left\{-\frac{1}{2}F_{mn}^2 - \frac{i\theta g^2}{16\pi^2}F_{mn}^*F_{mn} - 2\mathscr{D}_n\bar{\lambda}\bar{\sigma}_n\lambda - 2\mathscr{D}_n\bar{\psi}\bar{\sigma}_n\psi + \mathscr{D}_n\phi^\dagger\mathscr{D}_n\phi\right.$$
$$\left. + 2ig\bar{\psi}[\phi,\bar{\lambda}] + 2ig[\phi^\dagger,\lambda]\psi + \frac{1}{4}g^2[\phi,\phi^\dagger]^2\right\}\ . \tag{4.21}$$

In the following, we prefer the presentation of the theory in (4.19) since this will allow us to deal with the theories with different numbers of supersymmetries in a unified way.

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The equations-of-motion following from (4.19) are

$$\mathscr{D}_m F_{nm} = 2g[\phi_a, \mathscr{D}_n\phi_a] + 2g\bar{\sigma}_n\{\lambda^A, \bar{\lambda}_A\} , \tag{4.22a}$$

$$\not{\mathscr{D}}\lambda^A = g\Sigma_a^{AB}[\phi_a, \bar{\lambda}_B] , \tag{4.22b}$$

$$\not{\mathscr{D}}\bar{\lambda}_A = g\bar{\Sigma}_{aAB}[\phi_a, \lambda^B] , \tag{4.22c}$$

$$\mathscr{D}^2\phi_a = g^2[\phi_b, [\phi_b, \phi_a]] + g\bar{\Sigma}_{aAB}\lambda^A\lambda^B + g\Sigma_a^{AB}\bar{\lambda}_A\bar{\lambda}_B . \tag{4.22d}$$

The supersymmetry transformations in Euclidean space are given by (4.16a)–(4.16d) by replacing the sigma matrices with $-\mathrm{i}$ times their Euclidean space versions and by replacing Minkowski space inner products with Euclidean ones:

$$\delta A_n = \mathrm{i}\xi^A\sigma_n\bar{\lambda}_A + \mathrm{i}\bar{\xi}_A\bar{\sigma}_n\lambda^A , \tag{4.23a}$$

$$\delta\lambda^A = \mathrm{i}\sigma_{mn}\xi^A F_{mn} - \mathrm{i}g\Sigma_{ab}{}^A{}_B\xi^B[\phi_a, \phi_b] - \mathrm{i}\Sigma_a^{AB}\not{\mathscr{D}}\phi_a\bar{\xi}_B , \tag{4.23b}$$

$$\delta\bar{\lambda}_A = \mathrm{i}\bar{\sigma}_{mn}\bar{\xi}_A F_{mn} - \mathrm{i}g\bar{\Sigma}_{abA}{}^B\bar{\xi}_B[\phi_a, \phi_b] - \mathrm{i}\bar{\Sigma}_{aAB}\not{\mathscr{D}}\phi_a\xi^B , \tag{4.23c}$$

$$\delta\phi_a = \mathrm{i}\xi^A\bar{\Sigma}_{aAB}\lambda^B + \mathrm{i}\bar{\xi}_A\Sigma_a^{AB}\bar{\lambda}_B . \tag{4.23d}$$

4.2. The super-instanton at linear order

We will now attempt to find *super-instanton* configurations which solve the full coupled equations-of-motion (4.22a)–(4.22d). First notice that the original instanton solution of the pure gauge theory (2.49) is a solution of the full equations-of-motion when all other fields are set to zero. In fact, we can use $A_m(x;X)$ as a starting point to find the more general solutions where the fermion and scalar fields are non-vanishing. As explained in the introduction to this section we will proceed perturbatively order by order in the coupling. In this connection note the explicit powers of g appearing on the right-hand side of Eqs. (4.22a)–(4.22d).

The first step, following [38], is to expand to linear order in the fields around the bosonic instanton solution. To the next order, we must therefore solve the covariant Weyl equations

$$\not{\mathscr{D}}\lambda^A = 0 , \tag{4.24a}$$

$$\not{\mathscr{D}}\bar{\lambda}_A = 0 \tag{4.24b}$$

for the fermions, and the covariant Laplace equation

$$\mathscr{D}^2\phi_a = 0 \tag{4.25}$$

for the scalars. It then remains to be seen whether the original instanton solution needs to be modified due the source term on the right-hand side of (4.22a).

A key result follows from the fact that $\not{\mathscr{D}}$ has no zero modes in an instanton (rather than anti-instanton) background. Consequently, the solution to (4.24b) is $\bar{\lambda}_A = 0$. To prove this, (4.24b)

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One-loop effects

- running coupling

$$g^2_{eff} = g^2_{IR} = g^2(\mu = |a|)$$

$$\Lambda = \mu \exp\left(-\frac{2\pi^2}{g^2(\mu)}\right)$$

$$\Rightarrow g^2_{eff} = \frac{2\pi^2}{\log\left(\frac{|a|}{\Lambda}\right)} \quad - (\alpha)$$

- anomaly

$U(1)_R$ rotation: $a \to e^{2i\delta} a$

equivalent to shifting θ_{eff} by -8δ

$$\Rightarrow \theta_{eff} = -4\,\mathrm{Im}\left(\log\left(a/\Lambda\right)\right) \quad - (\beta)$$

$+ \text{cst}$

(a) + (B) $\Rightarrow$

$$\frac{\partial^2 F_{1\text{-loop}}}{\partial a^2} = \frac{4\pi i}{g_{eff}^2} + \frac{\theta_{eff}}{2\pi}$$

$$= \frac{2i}{\pi} \log\left(\frac{a}{\Lambda}\right)$$

$$\Rightarrow \mathcal{F}_{1\text{-loop}} = \frac{i}{2\pi} a^2 \log\left(a^2/\Lambda^2\right)$$

- holomorphic as required

Adler-Bardeen Theorem

$\Rightarrow$ no further renormalisation of chiral anomaly at any order in perturbation theory

$\Rightarrow$ no perturbative corrections to $\mathcal{F}_{1\text{-loop}}$

Non-perturbative prepotential

constraints:

- holomorphy
- RG invariance

$\Rightarrow \mathcal{F} = \mathcal{F}(a, \Lambda)$

- $U(1)_R$ anomaly

$\mathcal{L}_{eff}$ invariant under $U(1)_R$
except for the effects of anomaly

- $\theta_{eff} = -4\,\mathrm{Im}\left(\log\left(a/\Lambda\right)\right)$
  + single-valued

- $U(1)_R$ conservation violated
by $8k$ units in sector of
topological charge k

$\mathcal{L}_{\text{eff}} \sim \text{Im} \int d^4\theta\, \mathcal{F}$ invariant

$\Rightarrow$ $\mathcal{F}$ has charge 4 under $U(1)_R$

most general allowed form:

Seiberg

$$\mathcal{F}(a) = \frac{i}{2\pi} a^2 \log\left(a^2/\Lambda^2\right)$$

one-loop

$$- \frac{i}{\pi} \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{a}\right)^{4k} a^2$$

contribution from topological charge k

- $\Lambda^{4k} \sim e^{-\frac{8\pi^2 k}{g^2} + ik\theta}$

$\Rightarrow$ k'th term corresponds to leading semiclassical contribution of k instantons

$$\mathcal{F}(a) = \frac{i}{2\pi} a^2 \log\left(\frac{a^2}{\Lambda^2}\right) - \frac{i}{\pi} \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{a}\right)^{4k} a^2$$

= 1-loop + instantons

- no perturbative corrections in k-instanton background

<u>Seiberg</u>-<u>Witten</u> solution (1994) predicts exact function $\mathcal{F}$

$\Rightarrow$ prediction for coefs $\mathcal{F}_k$

$\mathcal{F}_1 = 1/2$, $\mathcal{F}_2 = 5/16$, ... $\mathcal{F}_k \in \mathbb{Q}$

Why???

- check against first principles calculation Finnel + Pouliot

ND + Khose + Mattis

Testing SW theory

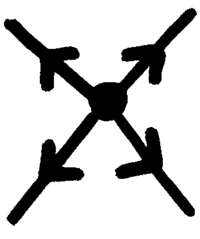
$$\mathcal{L}_{\text{eff}} \supset \frac{\partial^4 \mathcal{F}(a)}{\partial a^4} \psi^2 \lambda^2 \leftarrow$$ 4-fermion vertex

$\Rightarrow$ non-zero contribution to long-distance behaviour of correlator, $\qquad |x_i - x_j| \gg |a|$

$$\langle \bar{\Psi}(x_1) \bar{\Psi}(x_2) \bar{\lambda}(x_3) \bar{\lambda}(x_4) \rangle$$

$$\sim \frac{\partial^2 F}{\partial a^4} \int d^4 X \prod_{i=1}^{4} S_F(x_i - X)$$

$\nwarrow$ massless fermion propagator



- gets contributions from all numbers of instantons $\sim \mathcal{F}_k$

eg 1-instanton contribution to

$$G^{(4)} = \langle \bar{\psi}(x_1)\, \bar{\psi}(x_2)\, \bar{\lambda}(x_3)\, \bar{\lambda}(x_4) \rangle$$

$$\sim \frac{\partial^4}{\partial a^4} \mathcal{F}_1 \left(\frac{\Lambda^4}{a^4} \right) a^2$$

$$\sim \mathcal{F}_1 \left(\frac{\Lambda^4}{a^6} \right)$$

check:

R-charge of LHS $= -4$

R-charge of RHS $= -12$

$$\Delta Q_R = -4 - (-12) = 8 \quad ✓✓$$

note contribution of VEV a to counting reflects spontaneous breaking of $U(1)_R$

Semiclassical Calculation

Solve Euclidean e.o.m order by order in g^2

$$\mathcal{D}_m F_{mn} = 2g[\phi_a, \mathcal{D}_n \phi_a] + 2g\bar{\sigma}_n\{\lambda^A, \bar{\lambda}_A\}$$

$$\bar{\not{\mathcal{D}}}\lambda^A = g\,\Sigma_a^{AB}[\phi_a, \bar{\lambda}_B]$$

$$\not{\mathcal{D}}\bar{\lambda}_A = g\,\bar{\Sigma}_a^{AB}[\phi_a, \lambda_B]$$

$$\mathcal{D}^2\phi^a = g^2[\phi^b,[\phi^b,\phi^a]] + g\,\bar{\Sigma}_{aAB}\lambda^A\lambda^B + g\,\Sigma_a^{AB}\bar{\lambda}_A\bar{\lambda}_B$$

notation:

ϕ_a $\quad a = 1,2$ $\quad$ real adjoint scalars

$$\phi = \phi_1 + i\phi_2 \in \mathbb{C}$$

$$\lambda^1 = \lambda \qquad \lambda^2 = \psi$$

vacuum boundary condition:

$$\underset{\sim}{\phi}(x) \longrightarrow \frac{ia\tau^3}{2} \qquad |x| \to \infty$$

Zero'th order: $k=1$

$$\mathcal{D}_m \underset{\sim}{F}_{mn} = 0$$

$$\bar{\not{D}} \lambda^A = 0 \qquad \not{D} \bar{\lambda}^A = 0$$

$$\mathcal{D}^2 \underset{\sim}{\phi} = 0$$

<u>Bosons</u> $\underset{\sim}{A}_m = \underset{\sim}{A}_m^{cl}$ ← BPST instanton

collective coords: X_m, ρ, U

position size orientation

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^{cl} = \frac{(x-X)^2}{(x-X)^2+\rho^2} \cdot \frac{ia\tau^3}{2}$$

$\underset{\sim}{A}_m = \underset{\sim}{\hat{A}}{}_m^{cl} \quad \underset{\sim}{\phi} = \underset{\sim}{\phi}{}^{cl}$ is <u>not</u> a solution of <u>full</u> e.o.m.

<u>symptom</u>:

$$\frac{1}{g^2} S[\underset{\sim}{\hat{A}}{}_m^{cl}, \underset{\sim}{\phi}{}^{cl}] = \frac{8\pi^2}{g^2} + 4\pi^2 e^2 |a|^2$$

instanton unstable to scale transformation

<u>reflects</u> spontaneous breaking of conformal invariance

$\Rightarrow$ corresponding zero mode lifted

potential on instanton moduli space

fermions $\bar{\not{D}}_{cl}\lambda^A = 0$ 4 zero modes

$\not{D}_{cl}\bar{\lambda}_A = 0$ no zero modes

set

$$\underset{\sim}{\lambda}{}^A_\alpha = \underset{\sim}{\lambda}{}^{cl\,A}_\alpha = \sum_{\ell=1}^{4} \alpha^A_\ell \, \Psi^{(0)}_{\alpha\ell}(x : X, \varrho, U)$$

$\Uparrow$ Grassmann parameters

$$\underset{\sim}{\bar{\lambda}}_{\dot{\alpha}A} = 0$$

collective coordinates:

α^A_1
α^A_2 $\leftrightarrow \xi^A_\alpha$ SUSY zero modes

α^A_3
α^A_4 $\leftrightarrow \bar{\eta}_{\dot{\alpha}A}$ superconformal zero modes

$\underset{\sim}{A}_M^{cl}, \underset{\sim}{\phi}^{cl}, \underset{\sim}{\lambda}_\alpha^{A\,cl}$ is <u>not</u> a solution of the <u>full</u> e.o.m

<u>symptom</u>:

$$\frac{1}{g^2} S = \frac{8\pi^2}{g^2} + 4\pi^2 \rho^2 |a|^2 + c\,\bar{a}\,\bar{\eta}_1^A \bar{\eta}_{2A} \rho^2$$

⇒ superconformal zero modes "lifted" by VEV

selection rules altered

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quadratic fluctuation operators

<u>fermions</u>: $\Delta_F^{\dot\alpha\alpha} = \bar{\not{D}}_{cl}^{\dot\alpha\alpha}$ ← 4 zero modes

<u>gauge</u>: $(\Delta_B)_\alpha{}^\beta = (\not{D}_{cl})_{\alpha\dot\alpha} (\bar{\not{D}}_{cl})^{\dot\alpha\beta}$

also define ← no zero modes

$$(\bar{\Delta}_B)^{\dot\alpha}{}_{\dot\beta} = (\bar{\not{D}}_{cl})^{\dot\alpha\alpha} (\not{D}_{cl})_{\alpha\dot\beta}$$

yields 1:1 pairing of non-zero modes $\psi^{(\lambda)} \overset{1:1}{\longleftrightarrow} \bar{\psi}^{(\lambda)}$

$$\Rightarrow \mathcal{R} = \left(\frac{\det \bar{\Delta}_B}{\det' \Delta_B}\right)^{1/2} = 1 \;!!$$

major simplification true $\forall k$

<u>More precisely</u>. should regulate the theory with Pauli-Villars mass scale μ_{PV}

$$\mathcal{R}_{PV} = \left(\frac{\det \bar{\Delta}_B}{\det' \Delta_B}\right)^{1/2} \left(\frac{\det(\Delta_B + \mu_{PV}^2)}{\det(\bar{\Delta}_B + \mu_{PV}^2)}\right)^{1/2}$$

4 zero modes of $\Delta_B \Rightarrow$

$$\mathcal{R} = \mu_{PV}^4$$

total contribution of quadratic fluctuations:

= gauge + ghost + fermion + scalar

$$\mathcal{R} = \left(\frac{\det \bar{\Delta}_B}{\det' \Delta_B} \right)^{1/2}$$

zero eigenvalues removed

SUSY $\Rightarrow$ pairing of non-zero modes of Δ_B and $\bar{\Delta}_B$

<u>Exercise</u>:

suppose $\Delta_B \psi^{(\lambda)} = \lambda \psi^{(\lambda)}$, $\lambda \neq 0$

define $\bar{\psi}^{(\lambda)} = \bar{\not{D}}_{cl} \psi^{(\lambda)}$

then,

$$\bar{\Delta}_B \bar{\psi}^{(\lambda)} = \bar{\not{D}}_{cl} \not{D}_{cl} \bar{\not{D}}_{cl} \psi^{(\lambda)}$$

$$= \bar{\not{D}}_{cl} \Delta_B \psi^{(\lambda)} = \lambda \bar{\psi}^{(\lambda)}$$

Iterating E.O.M to $O(g^2)$

$\Rightarrow$ anti-fermions $\bar{\lambda}_{A\dot{\alpha}}$ turn on

$$\bar{\lambda}^{cl}_{A\dot{\alpha}} \to \xi_{\alpha A} S_F(x - X)^{*\alpha}_{\ \dot{\alpha}}$$

$|x - X| \gg |a|$ fermion propagator

The calculation:

$$\langle \bar{\lambda}(x_1) \bar{\lambda}(x_2) \bar{\Psi}(x_3) \bar{\Psi}(x_4) \rangle$$

$$= \int d^4X \prod_{i=1}^{4} S_F(x_i - X) \int d^4\xi \cdot \xi^4$$

$$\times \int d\rho \, d^2\bar{\eta} \, e^{-\rho^2|a|^2 - \bar{a}\rho^2\bar{\eta}\cdot\bar{\eta}}$$

finite $\int$

$$\underbrace{\times \mu_{PV}^4 \, e^{-\frac{8\pi^2}{g^2(\mu_{PV})} + i\Theta}}_{= \Lambda_{PV}^4}$$

$$\langle \bar\lambda(x_1)\,\bar\lambda(x_2)\,\bar\psi(x_1)\,\bar\psi(x_2)\rangle$$

$$= c_1 \left(\frac{\Lambda_{pv}^4}{a^6}\right) \int d^4X \prod_{i=1}^{4} S_F(x - X)$$

- after corefull matching of Λ_{SW} & Λ_{pv} coef c_1 precicely agrees with SW prediction

$\mathcal{F}_1 = 1/2$ Finel + Pouliot 1995

- test for $k=2$ (no matching required)
- works also

KD + Khoze + Mattis 1996

tests repeated for $k = 1, 2$ different gauge group + matter content

- Remarkalle all order calculation

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