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Introductory School on

RECENT DEVELOPMENTS IN SUPERSYMMETRIC GAUGE THEORIES

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NON-RENORMALIZATION THEOREMS IN SUSY GAUGE THEORIES (Part II)

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2. Symmetries & Selection Rules

- Continuous symmetry w/ parameter $\varepsilon \in \mathbb{R}$ & generator, or <u>charge</u> Q:

$$\mathcal{O}(0) \to \mathcal{O}(\varepsilon) = e^{i\epsilon Q}\,\mathcal{O}(0)\,e^{-i\epsilon Q}$$

any operator (field)

$$\simeq \mathcal{O}(0) + \underbrace{i\varepsilon\,[Q, \mathcal{O}(0)]}_{\equiv \epsilon\,\delta\mathcal{O}} + \cdots$$

- Main symmetry: <u>Poincaré</u>
  - <u>Space-time translations</u> generated by P_μ $\quad (x^\mu \to x^\mu + \varepsilon^\mu)$

$$\phi(x^\mu) \to \phi(x^\mu - \epsilon^\mu) \Rightarrow \delta\phi = -\epsilon^\mu \partial_\mu \phi \Rightarrow [P_\mu, \phi(x)] = i\partial_\mu \phi(x)$$

  - In local f.t. $P_\mu = \int d^3x\, T_{\mu 0}$ where $\partial^\mu T_{\mu\nu} = 0$ + $T_{\mu\nu} = T_{\nu\mu}$ is conserved energy-momentum tensor

$$T^{\mu\nu}_{(x)} \equiv 2\,\frac{\delta}{\delta g_{\mu\nu}(x)} \left[\int d^4x\, \mathcal{L} \right] \Big|_{g_{\mu\nu} = \eta_{\mu\nu}}$$

(generally covariantize $\mathcal{L}$ & put it in background metric $g_{\mu\nu}(x)$ ...)

  - <u>Lorentz transformations</u> generated by $M_{\mu\nu}$ ...

- Poincaré + locality $\Rightarrow^*$ $S = \int d^4x\, \mathcal{L}(\phi(x))$ w/ $\phi(x)$ transforming as finite dim'l rep'n of Lorentz + $\mathcal{L}$ a Lorentz scalar
(* at least at low enough energies.)

- <u>Other space-time symmetries</u>:
  - <u>Scale invariance</u>: $(x^\mu \to e^\varepsilon x^\mu)$

We used this in our RG discussion, with $e^\epsilon = \frac{\mu_0}{\mu}$.

Saw field of dimension Δ obeyed transformation

$$\phi(x) \to \tilde{\phi}(e^\epsilon x) = e^{-\epsilon\Delta}\phi(x)$$

or,

$$\tilde{\phi}(x) = e^{-\epsilon\Delta}\phi(e^{-\epsilon}x) \cong \phi(x) - \epsilon\left(\Delta\phi(x) + x\cdot\partial_x\phi(x)\right) + O(\epsilon^2)$$

$$\Rightarrow \quad \delta\phi = -\Delta\phi - x^\mu\partial_\mu\phi$$

- If we say it is generated by the dilatation operator D, then: $[D, \Phi(x)] = i\Delta\Phi(x) + ix^\mu\partial_\mu\Phi(x)$.
- 2^{nd} term suggests $D = \int d^3x\, x^\mu T_{0\mu}$ (since $i\partial_\mu \sim P_\mu \sim \int d^3x T_{0\mu}$)

Can show using Noether's theorem that dilatation current is given by

$$D_\mu = T_{\mu\nu}x^\nu \Rightarrow \boxed{\partial^\mu D_\mu = T^\mu_{\cdot\mu}}.$$

Thus scaling is a symmetry iff $T^\mu_{\cdot\mu} = 0$.

Exercise: In your favorite field theory, compute $T^{\mu\nu}$ and show $T^\mu_{\cdot\mu} \neq 0$ for any classically dimensionful couplings (use equations of motion).

- Can be generalized to conformal symmetry.

– Supersymmetry (& superconformal symmetry).
↳ Fermionic charges $Q_\alpha = \int d^3x\, S^0_\alpha$ w/ $\partial_\mu S^\mu_\alpha = 0$ where S^μ_α are supercurrents...

o By CM H-L-S, all other symmetries of (local, $d>2$, interacting) QFTs are internal symmetries:

Scalar conserved charges Q_i satisfy Lie algebra

$$[Q_i, Q_j] = i f_{ij}{}^k Q_k \qquad \text{(operator eqn.)}$$

- Simplest: U(1) symmetries generated by commuting charges:

$$[Q_i, Q_j] = 0 \qquad \forall i,j$$

- Examples: Free complex scalar: $\mathcal{L} \sim \partial_\mu\bar\Phi\partial^\mu\phi$;

Symmetry: $\phi(x) \to e^{i\epsilon q}\phi(x) \Rightarrow [Q, \phi(x)] = q\phi(x)$ $q \in \mathbb{R}$

q = "charge" of ϕ.

$Q = \int d^3x\, J^0$ w/ $\partial^\mu J_\mu = 0$ $J_\mu \sim i(\bar\Phi\partial_\mu\phi - \phi\partial_\mu\bar\Phi)$

Free spinor: $\mathcal{L} \sim \bar\Psi\not{\partial}\Psi$; $\Psi_\alpha \to e^{i\epsilon q}\Psi_\alpha$ etc.

$\Rightarrow J_\mu \sim \bar\Psi\sigma_\mu\Psi$.

Note: if Q_1 & Q_2 are 2 $U(1)$ symmetries then real lin. combos are too: $Q = a_1 Q_1 + a_2 Q_2$, $Q(\phi) = a_1 q_1 + a_2 q_2$.

$\therefore$ If many $U(1)$'s, get to pick basis & normalization.

- Non-abelian symmetries $f_{ij}{}^k \neq 0$. Once $f_{ij}{}^k$ specified, basis & normalization of Q_i's is fixed.
  - Example: Flavor symmetries of massless QCD

$$\mathcal{L} = \sum_{i=1}^{N_f \leftarrow \#\text{ flavors}} \left(\bar{\psi}^i \not{D} \psi_i + \bar{\tilde{\psi}}_i \not{D} \tilde{\psi}^i \right) + tr\left(F_{\mu\nu} F^{\mu\nu}\right)$$

w/ $\begin{cases} \psi_i \text{ transforming in rep'n } R \text{ of gauge group } G & \text{"quarks"} \\ \tilde{\psi}_i \quad " \quad " \quad " \quad \bar{R} \quad " \quad " \quad " \quad G & \text{"antiquarks"} \end{cases}$

($\bar{R}$ is complex conjugate of R; assume $R \neq \bar{R}$).

Then $U(N_f)_L = U(1) \times SU(N_f)_L$ mix ψ_i:

$$\begin{cases} \psi_i \to g_i{}^j \psi_j & \text{w/ } g_i{}^j = N_f \times N_f \text{ unitary matrix} \\ \tilde{\psi}^i \to \tilde{\psi}^i \end{cases}$$

and $U(N_f)_R = U(1)' \times SU(N_f)_R$ mix $\tilde{\psi}^i$:

$$\begin{cases} \psi_i \to \psi_i \\ \tilde{\psi}^i \to \tilde{g}^i{}_j \tilde{\psi}^j & \text{w/ } \tilde{g}^i{}_j \in U(N_f) \end{cases}$$

Note: L = "Left" + R = "Right" because if combine

$\Psi_i = \begin{pmatrix} \psi_i \\ \bar{\tilde{\psi}}_i \end{pmatrix}$ as Dirac 4-spinor, all transformin in Rep'n R of G.

So ψ_i is left-handed component of Ψ_i & $\bar{\tilde{\psi}}_i$ is right-handed component.

If Q^a_L & Q^a_R are generators of $SU_L(N_f)$ & $SU_R(N_f)$
& Q & Q' are " " $U(1)$ & $U(1)'$

Then usually define combinations

$$Q_A = Q - Q' \qquad \text{"axial } U(1)_A\text{"}$$

$$Q_B \equiv Q + Q' \qquad \text{"baryon number" } U(1)_B$$

$$\Rightarrow \begin{cases} U(1)_A: \Psi_i \to e^{i\epsilon\gamma_5}\Psi_i \Rightarrow \text{counts } \#\psi_i\text{'s} + \#\tilde{\psi}_i\text{'s}; \text{ 'chiral'} \\ U(1)_B: \Psi_i \to e^{i\epsilon}\Psi_i \Rightarrow \text{counts } \#\psi_i\text{'s} - \#\tilde{\psi}_i\text{'s}; \text{ non-chiral} \end{cases}$$

Note: Can also define combos:

$$Q_V^a \equiv (Q_L^a + Q_R^a) \quad \& \quad Q_A^a \equiv (Q_L^a - Q_R^a)$$

Algebra of Q_L, Q_R is

$$\left.\begin{array}{l} [Q_L^a, Q_L^b] = if^{ab}{}_c\, Q_L^c \\ [Q_R^a, Q_R^b] = if^{ab}{}_c\, Q_R^c \\ [Q_L^a, Q_R^b] = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} [Q_V^a, Q_V^b] = if^{ab}{}_c\, Q_V^c \\ \left.\begin{array}{l}[Q_A^a, Q_A^b] = if^{ab}{}_c\, Q_V^c \\ [Q_A^a, Q_V^b] = if^{ab}{}_c\, Q_A^c\end{array}\right] \leftarrow \text{don't close on } Q_A! \end{array}\right.$$

$\therefore$ Q_V^a generate "diagonal subgroup $SU(N_f)_V \subset SU(N_f)_L \times SU(N_f)_R$.

- More generally, If fields ϕ_{ir} $i = 1 \cdots N_r$ in rep R_r of gauge group, then have flavor symmetry $U(N_1) \times \cdots \times U(N_r)$.

- Mass terms in QCD break flavor symmetries: to be gauge invariant, ~~one~~ need to match R w/ $\bar{R}$ representations:

$$\mathcal{L}_{mass} = m_i^j \tilde{\psi}^i \psi_j + c.c.$$

With this term, the separate left & right flavor ~~symmetries~~ generators aren't conserved. E.g. if

$m_i^j = m\,\delta_i^j$, so all quarks have same mass, then only

$U(N_f)_V = U(1)_B \times SU(N_f)_V$ is still a symmetry.

Selection Rules

The main physical implication of continuous symmetries are conservation laws. However symmetries, even explicitly broken ones have subtler implications: selection rules.

- Example: Mass terms in QCD
  - Saw $m_i^j \tilde{\psi}^i_{(x)} \psi_{j(x)}$ breaks flavor symmetries, So the parameters m_i^j (numbers) govern the breaking.
  - "Localize" the parameters m_i^j; i.e. replace $m_i^j \to m_i^j(x)$, a field.
    - This is always possible: just turns mass term $\to$ Yukawa $m_i^j(x)\, \tilde{\psi}^i(x)\, \psi_j(x)$
    - Can recover original theory by simply setting $m_i^j(x) = \text{const}$.

- Now we can restore the flavor symmetry by the rule

$$\star\begin{cases} \psi_j \to g_j{}^k \psi_k \\ \tilde{\psi}^i \to \tilde{g}^i{}_\ell \tilde{\psi}^\ell \\ m_i{}^j \to (\tilde{g}^{-1})^k{}_i\, m_k^\ell\, (g^{-1})_\ell{}^j \end{cases} \qquad \text{where} \begin{cases} g \in U(N_f)_L \\ \& \ \tilde{g} \in U(N_f)_R \end{cases}$$

- On one hand, this invariance, even for constant m_k^ℓ, is just an identity (change of field variables). Thinking of m_k^ℓ as a field was just a crutch to come up with $\star$.
  So what is the use of $\star$?
  - Put m_k^ℓ in convenient form: e.g. diagonalize it.
  - If m_k^ℓ is only source of flavor symmetry breaking*, then $\star$ constrains form of possible symmetry-breaking effects.
  - E.g. some physical quantity, say a ~~scalar~~ mass, may get complicated quantum corrections, which will have some general dependence on the m_k^ℓ parameters:

$$M = M_0 + m_k^\ell (M_1)^k_\ell + c.c. + m_k^\ell m_i^j (M_2)^{ki}_{\ell j} + c.c. + m_k^\ell (m_i^j)^* (M_3)^{\ell j}_{ki} + \cdots \qquad (M_{0,1,2,\ldots} \ \text{constants, } m\text{-indep.})$$

But, to be invariant under $\star$ $\Rightarrow$

$$(M_1)^k_\ell = (M_2)^{\ell j}_{ki} = 0 \qquad (M_3)^{\ell j}_{ki} = \delta^{\ell j}_{\cdot}\,\delta_{ki}\, M_3 \quad \text{etc}\ldots$$

"Selection rules."

(* Note: assumed no other quantities in theory transform under $\star$. e.g. not dynamical breaking of flavor symm by vevs: $\langle \tilde{\psi}^\ell \psi_k \rangle$.)

- Will see more examples like this later.
- Generally, "localizing" trick promotes constants $\to$ fields. The reverse step, of taking these fields $\to$ constants is in field theory language, giving a vev to the fields.
  - If scalar vevs break symmetry, then is 'spontaneously' broken
  - If scalar vevs don't break symmetry, still have selection rules.

- **Noether's thm**: can be derived using the "localizing" trick:
  - Global symmetry: $\delta_\epsilon S = 0$ ϵ = constant parameter

    "localize" $\epsilon \to \epsilon(x)$. Stays symmetry of action if then gauge this new symmetry: (minimal coupling):

$$\partial_\mu \to D_\mu = \partial_\mu + q A_\mu(x) \quad \text{w/} \quad \delta A_\mu(x) = D_\mu \epsilon(x)$$

    Note: $A_\mu(x)$ is not dynamical $\Rightarrow$ no kinetic term.

    Then gauge invariance implies

$$0 = \delta S = \int d^4x\, D_\mu \epsilon \frac{\delta \mathcal{L}}{\delta A_\mu} = -\int d^4x\, \epsilon(x) D_\mu \frac{\delta \mathcal{L}}{\delta A_\mu}$$

$$\Rightarrow D_\mu \frac{\delta \mathcal{L}}{\delta A_\mu} = 0. \quad \text{So define current}$$

$$J^\mu \equiv \left.\frac{\delta \mathcal{L}}{\delta A_\mu}\right|_{A_\mu = 0} \quad \Rightarrow \quad \partial_\mu J^\mu = 0. \checkmark$$

  - Translation Invariance: $x^\mu \to x^\mu + \epsilon^\mu$ $\delta_{\epsilon^\mu} S = 0$.

    Localize: $\epsilon^\mu \to \epsilon^\mu(x)$ = diffeomorphism invariance

    & gauge $S \to S[g]$ by $\begin{cases} \eta^{\mu\nu} \to g^{\mu\nu}(x) \\ \partial^\mu \to D^\mu \quad g\text{-cov. derivative} \end{cases}$

    Again, $g^{\mu\nu}(x)$ not dynamical: no $R(g)$ term.

    Then diff. inv. $\Rightarrow$ $0 = \delta S = \int d^4x\, (D_\mu \epsilon_\nu + D_\nu \epsilon_\mu) \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}}$

$$= -2 \int d^4x\, \epsilon_\nu(x) D_\mu \left(\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} \right)$$

$$\Rightarrow D_\mu \left(2 \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} \right) = 0. \quad \text{Define } T^{\mu\nu} = 2 \left.\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}}\right|_{g_{\mu\nu} = \eta_{\mu\nu}} \Rightarrow \partial_\mu T^{\mu\nu} = 0.$$

  - Similar for scale invariance, conformal inv, susy ..

- **Weinberg-Witten thm**: a kind of selection rule for unbroken symmetries.
  - Global symmetry w/ group H, in theory w/ gauge group G.

    "localize" G : $\epsilon \to \epsilon(x)$, $\partial_\mu \to D_\mu^G = \partial_\mu + q_i A_\mu(x)$ (q_i = charges of ϕ_i)

    Localized theory now has $H \times G$ gauge group. But from our knowledge of how to write gauge-invariant kinetic terms for gauge group $G \times H$ $\Rightarrow$ gauge bosons of ~~H~~ G are not charged under ~~G~~ H.

∴ deduce: $\boxed{\text{No massless spin 1 particles can be charged under } H.}$ *

<u>NB</u>: • doesn't work if H is broken, or G higgsed: Higgs field can be charged under both H & G ⇒ massive spin 1's inherit charges.

• Higher spin ($j > 1/2$) fields all have gauge invariance (see Weinberg, QFT vol I), so implies (3): no massless $j \geq 1$ particles charged under global symmetries.

*• Different proof: Weinberg & Witten PLB <u>96</u> (1980) 59.

- <u>Translational invariance</u>: Generally covariantize translations $\epsilon_\mu \to \epsilon_\mu(x)$ by $\eta^{\mu\nu} \to g^{\mu\nu}(x)$ & $\partial_\mu \to D_\mu$ in kinetic term of spin $j \geq 3/2$ field. All these fields depend on extra constraints, which covariantize to mass terms.

<u>E.g.</u> $j = 3/2$: ψ_μ^α massless ⇒ connection depends on ψ_μ^α

$$\Rightarrow D^\mu T_{\mu\nu}\big|_{g=\eta} \not\Rightarrow \partial^\mu T_{\mu\nu} = 0.$$

<u>E.g.</u> $j = 2$: $h_{\mu\nu}$ massless ⇒ same story ... "no 2 graviton theory".

⋮

Deduce: $\boxed{\text{if } \partial^\mu T_{\mu\nu} = 0 \Rightarrow \text{only massless particles with } j < 3/2.}$ *

∘ <u>Gauge "symmetries"</u>: So far have only applied selection rule analysis to global symmetries. What about gauge symmetries?

• Gauge symmetries are <u>not</u> symmetries: they are redundancies in our field description. 2 field configurations related by a gauge transformation are identified as the <u>same</u> physical state. I.e., only gauge invariant operators are observables.

• But, at weak coupling (classically), gauge invariance looks like a local version of a global symmetry, so can treat the set of constant gauge transformations like a global symmetry and deduce similar results (charge conservation). For spont. broken gauge symm. (Higgs mechanism), selection rules automatic: ~~...~~ since breaking param = vev of field.

Supersymmetry selection rules

- Saw last lecture: IR eff. action:

$$\mathcal{L}_{eff} = \int d^2\theta \left[\mathcal{W}_{eff}(\tilde{\Phi}) + \tau_{eff}(\tilde{\Phi}) tr(\tilde{W}_\alpha \tilde{W}^\alpha) \right] + c.c.$$
$$+ \int d^2\theta \, d^2\bar{\theta} \; K_{eff}(e^V \tilde{\Phi}, \bar{\tilde{\Phi}}) \quad (+\text{ higher deriv. terms})$$

- Suppose we are given some AF SUSY g.t., so UV action is

$$\mathcal{L}_{UV} = \int d^2\theta \left[\mathcal{W}_{UV}(\Phi) + \tau \, tr(W^2) \right] + c.c. + \int d^4\theta \, \bar{\Phi} e^V \Phi$$

with $\mathcal{W}_{UV} = \mathcal{W}_{UV}(\Phi, \lambda_i) \overset{e.g.}{=} \lambda_1 \Phi^2 + \lambda_2 \Phi^3 + \cdots$

- Here $\{\Phi, W_\alpha\}$ in UV $\neq$ $\{\tilde{\Phi}, \tilde{W}_\alpha\}$ in IR, necessarily.
  & $\{\lambda_i, \tau\}$ are complex parameters (couplings) that enter into the theory.
- So question: how do $\mathcal{W}_{eff}$, τ_{eff}, & K_{eff} depend on the parameters $\{\lambda_i, \tau\}$ of the theory?
- A selection rule: "localize" $\{\lambda_i, \tau\}$ in superspace.
  I.e., since enter in same place as chiral superfields, treat $\lambda_i \to \lambda_i(y^\mu, \theta^\alpha)$ $\quad \tau \to \tau(y^\mu, \theta^\alpha)$
  $[y^\mu = x^\mu + i\bar{\theta}\sigma^\mu\theta]$ as chiral superfields. Then
  $\lambda_i = \langle \lambda_i(y^\mu, \theta^\alpha) \rangle$, $\tau = \langle \tau(y^\mu, \theta^\alpha) \rangle$: scalar vevs.
  - But, since chiral super-fields can only enter holomorphically in $\mathcal{W}_{eff}$ + τ_{eff} $\Rightarrow$ $\{\lambda_i, \tau\}$ can only enter holomorphically as well. Thus

$$\boxed{\frac{\partial}{\partial \bar{\lambda}_i} \mathcal{W}_{eff} = \frac{\partial}{\partial \bar{\tau}} \mathcal{W}_{eff} = \frac{\partial}{\partial \bar{\lambda}_i} \tau_{eff} = \frac{\partial}{\partial \bar{\tau}} \tau_{eff} = 0.}$$

Question: Are we getting something for nothing? No: all these arguments rely on the hard work of finding all local field representations of given symmetries. (w/ given number of derivatives). E.g. superfield formalism