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WORKSHOP ON

QUANTUM SYSTEMS OUT OF EQUILIBRIUM

(14 – 25 June 2004)

" Non-exponential quasiparticle decay and phase relaxation in low dimensional conductors "

presented by:

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Quasi-particle decay and phase relaxation in quasi-1D diffusive conductors

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Interacting electrons = quasi-particles interacting through a screened potential

1) Lifetime of quasiparticle

2) Time dependence of dephasing $\left\langle e^{i\Phi(t)} \right\rangle ?$ $\left\langle \Phi^2(t) \right\rangle ?$

Diffusive regime $l_e \ll L$ $P(r,r',t) = \frac{e^{-(r-r')^2/4Dt}}{\sqrt{4\pi Dt}}$ $|r-r'| \sim \sqrt{Dt}$

No other degrees of freedom (phonons, magnetic impurities, …)

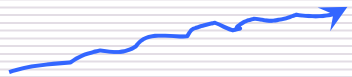
Interacting electrons = quasi-particles interacting through a screened potential

*** Lifetime of quasi-particle** $\tau_{ee}(\varepsilon, T)$

Quasiparticle injected at energy ε above the Fermi sea, at temperature T

- * Fermi golden rule
- * Fluctuating potential

$$\mathcal{P}(t) = e^{-t/\tau_{ee}}$$

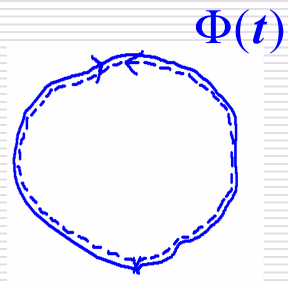


*** Phase coherence time** $\tau_{\varphi}(T)$

Lifetime of cooperon

Cooperon in a fluctuating field

$$\left\langle e^{i\Phi(t)} \right\rangle = e^{-t/\tau_{\phi}}$$



Outline

- Relation between $\tau_{ee}(T) = \tau_{ee}(\varepsilon = 0, T)$ and $\tau_\phi(T)$

- Non-exponential relaxations in quasi-1D

$$e^{-(t/\tau)^{3/2}}$$

Distribution of time scales

$$\tau(T) \propto T^{-2/3}$$

- Geometry effect, finite system (ring)

Ludwig, Mirlin 03 $\tau_\phi^{AB}(T) \propto T^{-1}$

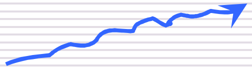
- Exponential relaxation on a ring

$$e^{-t/\tau_\phi^{AB}}$$

- From $e^{-(t/\tau_\phi)^{3/2}}$ to $e^{-t/\tau_\phi^{AB}}$

Quasi-1D

❑ *Relaxation of quasiparticle*



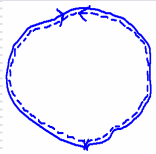
Schmid, Altshuler, Aronov

$$\mathcal{P}(t) = e^{-t/\tau_{ee}}$$

$$\tau_{ee}^{-1}(T) \propto T \int_{\tau_{ee(T)}^{-1}}^{T} \frac{d\omega}{\omega^{3/2}}$$

$$\tau_{ee}(T) \propto T^{-2/3}$$

❑ *Phase coherence*



$\Phi(t)$

Altshuler, Aronov, Khmelnitskii

$$\left\langle e^{i\Phi(t)} \right\rangle \neq e^{-t/\tau_\phi}$$

$$\int \frac{\left\langle e^{i\Phi(t)} \right\rangle e^{-\gamma t}}{\sqrt{4\pi D t}} dt = -\frac{1}{2}\sqrt{\frac{\tau_\phi}{D}} \frac{Ai(\gamma\tau_\phi)}{Ai'(\gamma\tau_\phi)}$$

$$\tau_\phi(T) \propto T^{-2/3}$$

Phase relaxation is not exponential

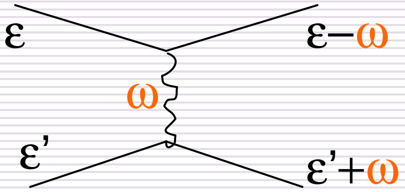
In quasi-1D diffusive conductors, quasi-particle decay and phase relaxation are not exponential

Both scale as

$$e^{-(t/\tau)^{3/2}}$$

1 - Lifetime of quasiparticle

$$\mathcal{P}(t) = e^{-t/\tau_{ee}} \quad ?$$



Fermi golden rule

Landau

$$\tau_{ee}^{-1}(\varepsilon, T=0) \propto \int_0^{\varepsilon} W^2 \omega \, d\omega \sim \varepsilon^2 \qquad \tau_{ee}^{-1}(\varepsilon=0, T) \propto T \int_0^{T} W^2 d\omega \sim T^2$$

W : matrix element of the interaction

Diffusive conductors : the typical matrix element of the interaction is energy dependent (Altshuler-Aronov)

$$W^2(\omega) \propto \frac{1}{\omega^{2-d/2}}$$

$$\tau_{ee}^{-1}(\varepsilon, T=0) \propto \int_0^{\varepsilon} W^2(\omega)\, \omega \, d\omega \sim \varepsilon^{d/2}$$

$$\tau_{ee}^{-1}(T) = \tau_{ee}^{-1}(\varepsilon=0, T) \propto T \int_0^{T} W^2(\omega)\, d\omega \;\propto\; T \int_0^{T} \frac{\mathrm{d}\omega}{\omega^{2-d/2}}$$

Saclay group, cond-mat/0404208

Infrared divergence for $d \leq 2$

Since τ_{ee} represents the lifetime of a quasiparticle state, energy exchange ω cannot be defined with a precision better than $1/\tau_{ee}$

Self-consistent relation $\tau_{ee}^{-1}(T) \propto T \int_{\tau_{ee(T)}^{-1}}^{T} \frac{\mathrm{d}\omega}{\omega^{3/2}}$ → ($d=1$) $\tau_{ee}(T) \propto T^{-2/3}$

Claim : this infrared divergence implies that the energy relaxation is not exponential

$$\mathcal{P}(t) \neq e^{-t/\tau_{ee}}$$

~~Since τ_{ee} represents the lifetime of a quasiparticle state, energy exchange cannot be defined with a precision better than $1/\tau_{ee}$~~

Fermi golden rule : For a given value of time t, transitions conserve energy within $1/t$

⇒ Energy transfer cannot be defined with a precision better than $1/t$

~~$\Delta\omega . \tau_{ee} \sim 1$~~ → $\Delta\omega . t \sim 1$

⇒ Relaxation rate $w(t) = \frac{1}{\mathcal{P}} \frac{d\mathcal{P}}{dt}$ cannot be constant $w(t) \neq \frac{1}{\tau_{ee}}$

~~$\Delta\omega . \tau_{ee} \sim 1$~~ $\Delta\omega . t \sim 1$

~~$\tau_{ee}^{-1}(T) \sim T \int_{\tau_{ee(T)}^{-1}}^{T} \frac{d\omega}{\omega^{3/2}}$~~ ⇒ $w(t) \sim T \int_{1/t}^{T} \frac{d\omega}{\omega^{3/2}} \sim T\, t^{1/2}$

Relaxation rate increases with time : relaxation is faster than exponential

$$\mathcal{P}(t) \sim e^{-T^{2/3}t}$$

$$w(t) = \frac{1}{\mathcal{P}}\frac{d\mathcal{P}}{dt} \propto T \int_{1/t}^{T} \frac{\mathrm{d}\omega}{\omega^{3/2}} \sim T\, t^{1/2}$$

$$\mathcal{P}(t) \sim e^{-T\, t^{3/2}} \sim e^{-\left(T^{2/3}\, t\right)^{3/2}}$$

$$\mathcal{P}(t) \sim e^{-(t/\tau_{ee})^{3/2}}$$

Characteristic time is unchanged

$$\tau_{ee}(T) \sim \left(\frac{\sigma S}{e^2\sqrt{DT}}\right)^{2/3}$$

The exponent « 2/3 » is indeed a signature of the non-exponential « 3/2 » relaxation

1bis - Lifetime of quasiparticle

e-e interactions → fluctuating electric potential

$$\mathcal{P}(t) = \left| \left\langle e^{i\phi(t)} \right\rangle^2 \right| = e^{-\left\langle \phi^2(t) \right\rangle}$$

$$\phi(t) = \frac{e}{\hbar} \int_0^t V(r(\tau),\tau)\, d\tau$$

$$\left\langle VV \right\rangle (q,\omega) = \frac{2T}{\sigma_0 q^2}$$

$$\left\langle \phi^2(t) \right\rangle \sim \frac{e^2 T}{\sigma} t \int_0 \frac{dq}{q^2}$$

Fluctuation dissipation theorem

$$\left\langle V(r,\tau) V(r',\tau') \right\rangle = \delta(\tau-\tau') \frac{T}{\pi\sigma_0} \int \frac{dq}{q^2} e^{iq(r-r')}$$

At time scale *t*, the quasiparticle cannot exchange energy with fluctuations of frequencies $\omega < 1/t$

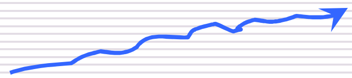
Select frequencies $\omega < 1/t$, $Dq^2 t < 1$

$$\left\langle \phi^2(t) \right\rangle \sim \frac{e^2 T}{\sigma} t \int_{1/\sqrt{Dt}} \frac{dq}{q^2} \sim \frac{e^2 \sqrt{D}\, T}{\sigma} t^{3/2} \sim \left(\frac{t}{\tau_{ee}} \right)^{3/2}$$

<u>Quasi-particle decay</u> is a non exponential process

$$\mathcal{P}(t) = e^{-(t/\tau_{ee})^{3/2}}$$

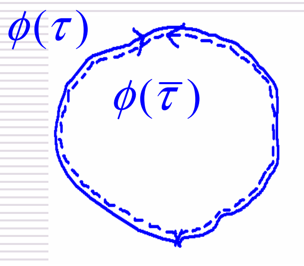
$$\tau_{ee}(T) \sim \left(\frac{\sigma S}{e^2 \sqrt{DT}} \right)^{2/3}$$



e-e interaction + Fermi golden rule

e in fluctuating potentiel

<u>Phase relaxation</u>



$\tau \in [0, t]$

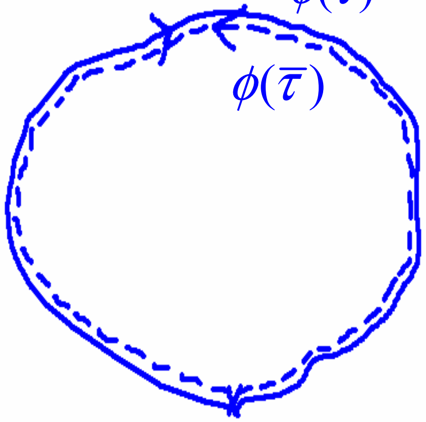
$\bar{\tau} \in [t, 0]$

$$\left\langle e^{i\Phi(t)} \right\rangle ?$$

$$\Phi(\tau) = \phi(\tau) - \phi(\bar{\tau}) \qquad \bar{\tau} = t - \tau$$

2 - Lifetime of cooperon

Altshuler,Aronov,Khmelnitskii, 82



Phase coherent contribution to the return probability

$$P(t) = P_0(t)\left\langle e^{i\Phi(t)}\right\rangle_{T,\mathcal{C}} \qquad \Phi(\tau) = \phi(\tau) - \phi(\overline{\tau})$$

$$P_0(t) = \frac{1}{\sqrt{4\pi D t}}$$

Replace e-e interaction by electric fluctuating potential → Fluctuating phase $\phi(t) = \frac{e}{\hbar}\int_0^t V(r(\tau),\tau)\, d\tau$

$$\left\langle \Phi^2(t)\right\rangle = 2\frac{e^2}{\hbar^2}\int_0^t \left[\left\langle V(r_\tau,\tau)V(r_\tau,\tau)\right\rangle - \left\langle V(r_\tau,\tau)V(r_\tau,\overline{\tau})\right\rangle\right] d\tau$$

Fluctuation dissipation theorem : $\left\langle V(r,\tau)V(r',\tau')\right\rangle_T = \delta(\tau-\tau')\frac{2TD}{\sigma}P(r,r')$

$P(r,r') \propto |r-r'|^{2-d}$ probability to diffuse from r to r'

Phase fluctuations

Diffusion $\propto \tau^{1/2}$ (Stern,Aharonov,Imry,90)

$$\left\langle \Phi^2(t) \right\rangle_T = \frac{4e^2 T}{\sigma\hbar^2}\int_0^t d\tau\left[P(r_\tau,r_\tau)-P(r_\tau,r_{\bar\tau})\right]$$

infinite wire →

$$\left\langle \Phi^2(t) \right\rangle_T = \frac{2e^2 T}{\sigma\hbar^2}\int_0^t d\tau\left|r_\tau - r_{\bar\tau}\right|$$

Gaussian fluctuations :

$$\left\langle e^{i\Phi(t)} \right\rangle_T = e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle_T}$$

Then average on diffusive trajectoires τ and $\bar\tau$ for $\tau \in [0,t]$

$$\left\langle e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle_T} \right\rangle_{\mathcal{C}}$$

Small times

$$e^{-\frac{1}{2}\left\langle\left\langle \Phi^2(t)\right\rangle_T\right\rangle_{\mathcal{C}}}$$

$$\int \frac{\left\langle e^{i\Phi(t)}\right\rangle_{T,\mathcal{C}} e^{-\gamma t}}{\sqrt{4\pi D t}}dt = -\frac{1}{2}\sqrt{\frac{\tau_\phi}{D}}\frac{Ai(\gamma\tau_\phi)}{Ai'(\gamma\tau_\phi)}$$

$$\left\langle \Phi^2(t) \right\rangle_{T,C} = \frac{\sqrt{\pi}}{2}\left(\frac{t}{\tau_\phi}\right)^{3/2}$$

Altshuler, Aronov, Khmelnitskii

$$\tau_\phi^{-3/2} = \frac{e^2\sqrt{DT}}{\sigma\hbar^2}$$

Time dependence of the phase relaxation

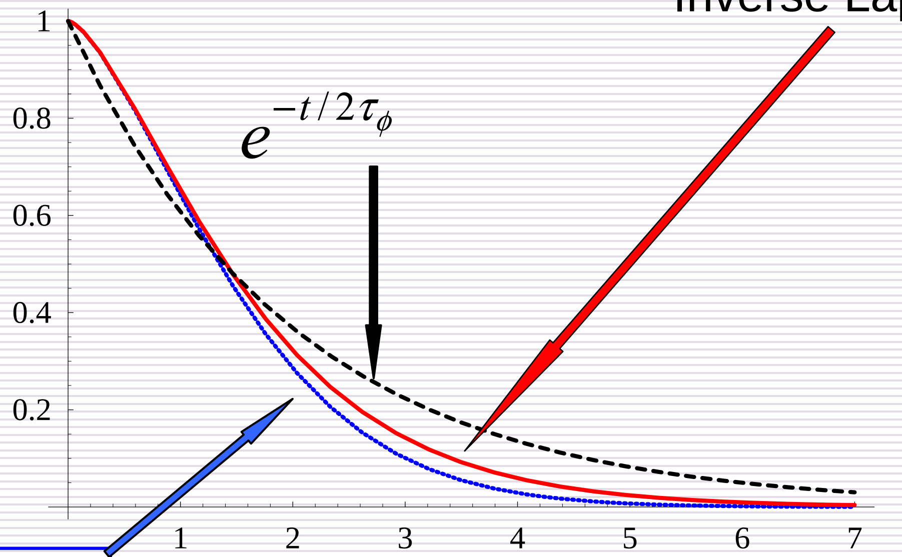
$$\int \frac{\left\langle e^{i\Phi(t)} \right\rangle_{T,\mathcal{C}} e^{-\gamma t}}{\sqrt{4\pi D t}} dt = -\frac{1}{2}\sqrt{\frac{\tau_\phi}{D}} \frac{Ai(\gamma\tau_\phi)}{Ai'(\gamma\tau_\phi)}$$

$$\tau_\phi(T) \sim \left(\frac{\sigma S}{e^2\sqrt{DT}} \right)^{2/3}$$

Inverse Laplace transform

$$\left\langle e^{i\Phi(t)} \right\rangle_{T,\mathcal{C}}$$

$$e^{-t/2\tau_\phi}$$



$$t/\tau_\phi$$

$$e^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_\phi}\right)^{3/2}}$$

Time dependence of the phase relaxation

Small times $t < \tau_\phi$

$$\left\langle e^{i\Phi(t)} \right\rangle_{T,\mathcal{C}} = e^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_\phi}\right)^{3/2}}$$

Exact result : inverse Laplace transform of AAK :

$$\left\langle e^{i\Phi(t)} \right\rangle_{T,\mathcal{C}} = \sqrt{\frac{\pi t}{\tau_\phi}} \sum_{n=1}^{\infty} \frac{e^{-\frac{u_n}{\tau_\phi} t}}{u_n}$$

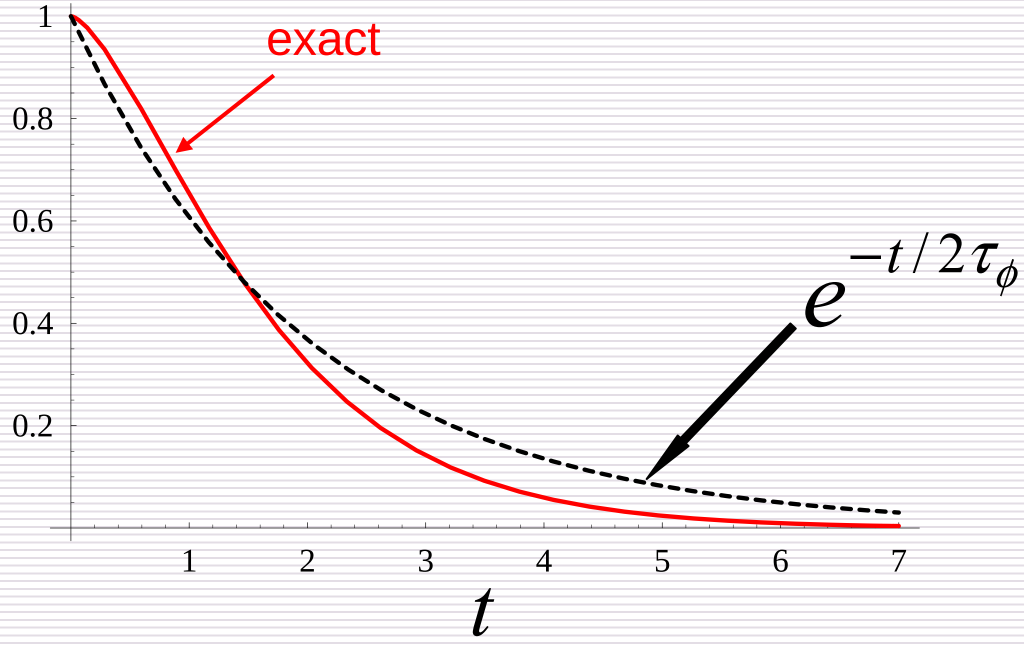
$$u_n \sim \left[\frac{3\pi}{2}\left(n - \frac{3}{4}\right)\right]^{2/3}$$

Distribution of relaxation times

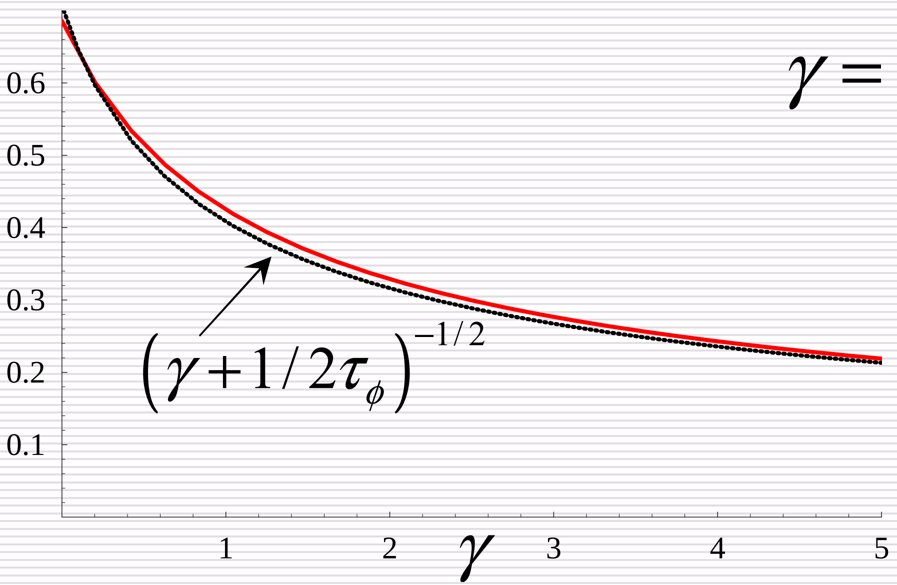
$$\tau_n \sim \frac{\tau_\phi}{n^{2/3}}$$

$$P(\tau_n) \sim \frac{1}{\tau_n^{5/2}}$$

$$\left\langle e^{i\Phi(t)} \right\rangle_{T,\mathcal{C}}$$



$$\int \frac{\left\langle e^{i\Phi(t)} \right\rangle e^{-\gamma t}}{\sqrt{4\pi D t}} dt$$



$$\gamma = \frac{1}{\tau_B} = \frac{e^2 D W^2 B^2}{3\hbar^2}$$

Relaxation almost exponential : addition of inverse times

Large field magnetoresistance

$$L_B = \sqrt{D\tau_B} = \frac{\sqrt{3}\hbar}{eWB}$$

$L_B \ll L_\phi$

$$\langle \Delta g \rangle = -2\frac{L_B}{L}\left(1 - \frac{1}{4}\frac{L_B^3}{L_\phi^3}\right) \quad \sim -\frac{1}{B} + \frac{T}{B^4}$$

If exponential decay :

$$\langle \Delta g \rangle = -2\frac{L_B}{L}\left(1 - \frac{1}{4}\frac{L_B^2}{L_\phi^2}\right) \quad \sim -\frac{1}{B} + \frac{T^{2/3}}{B^3}$$

Conclusion

E. Akkermans, G.M., cond-mat/0405523

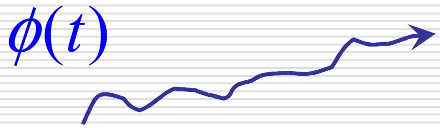
The decay of quasiparticle and the dephasing of time reversed trajectories are described by the same characteristic time

$$\tau_{ee} = \tau_{\phi}$$

Fermi golden rule + e-e interaction

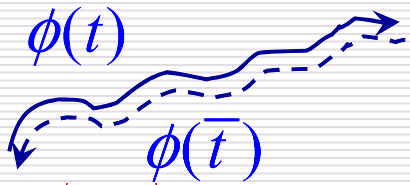
$$\mathcal{P}(t) = e^{-(t/\tau_{ee})^{3/2}}$$

fluctuating field



$$\mathcal{P}(t) = \left| \left\langle e^{i\phi} \right\rangle_{C(t)} \right|^2 = e^{-\left\langle \phi^2 \right\rangle_{C(t)}} = e^{-(t/\tau_{ee})^{3/2}}$$

Cooperon in a fluctuating field

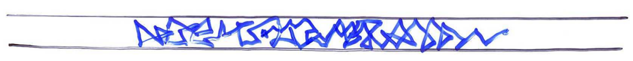


$$\left\langle e^{i\Phi} \right\rangle_{C(t)} = \left\langle e^{i(\phi(\tau)-\phi(\overline{\tau}))} \right\rangle_{C(t)} = e^{-(\left\langle \phi^2(\tau) \right\rangle - \left\langle \phi(\tau)\phi(\overline{\tau}) \right\rangle_{C(t)})}$$

$$\left\langle e^{i\Phi} \right\rangle_{C(t)} = e^{-(t/\tau_{\varphi})^{3/2}}$$

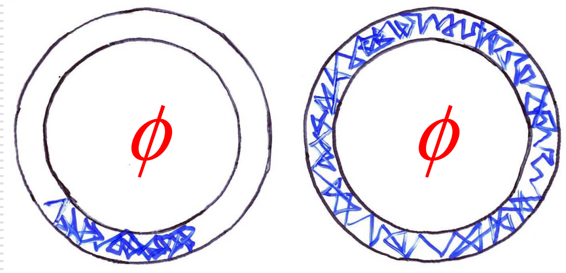
Dephasing on a ring

Ludwig, Mirlin 03



The dephasing depends on the nature of the diffusive trajectories,

Dephasing may differ for trajectories with different winding numbers



Conductance fluctuations

$\delta g = g - \langle g \rangle$ $$\delta g(\phi) = \delta g_0 + 2\sum_{1}^{\infty} \delta g_m \cos(2\pi m\phi/\phi_0 + \theta_m)$$

$$L_T = \sqrt{D/T} \ll L$$

Exponential relaxation e^{-t/τ_ϕ}

Fluctuating field

$$\left\langle \delta g_m^2 \right\rangle \sim \left(\frac{L_T}{L}\right)^2 \frac{L_\phi}{L} e^{-mL/L_\phi}$$

$$\left\langle \delta g_m^2 \right\rangle \sim \left(\frac{L_T}{L}\right)^2 \left(\frac{L_\phi}{L}\right)^{9/4} e^{-m(L/L_\phi)^{3/2}}$$

Aronov, Sharvin

Ludwig,Mirlin

$$L_\phi = \sqrt{D\tau_\phi} = \left(\frac{\sigma DS}{e^2 T}\right)^{1/3}$$

$$L_\phi \ll L$$

Dephasing on a ring

Conductance fluctuations and Weak-localization correction

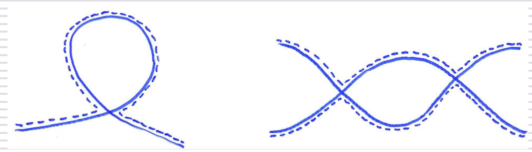
$$\delta g(\phi) = \delta g_0 + 2\sum_1^\infty \delta g_m \cos(2\pi m\phi/\phi_0 + \theta_m)$$

$$\langle \Delta g(\phi)\rangle = \langle \Delta g\rangle_0 + 2\sum_1^\infty \langle \Delta g\rangle_m \cos(2\pi m\phi/\phi_0)$$

Aleiner, Blanter, Ludwig, Mirlin

$$\langle \delta g_m^2\rangle = -\frac{4\pi}{3}\left(\frac{L_T}{L}\right)^2 \langle \Delta g\rangle_m$$

$$L_T \ll L$$



CF

$$\langle \delta g_m^2\rangle \sim \left(\frac{L_T}{L}\right)^2 \frac{L_\phi}{L} e^{-mL/L_\phi}$$

$$\langle \delta g_m^2\rangle \sim \left(\frac{L_T}{L}\right)^2 \left(\frac{L_\phi}{L}\right)^{9/4} e^{-m(L/L_\phi)^{3/2}}$$

$$e^{-t/\tau_\phi}$$

fluctuating field

WL

$$\langle \Delta g\rangle_m \sim \frac{L_\phi}{L} e^{-mL/L_\phi}$$

Altshuler, Aronov, Spivak

$$\langle \Delta g\rangle_m \sim \left(\frac{L_\phi}{L}\right)^{9/4} e^{-m(L/L_\phi)^{3/2}}$$

Ludwig, Mirlin

$$L_\phi \ll L$$

$$L_\phi = \sqrt{D\tau_\phi} \propto T^{-1/3}$$

$$e^{-mT^{1/3}} \rightarrow e^{-mT^{1/2}}$$

Dephasing on a ring

Analytical solution for WL correction all L/L_{ϕ}

Limiting cases

Time dependence of the phase relaxation in different geometries

C. Texier, G.M.

Dephasing on a ring

$$\langle \Phi^2(t) \rangle_T = \frac{4e^2 T}{\sigma \hbar^2} \int_0^t d\tau \left[P(r_\tau, r_\tau) - P(r_\tau, r_{\bar{\tau}}) \right]$$

Ring →

$$\langle \Phi^2(t) \rangle_T = \frac{2e^2 T}{\sigma \hbar^2} \int_0^t d\tau \left| r_\tau - r_{\bar{\tau}} \right| \left(1 - \frac{\left| r_\tau - r_{\bar{\tau}} \right|}{L} \right)$$

$$P(r,r,t) = P_0(r,r,t) \left\langle e^{i\Phi(t)} \right\rangle_{T,\mathcal{C}} = P_0(r,r,t) \left\langle e^{-\frac{1}{2}\langle \Phi^2(t) \rangle_T} \right\rangle_{\mathcal{C}}$$

$$\langle \Delta g(\phi) \rangle = -\frac{4}{L} \int P(r,r,t) dt$$

WL correction

$$\left\langle e^{-\frac{1}{2}\langle \Phi^2(t) \rangle_T} \right\rangle_{C_m} \qquad \left\langle \langle \Phi^2(t) \rangle_T \right\rangle_{C_m}$$

Phase accumulated after m loops ?

$$\int P(r,r',t) dt$$

- * is given by a path integral over diffusive trajectories

Ludwig, Mirlin, limit $L_\phi << L$

- * is solution of differential equation :

$$\left[-\left(\frac{d}{d\chi} - 2i\pi \frac{\phi}{\phi_0} \right)^2 + \frac{L^3}{L_\phi^3} \left(|\chi| - \chi^2 \right) \right] P_\phi(\chi, \chi') = \delta(\chi - \chi')$$

$$L_\phi = \sqrt{D\tau_\phi} = \left(\frac{\sigma D S}{e^2 T} \right)^{1/3}$$

$$\chi = r / L \in [0,1]$$

$$\left[-\left(\frac{d}{d\chi}-2i\pi\frac{\phi}{\phi_0}\right)^2+\frac{L^3}{L_\phi^3}(|\chi|-\chi^2)\right]P(\chi,\chi')=\delta(\chi-\chi')$$

* Exact solution

$$\langle\Delta g(\phi)\rangle=-4\,P_\phi(\chi,\chi)$$

* Results for return probability/weak-localization

$$\langle\Delta g(\phi)\rangle=-2F(\theta)=-2\sum_m F_m\cos m\theta$$

⟶ $F(\theta)=\dfrac{1}{a-b\cos\theta}$ $\quad F_m=A\,e^{-m\,B}$ $\quad m$: winding number

⟶ $L_\phi \ll L$ $\quad F_n=-\dfrac{Ai(0)}{Ai'(0)}\dfrac{L_\phi}{L}\,e^{-m\frac{\pi}{8}\left(\frac{L}{L_\phi}\right)^{3/2}}$ $\quad$ cf. Ludwig, Mirlin

⟶ $L_\phi \gg L$ $\quad F_n=\sqrt{6}\left(\dfrac{L_\phi}{L}\right)^{3/2}e^{-\frac{m}{\sqrt{6}}\left(\frac{L}{L_\phi}\right)^{3/2}}$

* Physical interpretation : time dependence of the phase relaxation ?

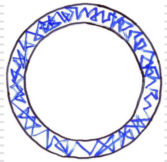
Time dependence of the phase relaxation

Thouless time $\tau_D = L^2 / D$

$$\left\langle \Phi^2(t) \right\rangle_{T,C_m} = \frac{2D}{L_\phi^3} \int_0^t d\tau \left\langle \left| r_\tau - r_{\bar{\tau}} \right| \left(1 - \frac{\left| r_\tau - r_{\bar{\tau}} \right|}{L} \right) \right\rangle_{C_m}$$

$t \gg \tau_D$

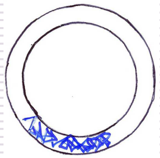
$$\left\langle \Phi^2(t) \right\rangle_{T,C_m} = \frac{\sqrt{\tau_D}}{3\tau_\phi^{3/2}}\, t$$



$t \ll \tau_D$

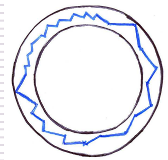
$m = 0$

$$\left\langle \Phi^2(t) \right\rangle_{T,C_{m=0}} = \frac{\sqrt{\pi}}{2} \frac{t^{3/2}}{\tau_\phi^{3/2}}$$



$m \neq 0$

$$\left\langle \Phi^2(t) \right\rangle_{T,C_m} = \frac{\sqrt{\tau_D}}{3\tau_\phi^{3/2}}\, t$$



Harmonics content of the phase relaxation → Harmonics of the WL correction

$$\langle \Delta g \rangle \propto \int P_0(t) \langle e^{i\Phi} \rangle_C \, dt$$

$$P_0(t) = \frac{1}{\sqrt{4\pi D t}} \sum_m e^{-m^2 L^2 / 4Dt} \cos m\theta$$

Harmonics expansion of the return probability

$$\langle \Delta g \rangle \propto \sum_m \int \frac{e^{-m^2 L^2 / 4Dt}}{\sqrt{4\pi D t}} \langle e^{i\Phi(t)} \rangle_{C_m} \, dt \quad \cos m\theta$$

$\tau_\phi \gg \tau_D$

$$\langle e^{i\Phi(t)} \rangle_{\mathcal{C}_m} \sim e^{-\frac{1}{2}\langle \Phi^2(t) \rangle_{\mathcal{C}_m}} \sim e^{-\frac{\sqrt{\tau_D}}{6\tau_\phi^{3/2}} t}$$

$$\int \frac{e^{-\frac{m^2 L^2}{4Dt} - \frac{\sqrt{\tau_D}}{6\tau_\phi^{3/2}} t}}{\sqrt{4\pi D t}} dt = \sqrt{6} \left(\frac{L_\phi}{L} \right)^{3/2} e^{-\frac{m}{\sqrt{6}} \left(\frac{L}{L_\phi} \right)^{3/2}}$$

$\longrightarrow \quad L_\phi \ll L$

$$F_m = -\frac{Ai(0)}{Ai'(0)}\,\frac{L_\phi}{L}\,e^{-m\frac{\pi}{8}\left(\frac{L}{L_\phi}\right)^{3/2}}$$

$$\to e^{-mT^{1/2}}$$

$\longrightarrow \quad L_\phi \gg L$

$$F_m = \sqrt{6}\left(\frac{L_\phi}{L}\right)^{3/2} e^{-\frac{m}{\sqrt{6}}\left(\frac{L}{L_\phi}\right)^{3/2}}$$

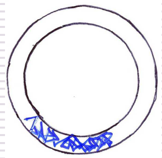
Same form as Altshuler, Aronov, Spivak oscillations

$$F_m = \frac{L_\phi^{AB}}{L}\; e^{-m\frac{L}{L_\phi^{AB}}} \qquad \text{with} \qquad \left(L_\phi^{AB}\right)^2 = \frac{L_\phi^3}{6L}$$

Conclusion : dephasing probes the nature of the diffusive trajectories

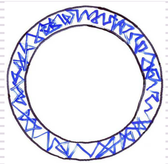
quasi-1D or $m = 0$

$t \ll \tau_D$



$$\left\langle \Phi^2(t) \right\rangle_{T,C} \propto \frac{t^{3/2}}{\tau_\phi^{3/2}}$$

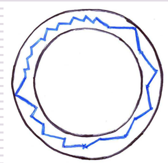
$t \gg \tau_D$



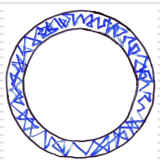
$$\left\langle \Phi^2(t) \right\rangle_{T,C} \propto \frac{\sqrt{\tau_D}}{\tau_\phi^{3/2}}\, t$$

$m \neq 0$

$t \ll \tau_D$



$t \gg \tau_D$



$$\left\langle \Phi^2(t) \right\rangle_{T,C} \propto \frac{\sqrt{\tau_D}}{\tau_\phi^{3/2}}\, t$$