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Introductory School on

**RECENT DEVELOPMENTS
IN SUPERSYMMETRIC GAUGE THEORIES**

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NON-RENORMALIZATION THEOREMS IN SUSY GAUGE THEORIES (Part IV)

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4. Anomalies

In computing quantum corrections, we need to regulate in UV. Some symmetries of classical theory are always broken by the regularization, & it can happen that, even in the limit when the regulator is removed, the classical symmetry remains broken. This is called an <u>anomaly</u>. Furthermore, we'll show that a class of these anomalies, the <u>chiral anomalies</u>, are not just UV artefacts, but persist & have effects at all scales.

Trace Anomalies

Most important anomaly is scale invariance anomaly. Recall dilatation current D^μ:

$$\partial^\mu D_\mu = T^\mu_{\cdot\mu} .$$

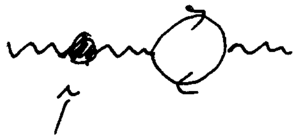
Even when $T^\mu_{\cdot\mu}$ vanishes classically, it is a composite operator (product of fields at same point), so must be regulated quantumly.

<u>Example</u>: QCD w/ dimensional regularization (dim'n d) $\Rightarrow$

$$T^\mu_{\cdot\mu} = \frac{d-4}{4} \operatorname{tr}(F_{\mu\nu}F^{\mu\nu}) + (1-d)\, i\bar\psi \not{D} \psi$$

Classically: $d=4$ & ψ e.o.m $\Rightarrow T^\mu_{\cdot\mu} = 0$. ✓

Quantumly: ~~compute~~ compute in background field

 $\Rightarrow \boxed{T^\mu_{\cdot\mu} = \frac{b}{32\pi^2}\operatorname{tr}(F^2)} + O(g^2)$

$\underbrace{T^\mu_{\cdot\mu}}_{\frac{d-4}{4}}$ $\quad \underbrace{\sim}_{\frac{1}{d-4}}$ $\rightarrow$ finite

∘ On other hand, under scaling

$$x \to e^{\epsilon} x \quad \Rightarrow \text{(Noether...)} \quad T^{\mu}{}_{\mu} = \left.\left(\frac{\partial \mathcal{L}}{\partial \epsilon} - 4\mathcal{L}\right)\right|_{\epsilon=0}$$

$$= \frac{\partial\left(\frac{-1}{4g^2}\right)}{\partial \epsilon}\, tr(F^2)$$

$$\Updownarrow \quad T^{\mu}{}_{\mu} = \frac{1}{4}\mu\frac{\partial}{\partial\mu}\left(\frac{1}{g^2}\right) tr(F^2)$$

$\therefore$ learn $\boxed{\frac{\partial g^{-2}}{\partial \varepsilon} = -\frac{b}{8\pi^2} + \mathcal{O}(g^2)}$

Under $x \to e^{\epsilon} x \Rightarrow \mu_0 \to e^{-\epsilon}\mu_0 \equiv \mu \Leftrightarrow \epsilon = \ln\left(\frac{\mu_0}{\mu}\right)$

$$\therefore \quad \frac{1}{g^2(\epsilon)} - \frac{1}{g^2(0)} = -\frac{b\epsilon}{8\pi^2} \Leftrightarrow \boxed{\frac{1}{g^2(\mu)} - \frac{1}{g^2(\mu_0)} = \frac{+b}{8\pi^2}\ln\left(\frac{\mu}{\mu_0}\right)}$$

$$\therefore \text{ if } \begin{cases} b < 0 \Rightarrow g \to 0 \text{ in IR} \\ b > 0 \Rightarrow g \to 0 \text{ in UV} \end{cases}$$

Also implies that combination

$$\boxed{|\Lambda| \equiv \mu\, e^{-\frac{8\pi^2}{b g^2(\mu)}} \quad \text{is } \mu\text{-independent (1-loop)}}$$

Strong-coupling scale "dimensional transmutation"

∘ Computation of <u>b coefficient</u> gives

$$\boxed{b = \frac{11}{6} T(adj) - \frac{1}{3}\sum_i T(R_i) - \frac{1}{6}\sum_a T(R_a)}$$

$$\begin{cases} i = \text{Weyl fermions in rep'n } R_i \text{ of gauge group} \\ a = \text{Complex scalars " " } R_a \text{ " " "} \end{cases}$$

$$T(R) \equiv \frac{C(R)}{C(\text{fund.})} \quad \text{index of representation}$$

& define Casimir by $\quad tr_R(T^a T^b) = C(R)\,\delta^{ab}$

For classical gauge groups:

dynkin	G	dim'n fund.	C(fund)	C(adj)	dim'n adj = dim G	rank G
C_N	$Sp(2N)$	$2N$	$1/2$	$N+1$	$N(2N+1)$	N
A_{N-1}	$SU(N)$	N	$1/2$	N	N^2-1	$N-1$
$B_{\frac{N-1}{2}}$ or $D_{N/2}$	$SO(N)$	N	1	$N-2$	$N(N-1)/2$	$[N/2]$

- Note $C(\text{fund})$ defines standard normalizations for gauge generators, if couplings defined by

$$\mathcal{L} = -\frac{1}{2g^2} \mathrm{tr}_f(F^2) + \frac{\theta}{16\pi^2} \mathrm{tr}_f(F\tilde{F})$$

[tr_f = trace in fundamental.]

- Thus, for $SU(N_c)$ QCD w/ N_f flavors in fundamental (N_c)

$\Rightarrow$ $2N_f$ Weyl fermions & 0 bosons } in fundamental

& $T(\text{adj}) = 2N_c$ $\quad T(\text{fund.}) = 1$

$$\therefore \quad b = \frac{11}{6} 2N_c - \frac{2N_f}{3} = \frac{11N_c - 2N_f}{3}.$$

- With many gauge groups: $G_1 \times G_2 \times \cdots \times G_n$

$\Rightarrow$ have n separate scales $\Lambda_1, \Lambda_2 \cdots \Lambda_n$.

- Works for $U(1)$ factors as well: representations all 1-dim'l, denoted by $T = q$ charge. Adjoint rep $\Leftrightarrow q = 0$.

& $T(R_q) = q^2$. $\Rightarrow$ All $U(1)$'s are IR free.

- Other dimensionless couplings can also run: $\lambda\phi^4$, $g\phi\psi^2$ etc.

E.g. Wv. fn. renorm $Z(\partial\phi)^2 \Leftrightarrow \frac{d\ln Z}{d\ln\mu} = \gamma$ anom. dimension.

- One other gauge coupling: theta angle $\sim \frac{\theta}{16\pi^2} \mathrm{tr}(F\tilde{F})$.
  - Covered in Dorey's lectures:
  - $\frac{1}{16\pi^2} \int d^4x\, \mathrm{tr}_f(F\tilde{F}) = n$ instanton #.

  $\therefore$ in path integral $\int D\phi \ldots e^{iS} \sim \int \ldots e^{i\theta n}$

  $\Rightarrow$ $\boxed{\theta \simeq \theta + 2\pi}$, hence angle.

  - Since $\mathrm{tr}(F\tilde{F})$ = total derivative $\Rightarrow \theta$ does not run perturbatively.
  - Recall $\tau(\mu) = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2(\mu)}$, $\quad (\tau \simeq \tau + 1)$,

then can define complex RG inv't scale:

$$\boxed{\begin{aligned} \Lambda &\equiv |\Lambda| e^{i\theta/b} = \mu e^{2\pi i \tau(\mu)/b} \\ \text{or} \quad \tau(\mu) &= \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) \end{aligned}}$$

- So dimensional transmutation trades

complex coupling $\tau(\mu)$ $\longleftrightarrow$ complex scale Λ.

- At 1-loop, write (SUSY) gauge kinetic terms as

$$\boxed{\mathcal{L}_\mu = \int d^2\theta \, \frac{+1}{32\pi^2} \ln\left(\frac{\Lambda^b}{\mu^b}\right) tr_f (W_\alpha W^\alpha)} + c.c.$$

~~o Selection rule: think of~~ Anomaly now manifest $-\mu\frac{\partial}{\partial\mu}$ ...

Chiral Anomalies

- The other kind of symmetry always broken by regulators are chiral symmetries. Chiral symmetries are those in which the left-handed fermions (ψ_α) transform differently than the right-handed fermions ($\bar\psi^{\dot\alpha}$). Since (in 4-dim) they are complex conjugates, if the ψ_α transf. in rep'n R, the $\bar\psi^{\dot\alpha}$ transf. in rep'n $\bar R$

- In terms of hermitian generators, if

$$g \equiv e^{iT} \qquad g^* \equiv e^{i\bar T}$$

$$\hookrightarrow \Rightarrow g^* = e^{-iT^*} = e^{i(-T^t)}.$$

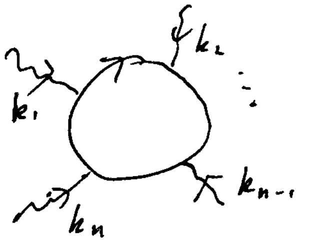
Thus if $\bar T^a \simeq -T^{a t}$ $\Rightarrow$ rep'n is <u>real</u> (or <u>pseudoreal</u>)
(up to change of basis)

Otherwise is <u>complex</u>. (Only $U(1)$, $SU(n)$, $SO(4n)$, E_6 have complex reps.)

- E.g., in Pauli-Villars regularization you give mass to all fields. But if ψ is in complex rep'n, then the $\psi\psi$ mass term cannot be invariant under chiral symmetry.

- So, want to compute quantum corrections to
$$\partial_\mu J^\mu = 0 \quad w/ \quad J^{a\mu} = \bar{\Psi}\,\sigma^\mu T^a \Psi + \cdots$$

Strategy: think of this symmetry as a gauge symmetry $\sim A^a_\mu J^{a\mu}$ (localize transf. parameter). Then we just need to check gauge invariance: @ 1-loop, look at n gauge boson amplitude: send in n on-shell gauge bosons w/ momenta k_i^μ & polarizations ξ_i^μ
$$k_i^2 = \xi_i \cdot k_i = 0, \quad \sum_i k_i^\mu = 0$$



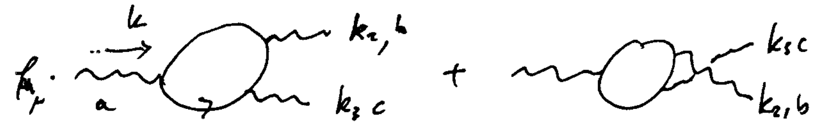
} Gauge invariance $\Rightarrow$ longitudinal ($\xi^\mu \propto k^\mu$) gauge bosons should decouple

$$A_n \sim \mathrm{Tr}(T^{a_1}\cdots T^{a_N}) \int d^4p \; \mathrm{tr}\left(\frac{\not p_1}{p_1^2}\not\xi_1 \frac{\not p_2}{p_2^2}\cdots \not\xi_n\right)$$

Put $(n-1)$ lines as background fields
& Longitudinal line $\sim \partial_\mu J^\mu_a$

So computes $\partial_\mu J_a^\mu = (\#)_{\cdots}\, F_2^{\mu\nu} F_3^{\rho\sigma} \cdots F_N^{\cdots}$

$(\#)_{\cdots} \propto \epsilon_{\mu\nu\rho\sigma}$ since if not, then can regularize.
So in d dim'ns, smallest $N = \frac{d+2}{2}$ to see an effect.
In 4-dim'n: look at 3-point diagram



$+ \quad = G$

$$G = \# \sum_i \mathrm{Tr}_{R_i}\left(T^a\{T^b, T^c\}\right) \epsilon_{\mu\nu\rho\sigma}\, k_2^\mu k_3^\nu \xi_2^\rho \xi_3^\sigma$$

$$\therefore \boxed{\partial_\mu J^\mu_a = (\#) \sum_i \mathrm{tr}_{R_i}\left(T_a\{T_b, T_c\}\right) \epsilon_{\mu\nu\rho\sigma} F_b^{\mu\nu} F_c^{\rho\sigma}} \quad \star$$

- Note: Only need to trace over massless fermions in loop, since massive ones (as saw above) don't contribute. (Massive in real rep'n.)
- Adler-Bardeen: higher loops don't contribute. (Higher-point 1-loop do contribute, but just related by gauge invariance.)

- So (★) is (perturbatively) exact. What does it mean?
  - if $\sum_r \mathrm{tr}_r(T_a\{T_b, T_c\}) = 0$, no chiral anomalies.

{ Only simple groups w/ $\mathrm{tr}(T_a\{T_b, T_c\}) \neq 0$ are $SU(N)$, $N \geq 3$. ($SO(6) = SU(4)$)
{ But when also $U(1)$ factors, can get non-zero.

- Now: we "localized" our global chiral symmetries to make them gauge symmetries, say with group G. There could also be a chiral gauge symmetry of group H. (E.g. $SU(2) \times U(1)$ EW ; not $SU(3)$ QCD, though.)

So chiral symmetries $\underset{\text{global}}{G} \times \underset{\text{gauge.}}{H}$

& $\{T^a\}$ all generators, split into $\{T^a_G, T^b_H\}$

But note that the global symmetries don't have associated field strengths (their gauging was a fake).

⇒ There are 2 cases.

(1.) Gauge anomalies : All $T^a \in H$

⇒ $\partial^\mu J_{\mu H} \neq 0$ ⇒ Gauge invariance is broken.

⇒ The theory is inconsistent (non-unitary).

E.g. Standard model: check that cancel. ✓ (see Weinberg.)

(Note: in 4-dim: $U(1)$ gauge theories also can have mixed gauge-gravitational anomalies. Also must cancel.)

(2.) Chiral anomalies : 1 $T^a \in G$, 2 $T^a \in H$

$$\Rightarrow \partial_\mu J^\mu_G = \frac{1}{32\pi^2} \sum_i \mathrm{tr}_i(T_G\{T^a_H, T^b_H\})\, F^{\mu\nu}_a \tilde{F}_{b\mu\nu}$$

⇒ Anomaly in (global) chiral symmetry: it is broken.

Evaluate r.h.s.: $\mathrm{tr}_{R_G R_H}(T_G\{T^a_H, T^b_H\}) = 2\,\mathrm{tr}_{R_G}(T_G)\,\mathrm{tr}_{R_H}(T^a_H T^b_H)$
$= 2\,\mathrm{tr}_{R_G}(T_G)\, \delta^{ab} C(R_H)$

Now, for semi-simple Lie algebras $tr_R(T) = 0$.

$\therefore$ Only anomalies in $U(1)$ chiral symmetries

$\Rightarrow Tr_{H_i}(T_G) = q_i$ ← charge of i^{th} fermion under $U(1)_G$

$$\therefore \quad \partial^\mu J_\mu = \frac{1}{16\pi^2} \sum_i q_i \, C(R_i) \, F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

$$\boxed{\partial^\mu J_\mu = \frac{1}{16\pi^2} \sum_i q_i \, T(R_i) \, tr_f (F\tilde{F})}$$ Chiral anomaly.

sum over L-weyl fermions

$\boxed{\psi_i \to e^{iq_i\epsilon}\psi_i}$ ← U(1) charges   gauge reps.

Notes:
- Also anom $\propto \left(\sum_i q_i\right)$ for gravitational theories.
- If 2 or 3 lines have global T_G's $\Rightarrow$ no anomaly. But will see has interesting consequence as low-energy selection rule.

Selection rule

Since anomaly $\propto tr_f(F\tilde{F})$ which is θ-term in action, can restore chiral invariance if treat θ parameter as field & let it transform as

$$\begin{cases} \psi^i \to e^{iq_i\epsilon}\psi^i \\ \theta \to \theta + \epsilon \sum_i q_i T(R_i) \end{cases}$$

For then $\overset{\text{adds to}}{\Rightarrow} \partial^\mu J_\mu = \frac{\delta \mathcal{L}}{\delta \epsilon} = \frac{\delta}{\delta\epsilon}\left(\frac{\theta}{16\pi^2} tr_f(F\tilde{F})\right)$ = r.h.s of anomaly.

- In this way chiral anomaly is now seen in classical action as explicit breaking.
- Since anomaly appears only through θ terms, $\Rightarrow$ at most 1 anomalous U(1) per simple gauge factor (by making appropriate linear combos of all U(1) generators).

• Other implications: anomalous $U(1)$ charge violation

$$\Delta Q = \int_{-\infty}^{\infty} dt\, \partial_0 Q = \int dt\, d^3x\, \partial_0 J_0$$
$$= \int d^4x \Big(\underbrace{\cancel{\vec{\nabla}\cdot\vec{J}}}_{\text{surface term}} + \frac{\sum_i q_i T(R_i)}{16\pi^2} \mathrm{tr}_f (F\tilde{F}) \Big)$$
$$= \Big[\sum_i q_i T(R_i)\Big]\, n \leftarrow \text{instanton number.}$$

But (Dorey's lectures), instanton ~~changing~~ processes suppressed at weak coupling by $e^{-\frac{8\pi^2}{g^2}|n|}$.
So though $U(1)$ broken, at weak gauge coupling the magnitude of breaking effects is tiny. (e.g. Baryon # violation in $SU(2)_{EW}$.)

• If $\exists$ massless chiral fermion, $\Rightarrow \exists$ chiral symmetry broken by anomaly. Can use to shift $\theta \to 0$.
$\therefore$ In such a theory, θ angle not observable.
(e.g. if u-quark massless (no Yukawa) $\Rightarrow$ no strong CP problem in QCD. Of course, naturalness for u Yukawa...)

• 't Hooft anomaly matching: skip.

Summary:

Scale & chiral anomalies in gauge theories summarized in classical action by

$$S = \int d^4x \Big[\frac{i\tau}{16\pi} \mathrm{tr}(\mathcal{F})^2 + c.c. \Big]$$

w/ $\tau(\mu) = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2(\mu)} = \frac{1}{2\pi i} \ln\left(\frac{\Lambda^b}{\mu^b}\right) \qquad \Lambda^b = \mu^b e^{2\pi i \tau}$

w/ $\theta \to \theta + \varepsilon\left(\sum_i q_i T(R_i)\right) \Rightarrow \Lambda^b \to e^{i\varepsilon\left(\sum_i q_i T(R_i)\right)} \Lambda^b$

$$\therefore \boxed{\begin{array}{l} S = \int d^4x \Big[\frac{1}{32\pi^2} \ln\left(\frac{\Lambda}{\mu}\right)^b \mathrm{tr}(\mathcal{F}^2) + c.c. \Big] \\ \text{w/ } b = \frac{11}{6} T(\mathrm{adj}) - \frac{1}{3}\sum_i T(R_i) - \frac{1}{6}\sum_a T(R_a) \\ \text{\& } \Lambda^b \text{ has charge } \sum_i q_i T(R_i) \text{ under } U(1) \text{ chiral.} \end{array}}$$

So scale & chiral breaking explicit.

- In $\underline{N=1 \text{ superspace language}}$ this becomes:

$$S = \int d^4x \left\{ \left[\int d^2\theta \, \frac{1}{32\pi^2} \ln\left(\frac{\Lambda^b}{\mu^b}\right) \mathrm{tr}(W^2) \right] + \text{c.c.} + \text{supo'l \& Kahler} \right\}$$

– $\underline{\text{Evaluate } b, \text{ change } \Lambda^b}$:

- Say have Φ_i in R_i rep of gauge group

$$\Rightarrow \quad \Phi_i \supset \{ \underset{\substack{\uparrow \\ \text{complex} \\ \text{scalar}}}{\phi_i} , \underset{\substack{\uparrow \\ \text{weyl} \\ \text{spinor}}}{\psi_i} \}$$

Also, have $W_\alpha \supset \{ \underset{\substack{\uparrow \\ \text{adjoint} \\ \text{Weyl}}}{\lambda_\alpha} , \underset{\substack{\uparrow \\ \text{gauge boson.}}}{F_{\mu\nu}} \}$

$$\therefore \quad b = \underset{\substack{\uparrow \\ F}}{\frac{11}{6} T(\mathrm{adj})} - \frac{1}{3}\Big(\underset{\substack{\uparrow \\ \lambda}}{T(\mathrm{adj})} + \underset{\substack{\uparrow \\ \psi_i}}{\sum_i T(R_i)} \Big) - \frac{1}{6}\Big(\underset{\substack{\uparrow \\ \phi_i}}{\sum_i T(R_i)} \Big)$$

or

$$\boxed{b = \frac{1}{2}\Big(\underset{\substack{\uparrow \\ W_\alpha}}{3\,T(\mathrm{adj})} - \underset{\substack{\uparrow \\ \Phi_i}}{\sum_i T(R_i)} \Big)}$$

- $\underline{\text{What is } \Lambda^b \text{ change}}$: $\sum\limits_{\substack{i \\ \forall \text{ Weyl}}} \hat{q}_i \, T(\hat{R}_i)$?

  $\Rightarrow$ depends on q_i i.e. on definition of chiral $U(1)$.

  - Since there are many $U(1)$'s in free (massless) theory, there are many choices...

(1.) <u>Axial flavor $U(1)$'s</u>:

Symmetry under which only one chiral supfld rotates: (class. symm if no superpot'l)

$$U(1)_i \equiv \begin{cases} \Phi_i \to e^{i\varepsilon}\Phi_i & \Rightarrow \psi_i \text{ has charge } 1 \\ \Phi_j \to \Phi_j \quad j\neq i & \Rightarrow \psi_j\ (j\neq i) \text{ has charge } 0 \\ W_\alpha \to W_\alpha & \Rightarrow \lambda_\alpha \text{ has charge } 0 \end{cases}$$

$$\Rightarrow \sum_{\text{ferms}} q\, T(R) = T(R_i)$$

So Λ^b has $U(1)_i$ charge $T(R_i)$

(2.) <u>$U(1)_R$ anomaly</u>:

- Under $U(1)_R$ $\quad \theta \to e^{i\varepsilon}\theta \quad$ (i.e. $R[\theta]=+1$, $\therefore R[d\theta]=-1$)

$\Rightarrow$ For symmetry, need $R[\tau\, \mathrm{tr}\, W^2] = +2$.

- An especially convenient such R symmetry is:

	$U(1)_R$		
Φ_i	$2/3$	$\Rightarrow R[\psi_i] = -1/3$	$(R[\phi_i] = 2/3)$
W_α	1	$\Rightarrow R[\lambda_\alpha] = +1$	$(R[F] = 0)$ ✓

This is classical symmetry because $R[\mathrm{tr}(W^2)] = +2$ ✓

and $R[\bar\Phi\Phi] = 0$. Kahler terms invariant. ✓

$$\Rightarrow \sum_{\text{ferms}} q\, T(R) = T(\text{adj}) - \frac{1}{3}\sum_i T(R_i) = \frac{2}{3} b$$

$\therefore$ Λ^b has $U(1)_R$ charge $\frac{2}{3}b$, $\sim R[\Lambda] = \frac{2}{3}$.

- Many other R symms possible: (e.g. $U(1)_R + \sum_i \alpha_i U(1)_i$)

but will call this one <u>the</u> $U(1)_R$.

Superspace form of anomalies:

(1.) Konishi anomaly $\quad U(1)_i: \quad \Phi_i \to e^{i\epsilon}\Phi_i$, rest don't change

$$\partial_\mu J_i^\mu = \frac{T(R_i)}{16\pi^2}\, tr(F\tilde{F})$$

Can compute J_i^μ from Noether's thm & supersymmetrize...
(Clark, Piguet, Sibold; Konishi)

Or: note that classically, $\partial^\mu J_\mu = 0$ follows from equations of motion:

$\phi \to \phi + \epsilon f(\phi)$ is symmetry $\Rightarrow$

$$\Big[\underbrace{-\partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial\,\partial_\mu\phi}\Big) + \frac{\partial\mathcal{L}}{\partial\phi}}_{\text{e.o.m.}}\Big]\cdot f = 0$$

Copy this for superspace. Superspace e.o.m.:

$$\frac{\delta S}{\delta\Phi(y,\theta)} = 0 \qquad S = \int d^4x\Big[\int d^4\theta\, \bar{\Phi} e^V \Phi + \int d^2\theta\,(\mathcal{W}(\Phi) + \tau W^2) + c.c.\Big]$$

$$= \int d^4y\, d^2\theta\,\big[\bar{D}^2(\bar{\Phi}e^V\Phi) + \mathcal{W}(\Phi) + \tau W^2\big] + \int d^4x\, d^2\bar{\theta}\,\big[\bar{\mathcal{W}}(\bar{\Phi}) + \tau\bar{W}^2\big]$$

Treat Φ, & $\bar{\Phi}$ independently

$\Rightarrow$ superfield e.o.m

$$0 = \frac{\delta S}{\delta\Phi_i(y,\theta)} = \bar{D}^2(\bar{\Phi}_i e^V) + \frac{\partial\mathcal{W}}{\partial\Phi_i}$$

For $U(1)_i$: $\Phi_i \to e^{i\epsilon}\Phi_i \simeq \Phi_i + i\epsilon\,\underset{\text{"}f\text{"}}{\Phi_i} \Rightarrow$

$$0 = \underbrace{\bar{D}^2(\bar{\Phi}_i e^V)\Phi_i}_{\partial_\mu J_i^\mu} + \underbrace{\frac{\partial\mathcal{W}}{\partial\Phi_i}\Phi_i}_{\text{explicit breaking terms in superpot}} \qquad (\text{no sum on } i)$$

Take θ^2 component:

$\therefore$ Superspace version of $\partial_\mu J_i^\mu = \cdots \Rightarrow$

$$-\bar{D}^2(\bar{\Phi}_i e^V\Phi_i) = +\,\Phi_i\frac{\partial\mathcal{W}}{\partial\Phi_i}$$

But, quantumly, has anomaly on r.h.s. $\approx \frac{T(R_i)}{32\pi^2}\underbrace{tr(\cancel{F^2})\cdot\theta^2}_{tr(W^2)}$

$$\therefore \quad \boxed{-\bar{D}^2(\bar{\Phi}_i e^V\Phi_i) = \Phi_i\frac{\partial\mathcal{W}}{\partial\Phi_i} + \frac{T(R_i)}{32\pi^2}\,tr(W^2)} \quad \text{Konishi anom.}$$

(2.) <u>$U(1)_R$ anomaly</u>: Can't use same trick as above to super-covariantize, because $U(1)_R$ does not commute w/ SUSY generators.

<u>Recall</u>: $U(1)_R$: $\begin{cases} \Phi_i \to e^{i\epsilon\frac{2}{3}}\Phi_i & \forall i \\ W_\alpha \to e^{i\epsilon}\, W_\alpha \end{cases}$

& got anomaly: $\partial_\mu J^\mu_R = \frac{\frac{2}{3}b}{32\pi^2}\,tr(F\tilde{F})$ + explicit breaking $W \sim \Phi^n$ $n \neq 3$

<u>But</u>: scale: $\begin{cases} \Phi_i \to e^{\delta}\,\Phi_i & \forall i \\ W_\alpha \to e^{\frac{3}{2}\delta}\,W_\alpha \end{cases}$

& got anomaly: $T_\mu{}^\mu = \partial_\mu D^\mu = \frac{b}{32\pi^2}\,tr(FF)$ + explicit breaking $W \sim \Phi^n$ $n \neq 3$

$$\Rightarrow \quad \frac{2}{3}T_\mu{}^\mu + i\,\partial_\mu J^\mu_R = \frac{2}{3}\cdot\frac{b}{32\pi^2}\,tr(\mathcal{F}^2) + \text{''}$$

$\downarrow$ supersymmetrize to θ^2 comp. of

$$= \frac{2}{3}\cdot\frac{b}{32\pi^2}\,tr(W^2) + 3W - \sum_i \Phi_i\frac{\partial W}{\partial \Phi_i}$$

This indicates that scale + $U(1)_R$ currents should appear together in same superfield as $\frac{2}{3}D^\mu + iJ^\mu_R$.

Turns out: real gen superfield V_μ ~ "current multiplet"

$$V_\mu = \left(\tfrac{2}{3}D_\mu + i J^R_\mu\right) + \theta^\alpha S_{\mu\alpha} + c.c. + \bar\theta\sigma^\nu\theta\left\{\tfrac{2}{3}T_{\mu\nu} + \partial_{[\mu}J^R_{\nu]}\right\} + \cdots$$

Defining $V_{\alpha\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha}V_\mu$, then compute ...

$$\bar D^{\dot\alpha}V_{\alpha\dot\alpha}\big|_\theta = \tfrac{1}{2}\left(\tfrac{2}{3}T^\mu{}_\mu + i\,\partial^\mu J^R_\mu\right)\theta_\alpha .$$

General computation:

$$\boxed{\bar D^{\dot\alpha}V_{\alpha\dot\alpha} = \frac{2}{3}D_\alpha\Big(\underbrace{3W - \sum_i \Phi_i\frac{\partial W}{\partial\Phi_i}}_{\text{explicit breaking}} + \underbrace{\frac{b}{32\pi^2}tr(W^2) - \frac{1}{2}\bar D^2\Big[\sum_i \gamma_i \bar\Phi_i e^V\Phi_i\Big]}_{U(1)_R-\text{scale anomaly}}\Big)}$$

This looks like a mess, but plugging Konishi anom. into this expression gives:

$$\bar D^{\dot\alpha}V_{\dot\alpha\alpha} = \frac{2}{3}D_\alpha\left(\Big[3W - \sum_i\big(1-\tfrac{1}{2}\gamma_i\big)\Phi_i\frac{\partial W}{\partial\Phi_i}\Big] + \Big[b + \sum_i \tfrac{1}{2}\gamma_i\,T(R_i)\Big]\frac{tr(W^2)}{32\pi^2}\right)$$

If $W = \lambda\mathcal{O}$, $\mathcal{O} = \prod_i \phi_i^{r_i}$ $\Rightarrow$ $3W - \sum_i\big(1-\tfrac{1}{2}\gamma_i\big)\Phi_i\frac{\partial W}{\partial\Phi_i} = \big(3 - \Delta + \tfrac{1}{2}\sum_i r_i\gamma_i\big)\mathcal{O}$

$$\therefore\ \bar D^{\dot\alpha}V_{\dot\alpha\alpha} = \frac{2}{3}D_\alpha\left(\sum_a\big(3-\Delta_a + \tfrac{1}{2}\sum_i r_{ai}\gamma_{ai}\big)\mathcal{O}_a + \Big(b + \sum_i\tfrac{1}{2}\gamma_i T(R_i)\Big)\frac{tr(W^2)}{32\pi^2}\right).$$

- Therefore, conditions for scale-invariance are:

$$\star \begin{cases} 0 = \beta^{CN}_{\lambda_a} \propto \Delta_a - 3 - \frac{1}{2}\sum_i r_{ai}\gamma_i & \checkmark \text{ supot'l non-renorm.} \\ 0 = \beta^{CN}_g \propto b + \frac{1}{2}\sum_i \gamma_i T(R_i) = \frac{3}{2}T(adj) - \frac{1}{2}\sum_i (1-\gamma_i)T(R_i) \end{cases}.$$

where $\left(\gamma_i \equiv \frac{d\ln Z_i}{d\ln\mu}\right)$.

- Note: expression for $U(1)_R$ anomaly was exact.
  But trace anomaly was only 1-loop.
  Since supersymmetry relates them, must be that, in fact, the β-functions computed above is exact.
  Just from $\star$ can deduce ~~se~~ existence of scale-invariant field theories when can eliminate or reduce γ_i's using symmetries [Leigh & Strassler NPB447(1995)95.]

- These scale invariance conditions have alot of the same information as a non-renormalization theorem.
  We will see how to recover them ~~using~~ from non-renormalization arguments for vector superfields.

5. Non-Renormalization theorems for Vector Superfields

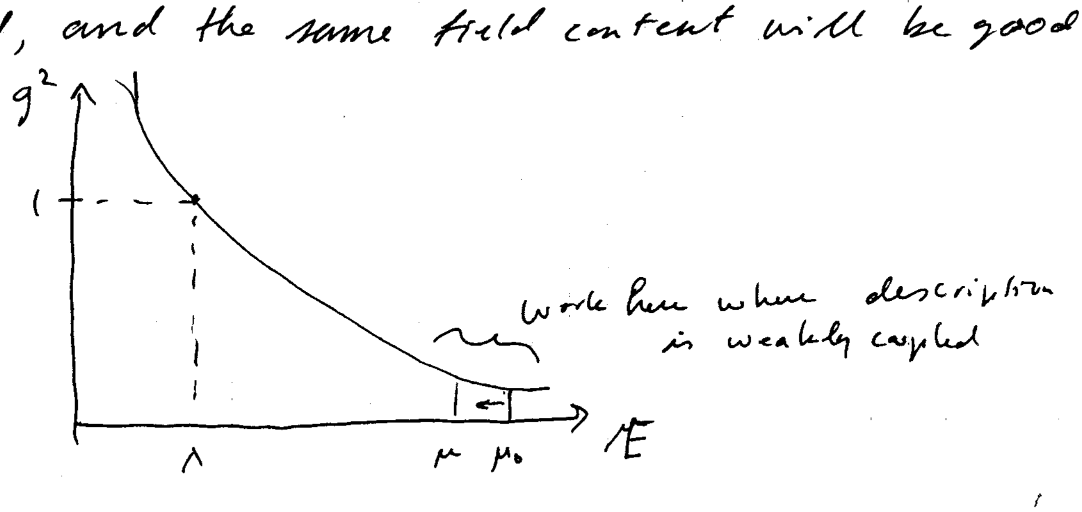
- UV action: $\mathcal{L}_{\mu_0} = \int d^4\theta \, \bar{\Phi}_i e^{V_0} \Phi_i + \int d^2\theta \, \tilde{W}_{\mu_0}$

  w/ gen. supot'l: $\tilde{W}_{\mu_0} = \frac{i\tau_0}{16\pi} tr(W^2) + W_{\mu_0}(\Phi_i, \lambda_0)$

- Want to find dependence of $\tilde{W}_\mu$ on τ_0, λ_0 in effective action at scales $\mu < \mu_0$.

- Let's consider the interesting case where $\mathcal{L}_{\mu_0}$ describes an <u>asymptotically free</u> theory; i.e. one that is weakly coupled if μ_0 is large enough. In general, we do not know what the relation is between the (free) UV field content, and the effective IR field content. (See next week's lectures...)

  But we can do something more modest: let's try to derive non-renormalization theorems for the effective action at a scale $\mu < \mu_0$, but only <u>slightly</u> less: $\frac{\mu_0 - \mu}{\mu_0} << 1$. Then, if μ_0 is large enough, the theory is weakly coupled, so we can be confident that if we run only slightly down in scale, the theory will remain weakly coupled, and the same field content will be good:



So the effective generalized superpotential:

$$\tilde{W}_\mu = \frac{i}{16\pi} \tau_\mu(\phi_i, \lambda_0, \tau_0)\, tr(W^2) + W_\mu(\phi_i, \lambda_0, \tau_0)$$

- Now, $\tau_0 \equiv \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu_0}\right)$. Under rotation of the θ angle,

$$\theta \to \theta + 2\pi \quad \Rightarrow \quad \Lambda^b \to e^{2\pi i} \Lambda^b \quad \Rightarrow \tau_0 \to \tau_0 + 1.$$

- The angularity of the θ angle should remain true as we flow down to μ, since it just depends on the topological quantization of the $tr(F\tilde{F})$ term. Thus, writing

$$\tau_\mu = \tau_\mu(\phi_i, \lambda_0, \Lambda^b),$$

we have selection rule:

$$\tau_\mu(\phi_i, \lambda_0, e^{2\pi i}\Lambda^b) = \tau_\mu(\phi_i, \lambda_0, \Lambda^b) + 1$$

(i.e., as rotate Λ^b, τ_μ shifts).

Since τ_μ is a holomorphic function of its arguments, it must have the form

$$\tau_\mu = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + h(\Lambda^b, \phi_i, \lambda_0)$$

where h is an arbitrary single-valued holomorphic function of its arguments.

- Since we are dealing with an AF theory, the weak-coupling limit is $\Lambda \to 0$. For this limit to have smooth physics, only positive powers of Λ^b should appear. Thus:

$$\boxed{\tau_\mu = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \Lambda^{nb}\, h_n(\phi_i, \lambda_0)}$$

- Recalling: $\underbrace{\ln \Lambda \sim \frac{1}{g^2}}_{\text{1-loop running}} \Rightarrow \Lambda^{nb} \sim \underbrace{e^{-\frac{8\pi^2 n}{g^2}}}_{\text{n-instanton (non-pert.) effect.}}$

So we learn:

The holomorphic coupling does not get any corrections in perturbation theory beyond 1-loop, though it may get non-perturbative corrections.

- The superpotential satisfies a similar "non-renormalization" theorem:

  If we turned off the gauge coupling ($\Lambda \to 0$) then we would just have our previous superpotential non-renormalization theorem: $\mathcal{W}_\mu = \mathcal{W}_{\mu 0}(\Phi_i, \lambda_0; \mu)$.

  Turning on Λ (the gauge interactions) can only add new terms holomorphic in Λ^b and vanishing as $\Lambda \to 0$, so

$$\boxed{\mathcal{W}_\mu = \mathcal{W}_{\mu 0} + \sum_{n=1}^{\infty} \Lambda^{nb} g_n(\Phi_i, \lambda_0)},$$

  again implying that $\mathcal{W}$ gets no perturbative corrections, but may get non-perturbative ones.

- We can tighten these results considerably using selection rules from global symmetries:

gauge invariant.

Say: $\mathcal{W}_{\mu 0} = \sum_r \lambda_r \mathcal{O}_r$, $\mathcal{O}_r = \prod_i \Phi_i^{n_{ir}}$ $\Rightarrow \Delta_r = \sum_i n_{ir}$

Consider $U(1)_i$ "symmetry"

$$\mathcal{U}(1)_i : \left\{ \Phi_i \to e^{i\epsilon} \Phi_i \right., \quad W_\alpha \to W_\alpha$$

Broken by $\mathcal{W}_{\mu 0}$, & anomalous:

$$\mathcal{O}_r \to e^{i\sum_i n_{ir}\epsilon} \mathcal{O}_r \quad \Rightarrow \underline{\text{assign}}: \ \lambda_r \to e^{-i\sum_i n_{ir}\epsilon} \lambda_r = e^{-i\Delta_r \epsilon} \lambda_r$$

Anomaly $\Rightarrow$ assign $\quad \theta \to \theta + \epsilon \sum_i T(R_i) \quad \Rightarrow \quad \Lambda^b \to e^{i\epsilon \sum_i T(R_i)} \Lambda^b$

Similarly, $U(1)_R$ symmetry:

$$\theta \to e^{i\epsilon}\theta \quad \& \quad \begin{cases} \Phi_i \to \Phi_i & \Rightarrow \psi_i \to e^{-i\epsilon}\psi_i \\ W_\alpha \to e^{i\epsilon} W_\alpha & \Rightarrow \lambda_\alpha \to e^{i\epsilon}\lambda_\alpha \end{cases}$$

Need: $\mathcal{W}_{\mu_0} \to e^{2i\epsilon}\mathcal{W}_{\mu_0}$ (R-charge +2)

$\Rightarrow$ since $\mathcal{O}_r \to \mathcal{O}_r$ $\therefore$ $\lambda_r \to e^{2i\epsilon}\lambda_r$

Anomaly: $\theta \to \theta + \epsilon\,(\overset{\lambda_\alpha}{T(adj)} - \sum_i T(R_i))$

$$\Rightarrow \Lambda^b \to e^{i\epsilon[T(adj) - \sum_i T(R_i)]}\Lambda^b$$

<u>Summarize charges:</u>

	$U(1)$	$U(1)_R$
Φ_i	1	0
W_α	0	1
λ_r	$-\Delta_r$	2
Λ^b	$\sum_j T(R_j)$	$T(adj) - \sum_i T(R_i)$

(gauge inv.)

Now say we are interested in a non-perturbative contribution to $\tau_\mu\, tr(W^2) \sim \Lambda^{b\alpha} \prod_r \lambda_r^{\alpha_r} \prod_i \Phi_i^{\beta_i}\, tr(W^2)$

Weak coupling limit $\Lambda, \lambda_r \to 0 \Rightarrow \alpha, \alpha_r > 0$.
Also, τ_μ single valued in this limit $\Rightarrow \alpha, \alpha_r \in \mathbb{Z}$.
Selection rule from "symmetries": Sum charges must vanish: $U(1)$; $= 2$ $U(1)_R$

$\mathcal{U}(1) \Rightarrow \quad 0 = \alpha \sum_i T(R_i) - \sum_r \alpha_r \Delta_r + \sum_i \beta_i$

$\mathcal{U}(1)_R \Rightarrow \quad 2 = \alpha\,[T(adj) - \sum_i T(R_i)] + 2\sum_r \alpha_r + 2$

Then $\frac{3}{2}U(1)_R + U(1) \Rightarrow \quad 0 = \alpha\,[\frac{3}{2}T(adj) - \frac{1}{2}\sum_i T(R_i)] + \sum_r (3-\Delta_r)\alpha_r + \sum_i \beta_i$

or, $\quad 0 = \alpha\, b + \sum_r (3-\Delta_r)\alpha_r + \sum_i \beta_i$

We were interested in AF theories:

$$\Rightarrow \quad b > 0 \quad + \quad 3 - \Delta_r \geq 0$$ (i.e. all microscopic couplings relevant or marginal)

Since $\alpha, \alpha_r > 0 \Rightarrow \sum_i \beta_i < 0$

- Only corrections involve Φ_i's to negative powers
- Furthermore, non-perturbative corrections to gauge running:

$$\sim \left(\Lambda^{b\alpha} \prod_r \lambda_r^{\alpha_r}\right) \mathrm{tr}(W^2) \Rightarrow \beta_i = 0$$

$$\Rightarrow \quad 0 = \alpha b + \sum_r (3 - \Delta_r) \alpha_r$$

$\Rightarrow$ no solution.

$\therefore$ The (holomorphic) gauge coupling is 1-loop exact.

- Can analyze contributions to superpot'l similarly: it is a bit more complicated...

Consider renormalization of an operator $\mathcal{O}_s$ in microscopic W_{tree}:

$$\sim \Lambda^{b\alpha} \prod_r \lambda_r^{\alpha_r} \mathcal{O}_s .$$

$$U(1) \Rightarrow \quad 0 = \alpha \sum_i T(R_i) - \sum_r \alpha_r \Delta_r + \Delta_s$$

$$U(1)_R \Rightarrow \quad 2 = \alpha \left[T(adj) - \sum_i T(R_i)\right] + 2 \sum_r \alpha_r$$

$$\tfrac{3}{2} U(1)_R + U(1) \Rightarrow \quad 3 - \Delta_s = \alpha b + \sum_r (3 - \Delta_r) \alpha_r$$

Again, for AF theory, $b > 0$, $3 - \Delta_r > 0$, $\alpha, \alpha_r \in \mathbb{Z}^+$, $\Rightarrow \Delta_s \leq 3$ only solution is $\alpha = 0$, $\alpha_s = 1$, $\alpha_r = 0$ $r \neq s$.

$$\sim \lambda_s \mathcal{O}_s = \text{tree level term.}$$

So tree-level superpotential is <u>not</u> renormalized.

(But non-perturbative corrections w/ inverse powers of Φ_i are allowed.) [See Intriligator lecture.]

- Let's interpret these non-renormalization theorems:
  - they applied to couplings in the holomorphic generalized superpotential only.
  - the Kahler term is not protected.
  - so there will be wave function renormalizations as well:

$$\mathcal{L}_\mu = \int d^4x \left[\int d^4\theta \; Z_i (\bar{\Phi}_i e^V \Phi_i) + \int d^2\theta \left\{ \frac{1}{16\pi^2} \ln\left(\frac{\Lambda}{\mu}\right)^b \mathrm{tr} W^2 + \sum_r \lambda_r \overset{\left(\frac{\mu_0}{\mu_0}\right)^{3-\Delta_r}}{\mathcal{O}_r} \right\} + \text{c.c.} \right]$$

$\left(+ \frac{\Lambda^{bn}}{\phi^N} \text{ corrections} \right)$

rewrite as

$$\mathcal{L}_\mu = \int d^4x \left[\int d^4\theta \; Z_i \bar{\Phi}_i e^V \Phi_i + \int d^2\theta \left\{ \left(\frac{1}{g^2(\mu)} - \frac{i\theta}{8\pi^2} \right) \mathrm{tr} W^2 + (\overset{\left(\frac{\mu_0}{\mu}\right)^{3-\Delta_0}}{\lambda_0} \prod_i \phi_i^{n_i} \right\} + \text{c.c.} \right]$$

Need to rescale fields to canonically normalize:

Want coefficient of $\bar{\Phi}_i e^V \Phi_i = 1$ $\quad \bar{\Phi}_i e^{\hat{g} V} \Phi_i$

" " " (Re) $\mathrm{tr} W^2 = 1$ $\Rightarrow$ (put $V \to \hat{g} V$)

Wave function:

$$\Phi_i = \frac{1}{\sqrt{Z_i}} \Phi_i^{\mathrm{cN}}$$

but this change of variables rescales the fermion as

$$\psi_i \to \frac{1}{\sqrt{Z_i}} \psi_i .$$

In our supersymmetric scheme, this rescaling may have an anomaly (rather, a non-trivial Jacobian in path integral). Can see as follows: think of $\frac{1}{\sqrt{Z_i}}$ as χsf; then is complex & includes phase rot'n as well as scaling. The anomaly in the phase rot'n then must also apply to rescalings:

$$\psi_i \to e^{\epsilon} \psi_i = e^{i\epsilon_i(-i)} \psi_i \qquad \epsilon_i = \ln\left(\frac{1}{\sqrt{Z_i}}\right)$$

$$\Rightarrow \quad \theta \to \theta + (-i\epsilon_i) \sum_i T(R_i)$$

Since coupling appears as $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$,

$$\Rightarrow \text{ shifts } \quad \frac{4\pi}{g^2} \to \frac{4\pi}{g^2} - \frac{\epsilon_i \sum T(R_i)}{2\pi} , \quad \text{gauge coupling} \propto \mathrm{tr}(F^2)$$

(compare w/ Konishi anomaly).

- So, WV-function renormalization $\Rightarrow \quad \theta \to \theta + i\frac{1}{2}\left(\sum_i \ln Z_i T(R_i)\right)$

- Similarly $\quad W_\alpha \to W_\alpha^{CN} = \frac{1}{g_{CN}} W_\alpha \qquad \left(V^{CN} = \frac{1}{g_{CN}} V \right.$

$\qquad\qquad \simeq e^{\frac{1}{2}\ln\left(\frac{1}{g_{CN}^2}\right)} W_\alpha \qquad$ so that $\quad \left. e^{V} \to e^{g_{CN} V_{CN}} \right)$

$$\Rightarrow \quad \theta \to \theta - \frac{i}{2}\ln\left(\frac{1}{g_{CN}^2}\right) T(adj).$$

- Net result: $\quad \theta \to \hat{\theta} = \theta - \frac{i}{2}\left[T(adj)\ln\left(\frac{1}{g_{CN}^2}\right) - \sum_i \ln Z_i T(R_i)\right]$

$$\mathcal{L}_\mu \to \mathcal{L}_\mu^{CN} = \int d^4\theta\, \bar{\Phi}_i^{CN} e^{g_{CN}V_{CN}} \Phi_i^{CN} - \frac{1}{4}\int d^2\theta \left\{\left[\frac{g_{CN}^2}{g^2(\mu)} - \frac{i\hat{\theta} g_{CN}^2}{8\pi^2}\right] \mathrm{tr}\, W_{CN}^2 \right.$$

$$\left. + \lambda_0 \left(\frac{\mu_0}{\mu}\right)^{3-\Delta} \prod_i Z_i^{-n_i/2}\, \mathcal{O}\right\} + c.c.$$

$$\therefore \quad \lambda^{CN}(\mu) = \lambda_0\left(\frac{\mu_0}{\mu}\right)^{3-\Delta} \prod_i Z_i^{-n_i/2} \qquad \left(\sum_i n_i = \Delta\right) \quad (\star)$$

& g_{CN}^2 s.t. ~~real~~ coefficient in front of $\mathrm{tr}\, W_{CN}^2 = \left(1 - \frac{i\theta g_{CN}^2}{8\pi^2}\right)$

$$\Rightarrow \quad \frac{g_{CN}^2}{g^2(\mu)} - \frac{g_{CN}^2}{16\pi^2}\left(T(adj)\ln\left(\frac{1}{g_{CN}^2}\right) - \sum_i \ln Z_i\, T(R_i)\right) = 1$$

$$\text{or} \quad \frac{1}{g_{CN}^2} = \frac{1}{g^2(\mu)} + \sum_i \frac{\ln Z_i\, T(R_i)}{16\pi^2} - \frac{T(adj)}{16\pi^2}\ln\left(\frac{1}{g_{CN}^2}\right)$$

Take $\frac{d}{d\ln\mu}$ both sides; use $\frac{d}{d\ln\mu}\left(\frac{1}{g^2(\mu)}\right) = \frac{b}{8\pi^2}$ (exact 1-loop running)

$$\Rightarrow \cdots \quad \boxed{\frac{d\left(\frac{1}{g_{CN}^2}\right)}{d\ln\mu} = \frac{b + \frac{1}{2}\sum_i T(R_i)\gamma_i}{8\pi^2 - \frac{1}{2}T(adj)\, g_{CN}^2}} \quad \text{NSVZ exact } \beta\text{-fnc.}$$

$$\left(\gamma_i \equiv \frac{d\ln Z_i}{d\ln\mu}\right).$$

Also $\frac{d}{d\ln\mu}$ of $(\star)$ $\Rightarrow$ $\boxed{\frac{d\lambda^{CN}(\mu)}{d\ln\mu} = \left(\Delta - 3 - \frac{1}{2}\sum_i n_i\gamma_i\right)\lambda^{CN}(\mu)}$ Exact RG eqn for superpotl.

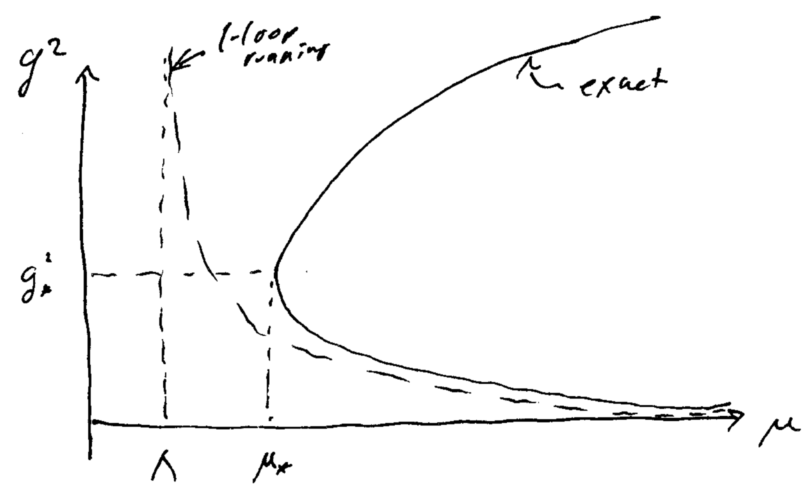
Since can't compute γ_i non-pert, limited use.
(Cf. Leigh & Strassler NPB, 1995).

But in one case of interest, can solve for g_{cw} exactly:

$\gamma_i = 0$ if there are no Φ_i's $\Rightarrow$ Pure superYM:

$$\frac{d\left(\frac{1}{g_{cw}^2}\right)}{d\ln\mu} = \frac{\frac{3}{2}\,T(\text{adj})}{8\pi^2 - \frac{1}{2}T(\text{adj})\,g_{cw}^2}$$

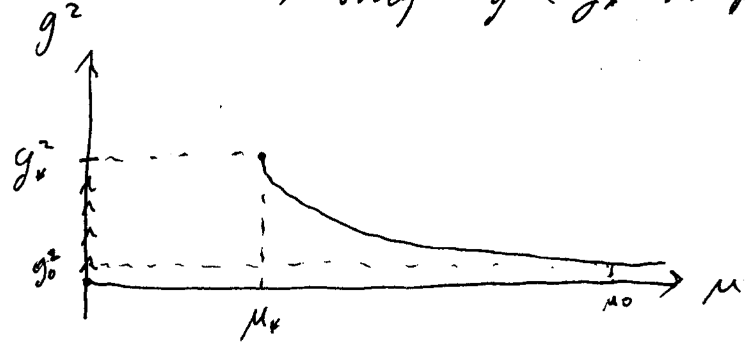
Easy to solve this differential equation. Get



What does this mean?

1) Can only flow down in μ, so 2 branches:
$g^2 > g_*^2$ & $g^2 < g_*^2$.

2) Made assumption that as $\mu \to \infty$ $g^2 \to 0$ (AF)
$\Rightarrow$ only $g^2 < g_*^2$ is physical:



What does flow stopping mean?

- Not fixed point, since doesn't go to $\mu = 0$.
- Means operator $\mathrm{tr}(W^2)$ decouples $\to$ get integrated out
$\Rightarrow$ glue ball gets mass $\sim \mu^*$

$$\left[\mu_* = \left(\frac{16\pi^2 e}{T_{\text{adj}}\,g_0^2}\right)^{1/3} \underbrace{\mu_0 e^{-\frac{8\pi^2}{6 g_0^2}}}_{\Lambda}\right] = \Lambda\left[e\ln\left(\frac{\mu_0}{\Lambda}\right)^3\right]^{1/3}$$

Lesson: Non-renormalization theorems, by themselves, do not solve strong coupling problem: they can't tell you what the new effective low energy degrees of freedom will be.

Next weeks lectures will concern cases where progress can be made by assuming certain low energy field content (based on some other physical inputs), and then applying the non-renormalization style arguments to find unique results.