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Introductory School on

RECENT DEVELOPMENTS IN SUPERSYMMETRIC GAUGE THEORIES

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NON-RENORMALIZATION THEOREMS IN SUSY GAUGE THEORIES (Part III)

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3. Non-Renormalization Theorem for Chiral Superfields

- Turn off vectors. Then susy selection rule gives:

  if $\mathcal{L}_{UV} = \int d^2\theta \, W_0(\Phi;\lambda) + c.c. + \int d^4\theta \, \Phi^\dagger \Phi$

  then $\mathcal{L}_{eff} = \int d^2\theta \, W_{eff}(\tilde{\Phi};\lambda) + c.c. + \int d^2\theta \, K_{eff}(\tilde{\Phi},\bar{\tilde{\Phi}};\lambda,\bar{\lambda})$

  + higher derivatives

  where in particular $\frac{\partial}{\partial\bar{\lambda}} W_{eff} = 0$.

- Note that don't have to assume $\{\tilde{\Phi}\}$ are same set of fields as UV fields $\{\Phi\}$.

- To get a non-renormalization theorem, we need to make some guess about the low-energy field content $\{\tilde{\Phi}\}$.

  Since this is just a theory of scalars + spinors, which are IR free for weak enough coupling, <u>guess</u> that

  $$\{\tilde{\Phi}\} = \{\Phi\}$$

  I.e. that IR field content is the same as the UV field content.

- Now we can see the "power of holomorphy":
  - Say $W_0 = \lambda \Phi$. Then can say that there is a $U(1)$ global symmetry $\Phi \to e^{i\epsilon}\Phi$ (under which Φ has charge $+1$), broken by λ. Thus, thinking of λ as a superfield, can restore this symmetry by assigning charge -1 to λ.

  - Without holomorphy, we could write ∞-ly many possible contributions to the effective superpot'le:

$$W_{eff} = \cdots + \cancel{\frac{\bar{\lambda}^2}{\Phi^2}} + \cancel{\frac{\bar{\lambda}}{\Phi}} + \lambda\Phi + \cancel{\lambda^2\bar{\lambda}\Phi} + \cdots + \cancel{\lambda e^{-1/|\lambda|^2}\Phi} + \cdots$$

– But with holomorphy, the only terms allowed are of the form

$$W_{eff} = \sum_\alpha (\lambda\Phi)^\alpha \qquad \alpha > 0$$

- This already rules out non-perturbative contributions,
- $\alpha > 0$ since as $\lambda \to 0$, theory is free, so $W_{eff} \to 0$.

– Note, however, that Kahler pot'l is ~~not~~ holomorphic, so don't get these restrictions.

– Seiberg summarized this argument prescriptively:

Constrain the effective superpot'l by

- holomorphy in UV couplings
- selection rules from global symmetries broken by the couplings,
- smoothness of the physics in weak coupling limits.

Non-renormalization of the superpotential

- Start w/ Wess-Zumino model of 1 Φ:

$$W_{\mu_0} = \frac{1}{2}\lambda_2 \Phi^2 + \frac{1}{3}\lambda_3 \Phi^3 .$$

∴ Eff. supot'l:

$$W_\mu = W_\mu(\Phi, \lambda_2, \lambda_3) \qquad \text{(holomorphy)}$$

- UV Kahler term invariant under $\boxed{U(1) \times U(1)_R}$ symmetry under which Φ has charge: $+1 \quad +1$.

Recall $U(1)_R$: rotates θ : $\begin{cases} \theta : 0 & +1 \\ d\theta : 0 & -1 \end{cases}$

∴ Supot'l inv't if W_{μ_0} : $0 \quad +2$ $\qquad (\int d^2\theta\, W_{\mu_0})$

$\Rightarrow$ Assign charges to $\boxed{\begin{matrix} \lambda_2 : -2 & 0 \\ \lambda_3 : -3 & -1 \end{matrix}}$

$\therefore \quad \mathcal{W}_\mu = \lambda_2 \Phi^2 \cdot g\left(\frac{\lambda_3 \Phi}{\lambda_2}\right)$ (selection rules),

Since under $U(1)$ $\mathcal{W}_\mu$ neutral; $U(1)_R$ $\mathcal{W}_\mu$ charge $+2$.

- In $\lambda_3 \to 0$ limit (w/ λ_2 fixed) theory is free,

  $\therefore$ only terms w/ non-negative powers of λ_3 can appear

$$\mathcal{W}_\mu = \sum_{n \geq 0} g_n \lambda_2^{1-n} \lambda_3^n \Phi^{n+2} \qquad \text{(weak cplg).}$$

- Can also take $\lambda_2 \to 0$ limit at same time* $\Rightarrow$ $n > 1$ not allowed.

$$\Rightarrow \mathcal{W}_\mu = g_0 \lambda_2 \Phi^2 + g_1 \lambda_3 \Phi^3 \qquad \text{(weak cplg)}$$

- At $\lambda_3 = 0$, theory free $\therefore$ ~~$Z=1$ (no wfn renorm) $\Rightarrow$~~ mass ~~doesn't run~~ runs canonically

  $\therefore$ ~~must have $g_0 \lambda_2$ = UV value $= \frac{1}{2}\lambda_2$ $\Rightarrow g_0 = \frac{1}{2}$~~.

  Really: in our conventions, mass runs as $\left(\frac{\mu_0}{\mu}\right)^*$ in free theory

  So get $g_0 \lambda_2 = \frac{1}{2}\left(\frac{\mu_0}{\mu}\right)^* \lambda_2 \Rightarrow g_0 = \frac{1}{2}\left(\frac{\mu_0}{\mu}\right)^*$ (weak cplg)

- Take λ_3 small; use perturbation theory to match $\mathcal{W}_\mu \Leftrightarrow \mathcal{W}_{\mu_0}$.

  Since the Φ^3 vertex appears in both proportional to the same coupling λ_3, they must match at tree level (classically) $\Rightarrow$ $g_1 = \frac{1}{3}\left(\frac{\mu_0}{\mu}\right)^{\circledast}$ (canonical running).

$$\therefore \quad \boxed{\mathcal{W}_\mu = \frac{1}{2}\left(\frac{\mu_0}{\mu}\right)^* \lambda_2 \Phi^2 + \frac{1}{3}\lambda_3 \Phi^3}$$

I.e. couplings are not renormalized:

no quantum corrections enter. (Only classical scaling).

- Exercise: generalize this argument to:

$$\mathcal{W}_{\mu_0} = \lambda_1 \Phi^1 + \lambda_2 \Phi^2 + \cdots + \lambda_r \Phi^r + \cdots$$

  & show all couplings not renormalized.

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* $\lambda_2 \to 0$ limit: If keep λ_3 finite, not free. But take both $\lambda_2, \lambda_3 \to 0$ s.t. $\lambda_2/\lambda_3 \to 0$, achieve desired result. (Since Wilsonian eff. act., don't need to worry about possible IR divergences as $\lambda_2 \to 0$.)

- Clever general proof (Weinberg)

$\mathcal{W}_{\mu_0} = \mathcal{W}_{\mu_0}(\Phi_n)$ arbitrary superpotential with arbitrary # chiral superfields

Write as $\mathcal{W}_{\mu_0} \leftrightarrow Y \cdot \mathcal{W}_{\mu_0}(\Phi_n)$

where Y is a new superfield. When $\langle Y \rangle = 1$, recover original theory. This enlarged theory is invariant under $U(1)_R$ where $R(Y) = +2$ + $R(\Phi_n) = 0$ $\forall n$.

This symmetry + holomorphy $\Rightarrow$ $\mathcal{W}_\mu = Y \cdot g(\Phi_n)$.

Then $Y \to 0$ limit $\Rightarrow$ theory is free. Match UV + IR effective actions in pert. theory $\Rightarrow$

$g(\Phi_n) = \mathcal{W}_{\mu_0}(\Phi_n)$ [w/ canonical scalings]

$\therefore$ $\mathcal{W}_\mu = Y \mathcal{W}_{\mu_0}$: Set $Y = 1$ to get original theory $\Rightarrow$

$\mathcal{W}_\mu = \mathcal{W}_{\mu_0}$: not renormalized.

- This non-renorm. result shows no contradiction w/ our original assumption that the IR d.o.f. = the UV d.o.f. In fact, holomorphy now proves it:

For $\mathcal{W}_{\mu_0}$ couplings small (so 2-loops << 1-loop) Coleman-Gross $\Rightarrow$ $\mathcal{W}_{\mu_0}$ IR-free. $\Rightarrow$ $\mathcal{W}_\mu = \mathcal{W}_{\mu_0}$ by above. But then analytic continuation in the couplings shows this for all couplings..

$\therefore$ WZ model has no non-trivial fixed points in RG flow at strong coupling.

- NB: This argument does not preclude the quantum generation of higher-derivative generalized superpotential terms.

Kahler term renormalization

- Since Kahler term not protected by this argument, it will get wave function renorm (+ higher derivs). So to see how the canonical couplings renormalize, define canonical xsf by

$$\Phi_n \to \Phi_n^{CN} \equiv \sqrt{Z_n(\mu)}\, \Phi_n$$

Then if supot'l is

$$\mathcal{W}_\mu = \left(\frac{\mu_0}{\mu}\right)^{3-\Delta_r} \lambda_r \mathcal{O}_r$$

w/ $\mathcal{O}_r = \prod_n \Phi_n^{r_n} \Rightarrow \Delta_r = \sum_n r_n$

Then

$$\mathcal{W}_\mu = \underbrace{\left(\frac{\mu_0}{\mu}\right)^{3-\Delta_r} \left(\prod_n Z_n^{-r_n/2}\right) \lambda_r}_{\equiv\ \lambda_r^{CN}(\mu)} \mathcal{O}_r$$

$$\therefore \boxed{\mu \frac{d\lambda_r^{CN}(\mu)}{d\mu} = \lambda_r^{CN}(\mu)\left(\Delta_r - 3 - \frac{1}{2}\sum_n r_n \gamma_n(\mu)\right)} \quad \star$$

where we defined the anomalous dimension of Φ_n as

$$\gamma_n(\mu) \equiv \frac{d \ln Z_n}{d \ln \mu}.$$

- ($\star$) is an exact RG equation — but we don't know how to compute the $\gamma_n(\mu)$ (exactly), so not much use.

Some physics (an example):

- Theory w/ 2 χsfs: Φ_1, Φ_2 & $\mathcal{W}_{\mu_0} = \lambda \Phi_1 \Phi_2^2$.
- Find, by extremizing $\mathcal{W}_{\mu_0}$ SUSY vacua when

$$\Phi_2 = 0 \quad \& \quad \Phi_1 = \text{arbitrary}.$$

$\Rightarrow$ Whole moduli space ($\mathbb{C}$-plane) of vacua $\mathcal{M}$: degenerate in energy, but inequivalent classical ground states.

- By non-renormalization theorem of superpot'l, this conclusion does not change once quantum effects are taken into account.
- But quant. effects can renormalize Kahler pot'l & thus change metric on $\mathcal{M}$ from its classical value.
  - Since $\mathcal{K}_{\mu_0} = \bar{\Phi}_1 \Phi_1 + \bar{\Phi}_2 \Phi_2$

    $\Rightarrow$ Kahler metric induced on $\mathcal{M}$ is $ds^2_{class} = d\Phi_1 d\bar{\Phi}_1$ (flat) $\mathbb{C}$.
  - Spectrum at any $\langle \Phi_1 \rangle$ is: 1 massless Φ_1

    & 1 massive χsf Φ_2 w/ mass $\sim |\lambda \langle \Phi_1 \rangle|$.

  $\therefore$ At scales $>$ mass Φ_2, virtual Φ_2 loops contribute to Φ_1 propagator $\Rightarrow$ (1-loop p.t.)

$$\mathcal{K} = \bar{\Phi}_1 \Phi_1 - (\#)\, \bar{\Phi}_1 \Phi_1 |\lambda|^2 \ln \left| \frac{\Phi_1}{\mu_0} \right|^2 + \cdots$$

(for IR ~~xxxx~~ effective action scale $\mu <$ mass Φ_2.)

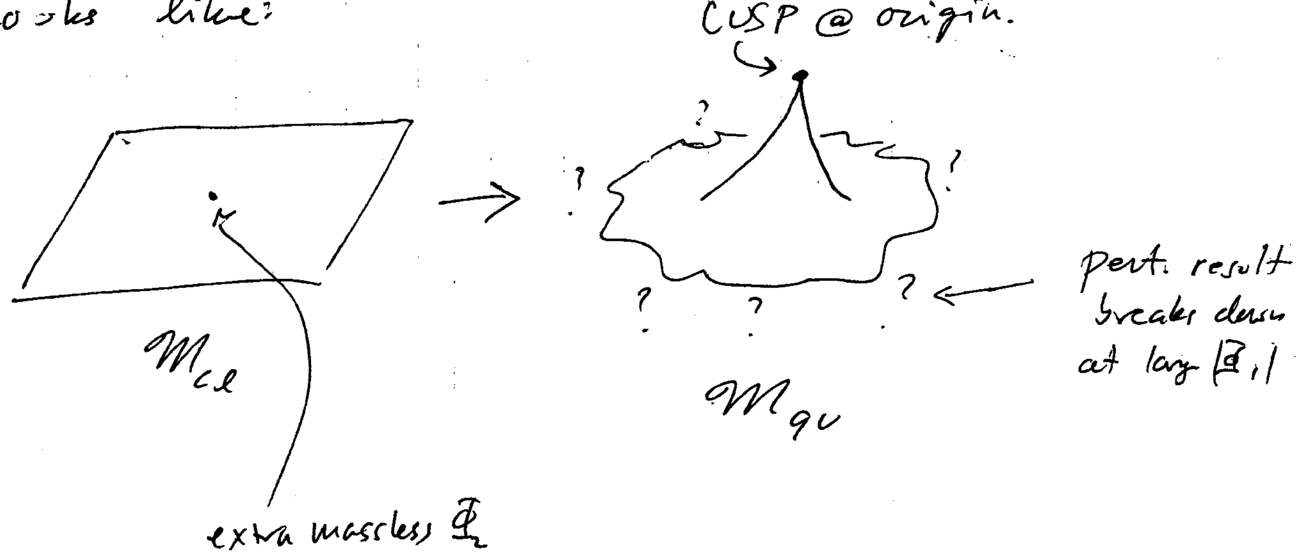
  - As $\Phi_1 \to 0$, $\mathcal{K} \to \infty$, $\Rightarrow$ canonical couplings $\to 0$

    $\Rightarrow$ 1-loop becomes better near origin of $\mathcal{M}$
  - Kahler metric

$$ds^2_{\mu} = \partial_1 \partial_{\bar{1}} \mathcal{K}\, d\Phi_1 d\bar{\Phi}_1 \simeq \left(-|\lambda|^2 \log \Phi_1 \bar{\Phi}_1 + \text{const}\right) d\Phi_1 d\bar{\Phi}_1$$

$\Rightarrow$ metric singularity @ $\Phi_1 = 0$.

Looks like:



What does the cusp singularity mean? Just that the assumption under which we computed the IR eff. action broke down: we assumed $\mu <$ mass Φ_2. But as $\Phi_2 \to 0$ this can't remain true for finite μ.

General lesson: singularities in IR eff. actions are sign of new massless degrees of freedom. To get a smooth description of the physics there, you need to include them.