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WORKSHOP ON

QUANTUM SYSTEMS OUT OF EQUILIBRIUM

(14 – 25 June 2004)

" The glass transition and the Coulomb gap in electron glasses "

presented by:

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The Glass Transition and the Coulomb gap in Electron Glasses

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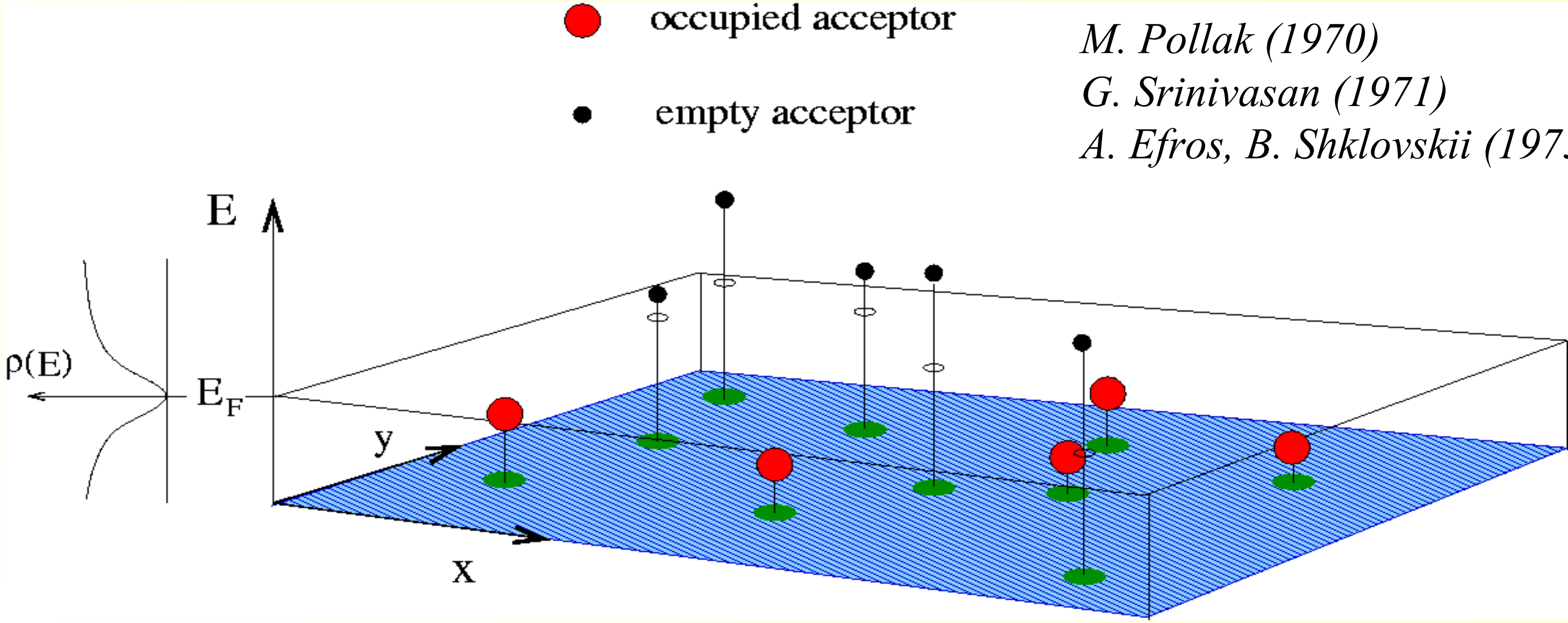
Workshop on Quantum Systems out of Equilibrium
21st June, 2004, ICTP, Trieste

Electron glasses



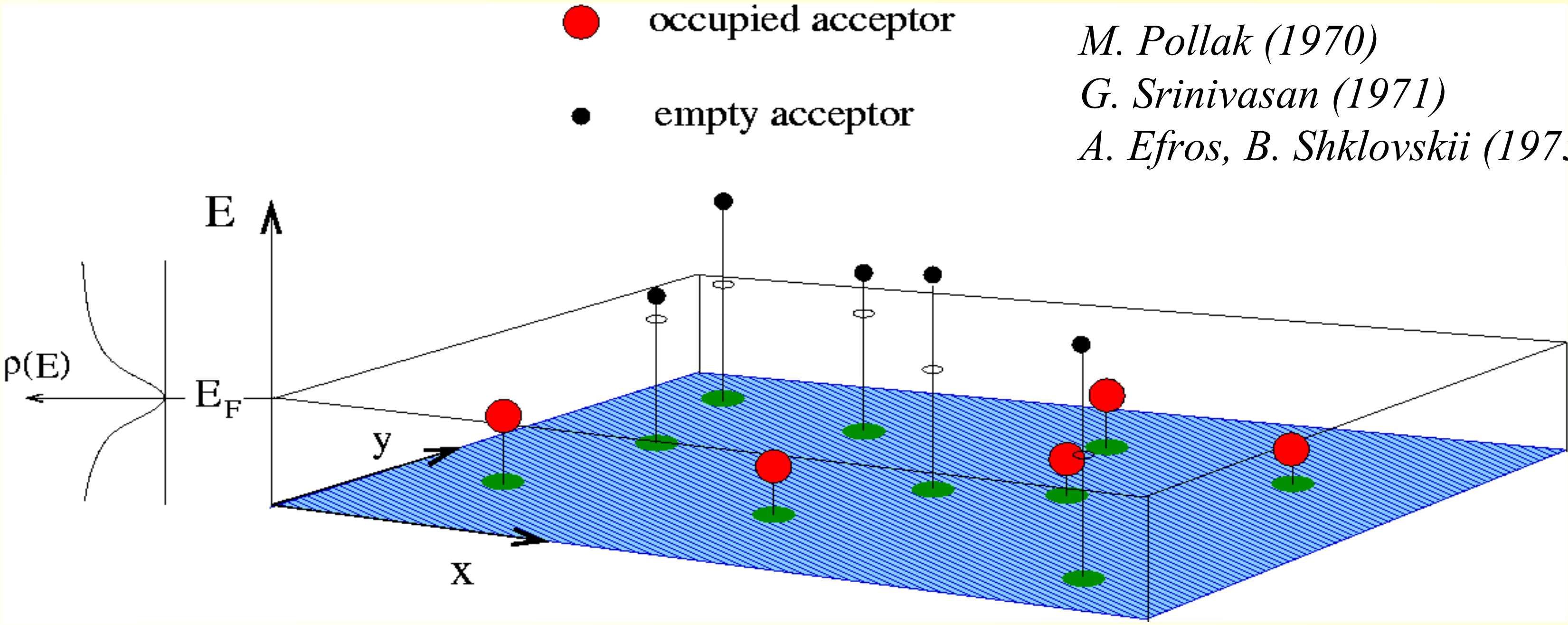
M. Pollak (1970)
G. Srinivasan (1971)
A. Efros, B. Shklovskii (1975)

$$H = \frac{1}{2}\sum_{i \neq j} n_i J_{ij} n_j + \sum_i n_i (\varepsilon_i - \mu)$$

$$J_{ij} = \frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$

Unscreened Coulomb interaction

Electron glasses



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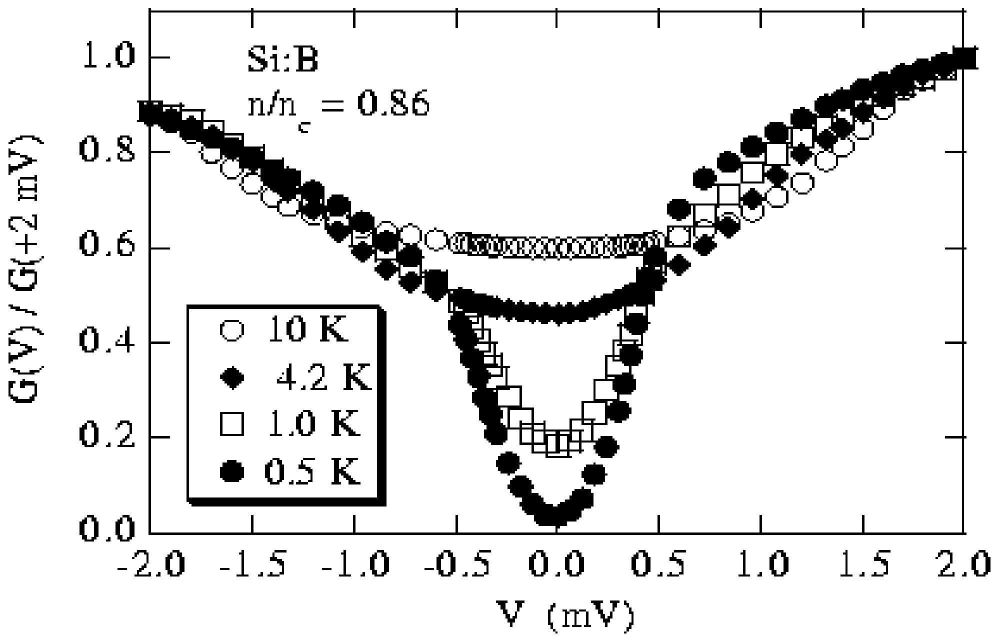
Unscreened Coulomb interaction

Spin representation

$\mu = 0 \;\rightarrow\; s_i = n_i - 1/2$ ⟷ Long range spin glasses (SK-model)

Coulomb gap : Tunneling



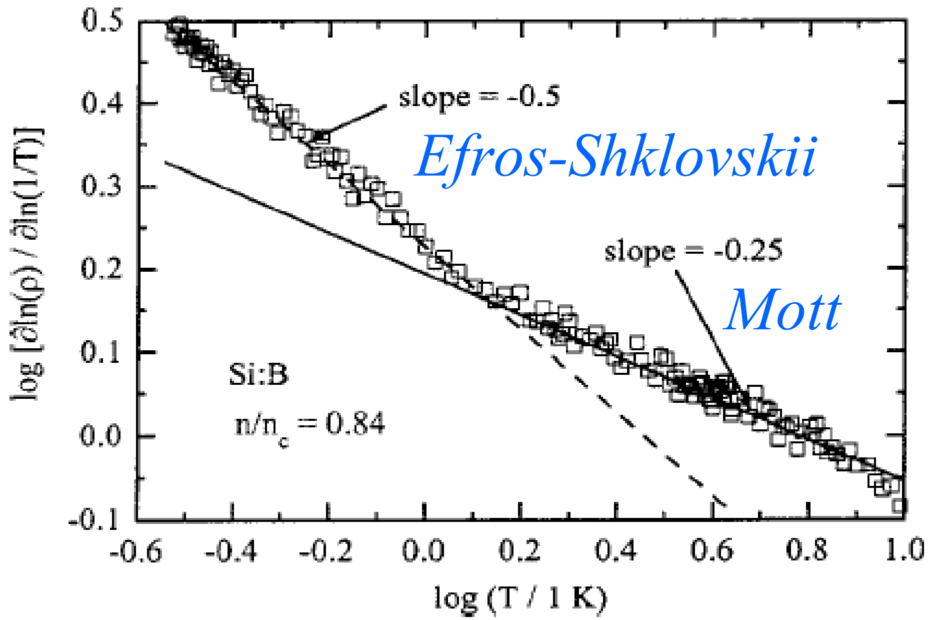
Boron-doped silicon matrix

- $n_c = 4.0 \cdot 10^{18}\, cm^{-3}$
- $\kappa = 100 \pm 10$
- $E_{Cb} = e^2 n^{1/3} / \kappa$

  $\approx 27\, K$

J. G. Massey and M. Lee, PRL ***75****, 4266 (1995)*

Coulomb gap : DC conductivity



Mott conductivity:

Constant density of states

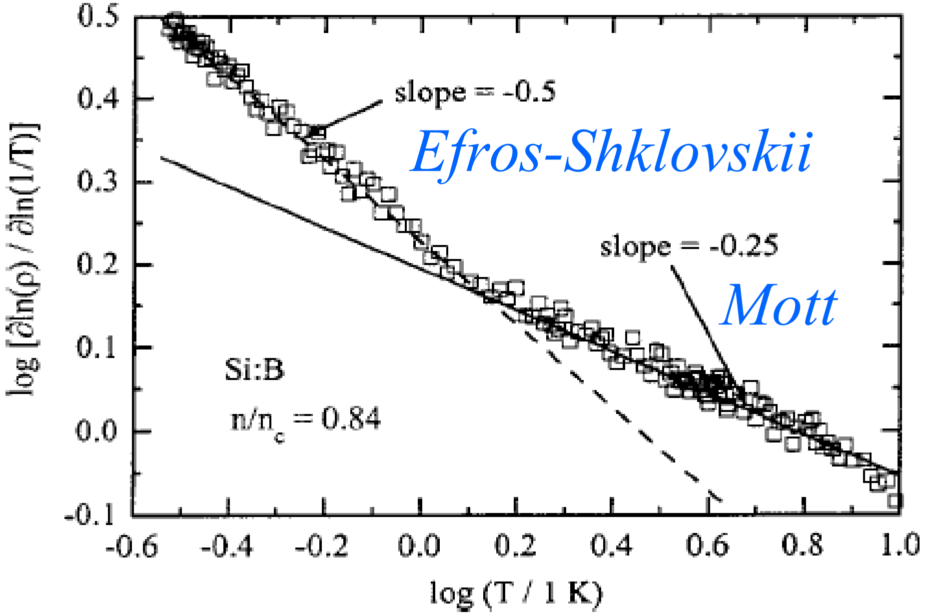
$\rho(E=0) = N_0$

$\sigma(T) \propto \exp\left[-(T_M/T)^{1/4}\right]$

$T_M = C/\xi^3 N_0$

J. G. Massey and M. Lee, PRL ***87****, 056402 (2001)*

Coulomb gap : DC conductivity



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Mott conductivity:
Constant density of states

$$\rho(E=0)=N_0$$

$$\sigma(T)\propto \exp\left[-\left(T_M/T\right)^{1/4}\right]$$

→ $$T_M = C/\xi^3 N_0$$

Efros-Shklovskii conductivity:
Coulomb gap $\rho(E)=\alpha E^2$

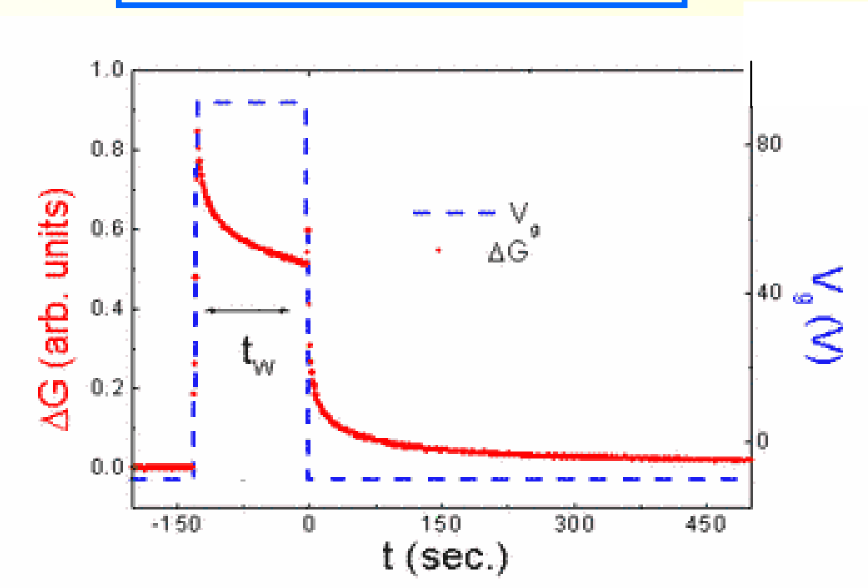
$$\sigma(T)\propto \exp\left[-\left(T_{ES}/T\right)^{1/2}\right]$$

→ $$T_{ES} = C/\xi\alpha^{1/3}$$

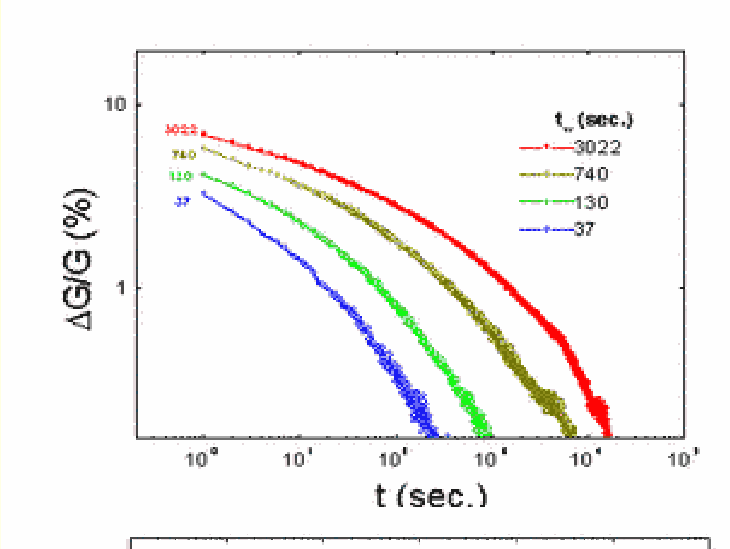
Glassy behaviour : Slow relaxation and aging

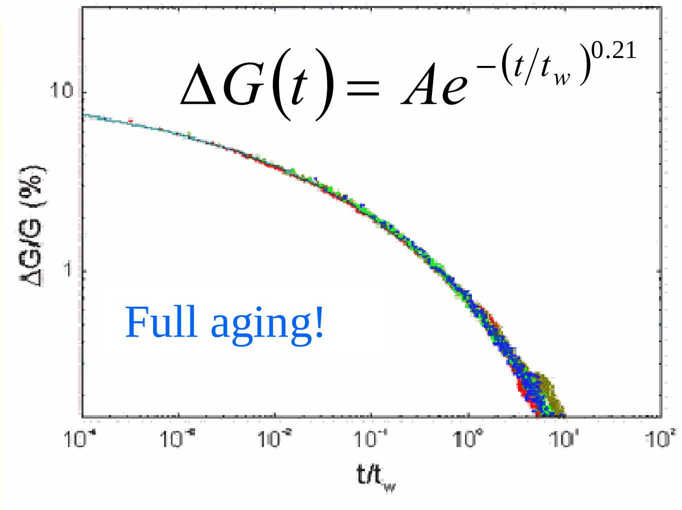
Indium-oxides In_2O_{3-x},

$$n = \left(10^{19} - 10^{21}\right) cm^{-3}$$



A. Vaknin, Z. Ovadyahu,
and M. Pollak, PRL ***84****, 3402 (2000)*



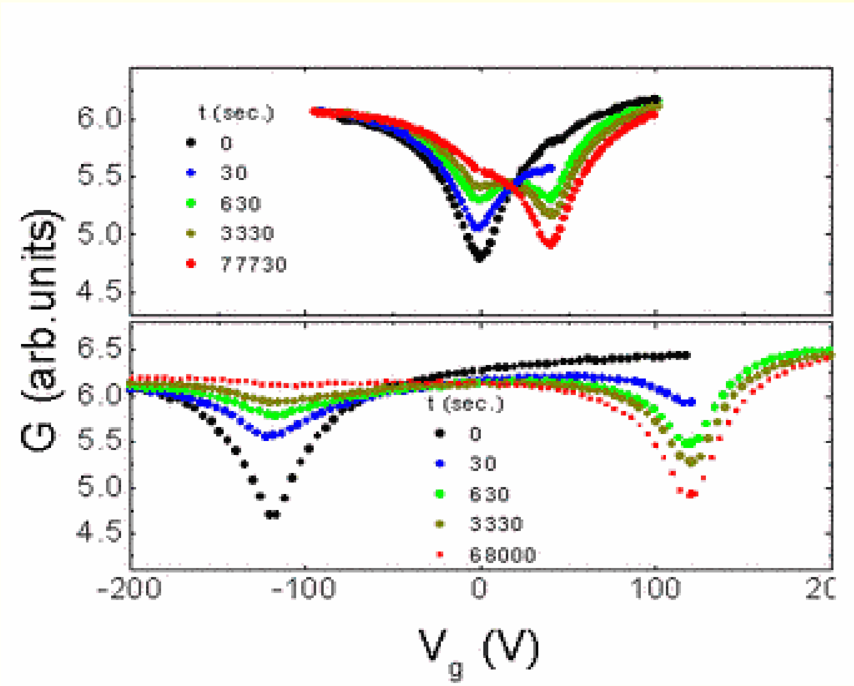


No trace of fast β-relaxation!

Glassy behaviour: Memory effect

Protocol:

- Imprint gate voltage $V_g = 0$.
- Change V_g to new value.
- Sweep V_g, record conductivity $G(V_g)$.



*A. Vaknin, Z. Ovadyahu, and M. Pollak, PRL **81**, 669 (1998)*

Summary of experiments

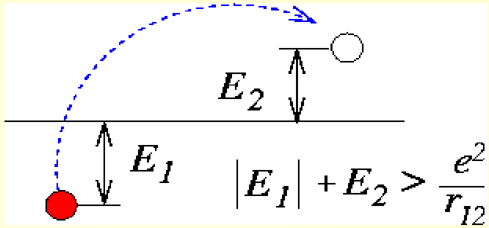
- Experimental evidence for the Coulomb gap
- Coulomb gap is due to electron-electron interactions
- Out of equilibrium behaviour of Coulomb glasses:
    - Slow relaxation
    - Aging (relaxation time scales with t_w)
    - Memory effect

Single particle theory

A. Efros, B. Shklovskii (1975)

Stability of ground state with respect to one particle hop:

The density of states at the Fermi level must vanish at T = 0.



Self-consistent argument for the density of states:

$$R_E = \frac{e^2}{E} \ ; \quad R_E^D \cdot \int_0^E \rho(E)\, dE \le 1$$

$$\longrightarrow \quad \rho(E) \propto E^{D-1} / e^{2D}$$

Parabolic pseudogap in D = 3.

?

- Why is this bound saturated ?
- Why is it universal (exponent) ?
- Many-particle constraints (prefactor) ?

?

Beyond the single particle theory

?

- Relation between glass phase and Coulomb gap ?
- What are the soft modes in the glassy relaxation due to ?
- Screening and charge response ?
- How does hopping conductivity work ?
- Importance of correlations ?

(*M. L. Knotek, M. Pollak, M. Ortuño)*

?

New approach

(MM and L.B. Ioffe, cond-mat/0406324)

- Locator approximation based on a systematic diagrammatic technique.
- Prediction of a glass transition and its physical signatures.
- The central role of marginal stability and its relation to the Efros-Shklovskii Coulomb gap.
- Quantitative prediction for different density of states relevant for tunneling and transport.

Diagram technique

G. Srinivasan, PRB ***4****, 2581 (1971)*

Hamiltonian $$H = \frac{1}{2}\sum_{i \neq j} s_i J_{ij} s_j + \sum_i s_i \varepsilon_i \quad , \qquad P(\varepsilon) = \frac{\exp\left[-(\varepsilon/W)^2/2\right]}{\sqrt{2\pi W^2}}$$

Energy scales $E_{Cb} = e^2/a \equiv 1, \; T, \; W$

Strong disorder limit $E_{Cb}/W << 1$ Low temperature $T \leq W$

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Hubbard-Stratonovich $\rightarrow$ Mean Coulomb potentials φ_i

$$Z = \int \prod_i d\varphi_i \sum_{\{s_i\}} \exp\left\{-\frac{1}{2}\sum_{i\neq j}\varphi_i (\beta J)^{-1}_{ij}\varphi_j + \sum_i (\beta\varepsilon_i + i\varphi_i)s_i\right\}$$

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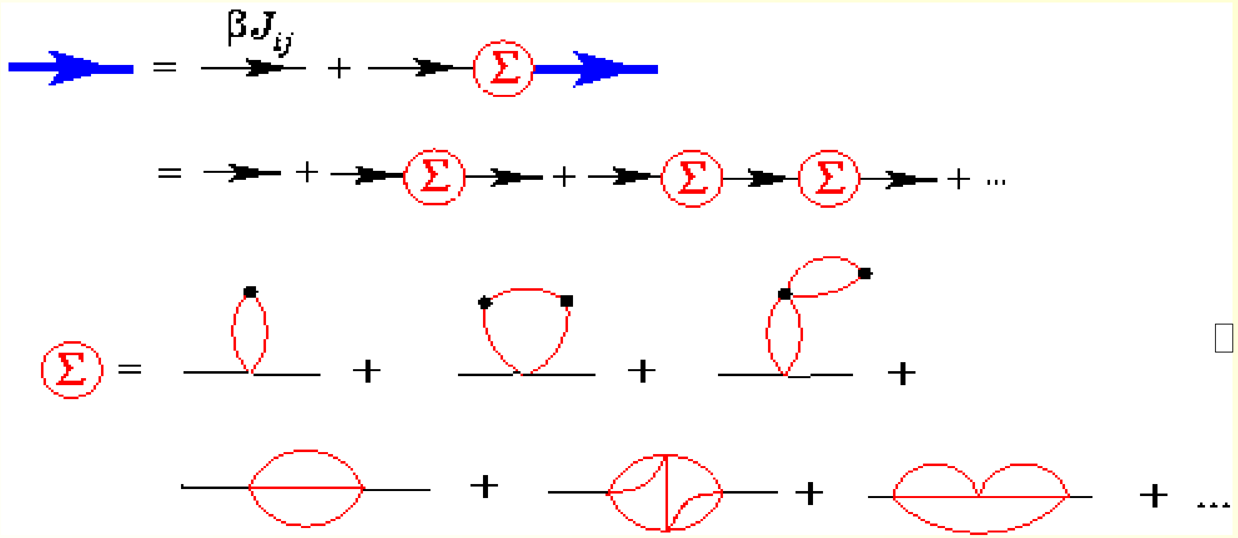
Hubbard-Stratonovich → Mean Coulomb potentials φ_i

$$Z = \int \prod_i d\varphi_i \sum_{\{s_i\}} \exp\left\{-\frac{1}{2}\sum_{i \neq j} \varphi_i (\beta J)^{-1}_{ij} \varphi_j + \sum_i (\beta\varepsilon_i + i\varphi_i) s_i\right\}$$

Replica trick

$$\overline{Z^n} = \int \prod_{i,a} d\varphi_i \overline{\exp\left\{-\frac{1}{2}\sum_{i \neq j, a} \varphi_i^a (\beta J)^{-1}_{ij} \varphi_j^a + \sum_{i,a} \ln\left[\cosh\left(\frac{\beta\varepsilon_i + i\varphi_i}{2}\right)\right]\right\}}$$

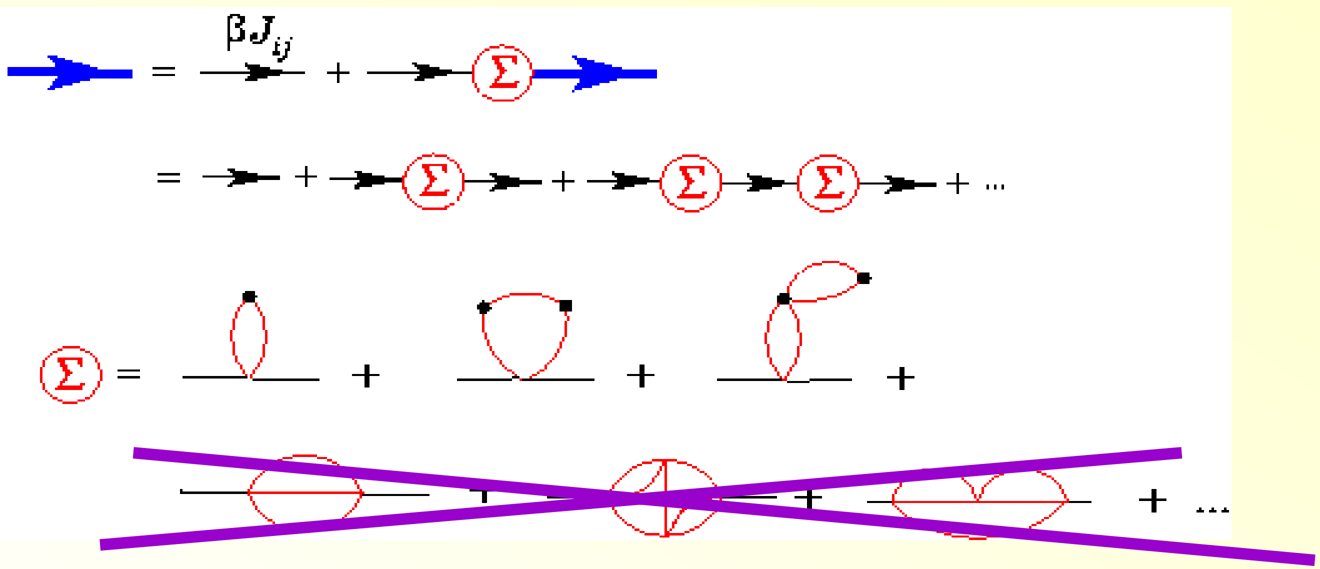
Locator approximation



$$\sim \left(\frac{\beta^2 E_{Cb}}{W}\right)^{1/2} \frac{1}{(\beta W)^3}$$

$$\sim \left(\frac{E_{Cb}}{W}\right)^{3} \frac{1}{(\beta W)^3}$$

Locator approximation



Local self-energy with non-trivial replica structure

$$\Sigma_{ab}(k) \approx \Sigma_{ab}(0)$$

- Map to an effective single-site model.
- Describe environment by selfconsistent Σ (local field distribution).

Mapping to single-site model

$$H[\{s_i\}] = \frac{1}{2}\sum_{i\neq j} s_i J_{ij} s_j + \sum_i s_i \varepsilon_i \longrightarrow H_0\left[\left\{s^a\right\}\right] = \frac{1}{2}\sum_{a.b} s^a B_{ab} s^b + \sum_a s_a \varepsilon_0$$

Self-consistency condition on B_{ab}

$$\left\langle s^a s^b \right\rangle_c \equiv G_{ab}^{-1} = \left[\frac{1}{\beta B - \Sigma(B)}\right]_{ab} = \frac{1}{V}\sum_i \left\langle s_i^a s_i^b \right\rangle_c = \frac{1}{V} Tr\left[\frac{1}{\beta J - \Sigma(B)}\right]_{ab}$$

→ Free energy of the single-site model

$$n\beta F(B) = -\ln\left[\sum_{s^a} \exp\left(-\frac{\beta}{2}\sum_{a,b} s^a B_{ab} s^b\right)\right] + \frac{1}{2} tr\left\{\ln\left[\beta B - \Sigma(B)\right] - \frac{1}{V} Tr \ln\left[\beta J - \Sigma(B)\right]\right\}$$

Glass transition

Original lattice model

Disorder-averaged correlations

$$\overline{\left\langle \varphi_i \varphi_j \right\rangle_c} = C \frac{e^{-r/\xi_1}}{r} , \quad \xi_1 \propto \sqrt{W}$$

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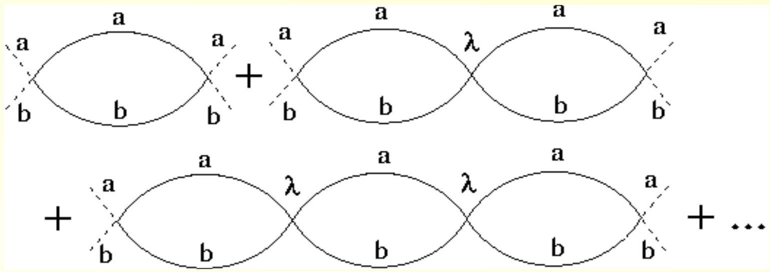
Disorder-averaged correlations

$$\overline{\langle \varphi_i \varphi_j \rangle_c} = C \frac{e^{-r/\xi_1}}{r}, \quad \xi_1 \propto \sqrt{W}$$

Fluctuations

$$\overline{\langle \varphi_i \varphi_j \rangle_c^2} = C \frac{e^{-r/\xi_2}}{r}, \quad \xi_2 \to \infty \text{ for } T \to T_c$$

$$= \langle \varphi_i^a \varphi_i^b \varphi_j^a \varphi_j^b \rangle =$$



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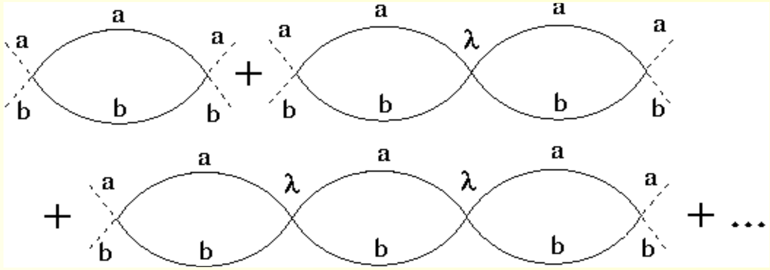

Single site model

Replicon instability

$$\partial^2 \mathrm{F}(\mathrm{B}) / \partial^2 \mathrm{B} = 0$$

at the same

$$T_c = \frac{1}{6(2/\pi)^{1/4}\sqrt{W}}$$

Experiment

Detect long range charge correlations by measuring $\sigma(V_g + \delta V \cdot \cos[\omega t])$.

Glass phase

Generic properties of a glass phase

- Large number of pure states with self-generated disorder.
- Spontaneous expectation values $\langle \varphi_i \rangle$
- Broken ergodicity.

Landau expansion around T_c

$$n\beta F \approx \frac{1}{W^{3/2}}\left[Tr\left(-\tau\delta B^2 + \delta B^3\right) + \sum \delta B_{ab}^4 \right]$$

Same form as the Sherrington Kirkpatrick (SK) spin glass!

→

- Expect similar glassy dynamics
- Ultrametricity in real space?

Marginal stability

Persistent long range correlations in the glass phase

$$\overline{\langle s_i s_j \rangle_c^2} \propto \frac{1}{r_{ij}}$$

→ Wide distribution of charge response (screening)

$$\langle s_j | s_i = S \rangle - \langle s_j \rangle \propto \langle s_i s_j \rangle_c$$

↔ The system is permanently in a marginally stable (critical) state:

$$\partial^2 F(B) / \partial B^2 = 0$$

→ Soft modes and slow dynamics.

Coulomb gap and marginal stability

Marginal stability $\partial^2 F(B)/\partial B^2 = 0$ ⟶

Condition on the distribution of thermodynamic fields y_i

$$\langle s_i \rangle = \frac{1}{2}\tanh\left(\frac{\beta y_i}{2}\right)$$

$$\int \frac{dyP(y)}{[2\cosh(\beta y/2)]^4} = \frac{1}{g_1^{-2}(\Sigma_0) - g_2^{-1}(\Sigma_0)}$$

$$g_m(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\left(\frac{4\pi\beta}{k^2} + x\right)^m}$$

Coulomb gap and marginal stability

Marginal stability $\partial^2 F(B)/\partial B^2 = 0$ →

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+ Susceptibility (spin-spin correlator)

$$\chi \equiv \beta\left[\frac{1}{4} - \langle S_a S_b \rangle\right] = \beta g_1(\Sigma_0) = \beta \int \frac{dyP(y)}{\left[2\cosh\left(\beta y/2\right)\right]^2}$$

Coulomb gap and marginal stability

Marginal stability $\partial^2 F(B)/\partial B^2 = 0$ →

Condition on the distribution of thermodynamic fields y_i

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$$g_m(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\left(\frac{4\pi\beta}{k^2} + x\right)^m}$$

\+ Susceptibility (zero-field cooled!)

$$\chi \equiv \beta\left[\frac{1}{4} - \langle S_a S_b \rangle\right] = \beta g_1(\Sigma_0) = \beta \int \frac{dyP(y)}{\left[2\cosh(\beta y/2)\right]^2}$$

$$2 = D - 1$$

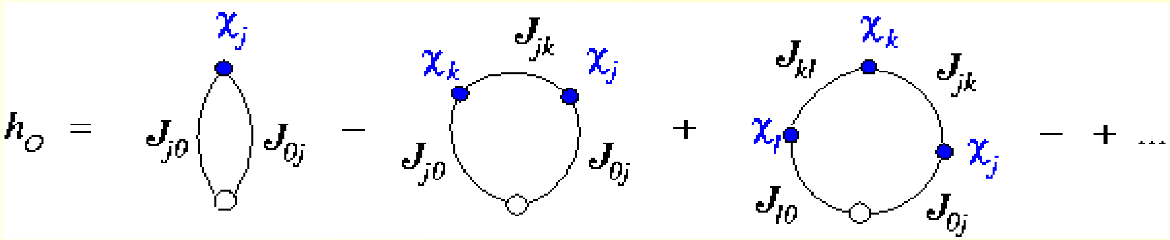
→ $P(y) \xrightarrow{T \to 0} T^2 \Psi(y/T)$ and $\chi \approx \beta/\Sigma_0 \propto T^2$

Parabolic Coulomb gap is a consequence of marginal stability!

“TAP” equations

Relation between thermodynamic field y_i and average local field $\langle h_i \rangle$

$$\langle h_i \rangle = y_i + \langle s_i \rangle h_O$$



Onsager back reaction

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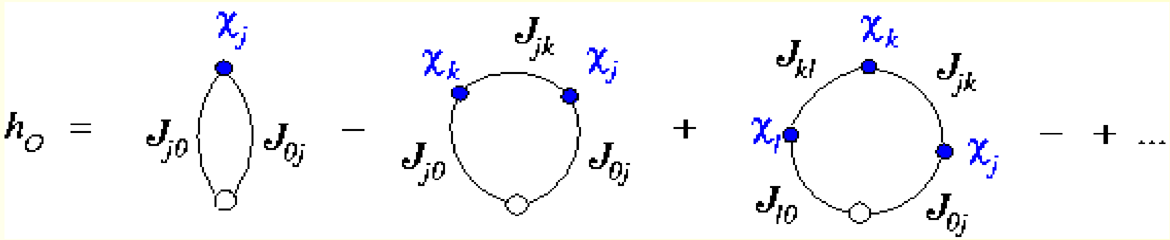

Onsager back reaction

$$h_O = \beta \int \frac{d^3k}{(2\pi)^3} \frac{J^2(k)}{\beta J(k) + \Sigma_0} \approx 2\sqrt{\pi\chi}$$

$$J(k) = 4\pi / k^2$$

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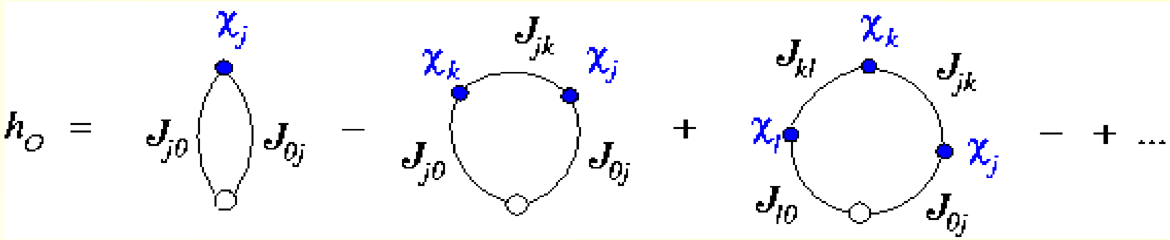

Onsager back reaction

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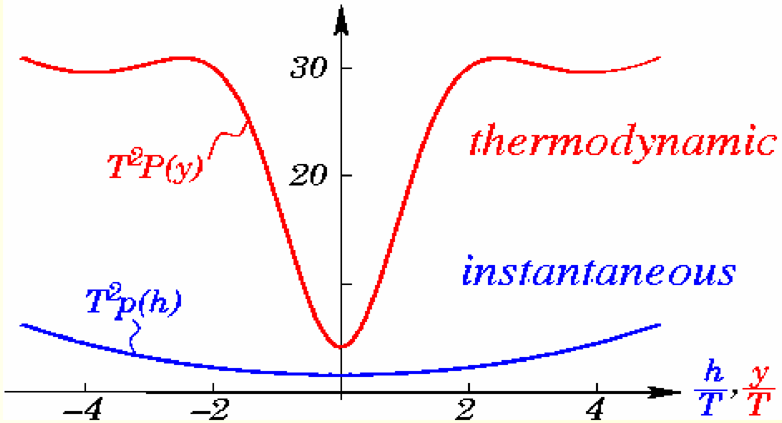
$$J(k) = 4\pi / k^2$$

Distribution of instantaneous local fields h_i

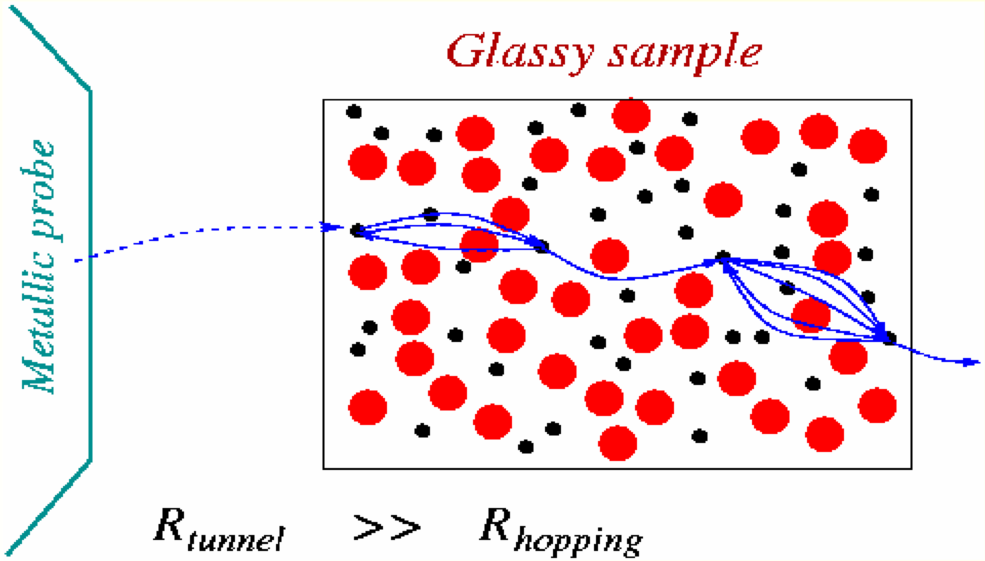
$$p(h) = \cosh(\beta h/2) \int \frac{dy P(y)}{\cosh(\beta y/2)} \frac{\exp\left(-\frac{\beta (h-y)^2}{2h_O}\right)}{\sqrt{\frac{2\pi h_O}{\beta}} \exp\left(-\frac{\beta h_O}{8}\right)}$$

Tunneling versus hopping transport

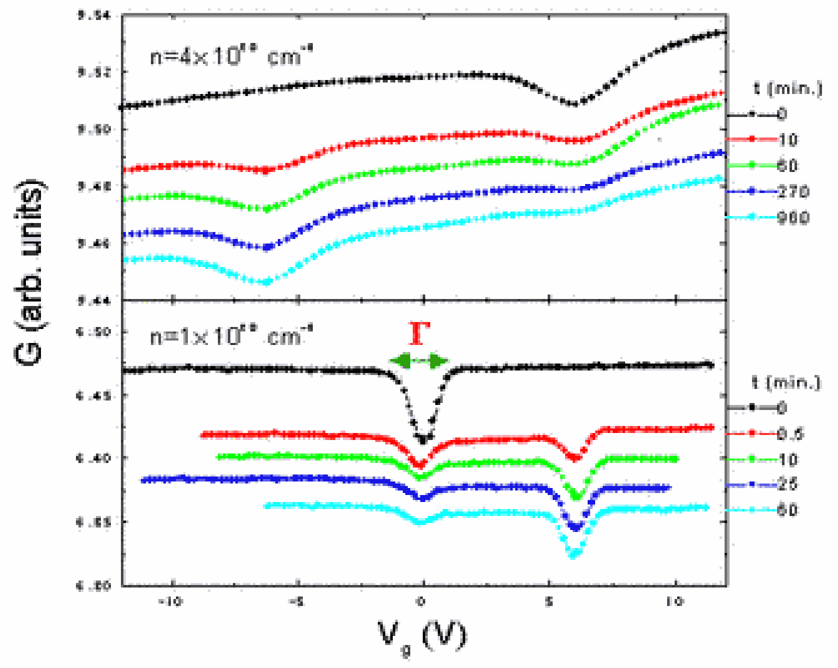


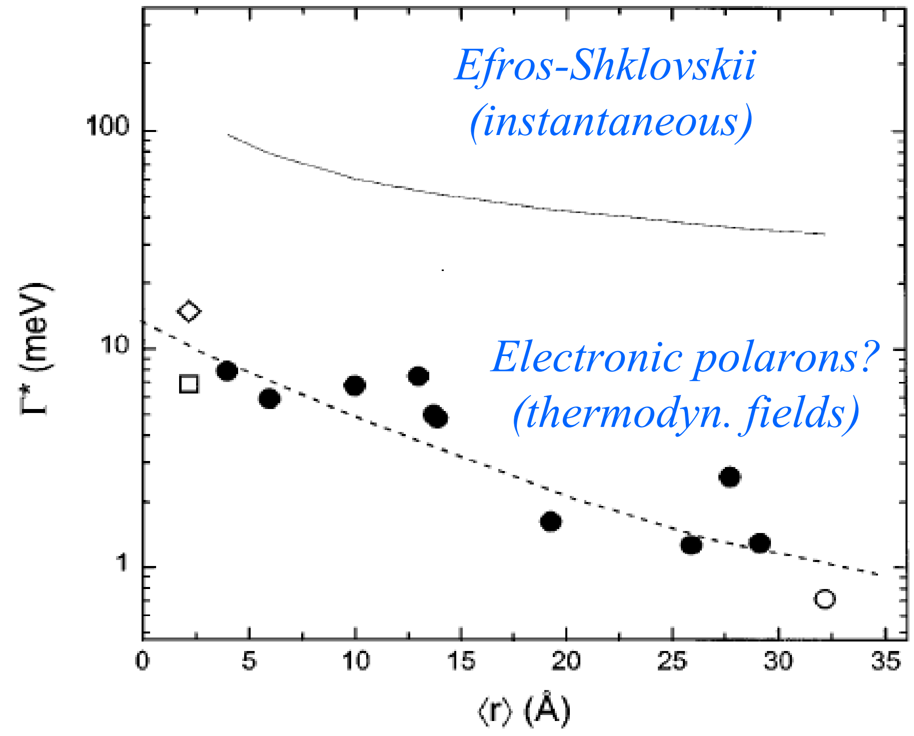
$$v_0 = \int dh \, \frac{p(h)}{\left[2\cosh(\beta h/2)\right]^2} \approx 2.2 \; T^2$$

$$\chi = \int dy \, \frac{P(y)}{\left[2\cosh(\beta y/2)\right]^2} \approx 20 \, T^2$$



Width of cusp in the memory effect





A. Vaknin, Z. Ovadyahu, and M. Pollak, PRL ***81****, 670 (1998)*

Conclusions

- Evidence for a continuous glass transition (SK-type) in Coulomb glasses.
- Prediction for its experimental and numerical observation.
- Marginal stability of the glass phase
  $\rightarrow$ Soft mode in dynamics
  $\rightarrow$ Coulomb gap in the density of states
- Long range correlations of charge fluctuations in the glass phase. $\rightarrow$ Collective behaviour, at the basis of memory effect and aging phenomena.
- Quantitative predictions on the distribution of instantaneous fields (tunneling) and thermodynamic fields (transport, screening).

Outlook and open questions

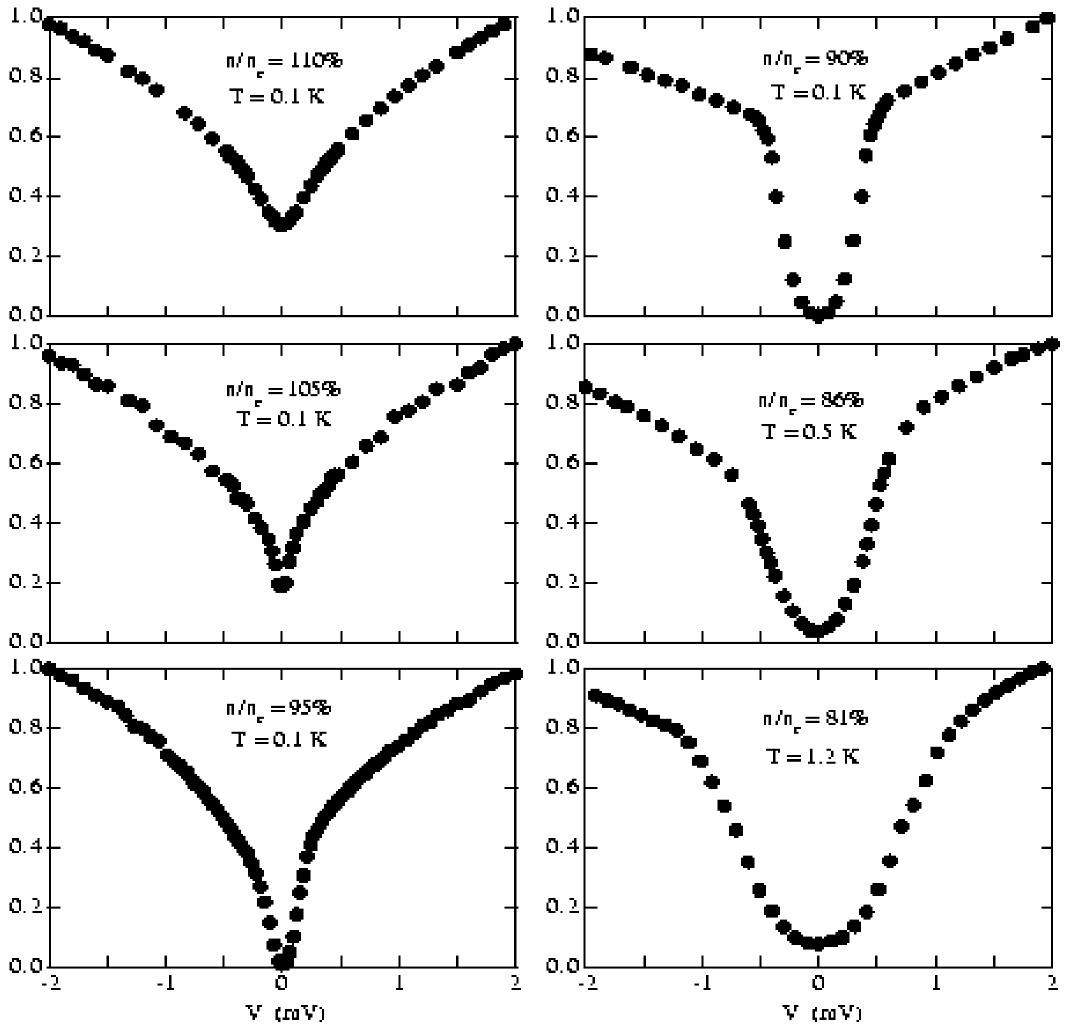
- Hopping transport:
  - Successive correlations ? (*Knotek, Pollak, Ortuño*)
  - Interaction-assisted tunneling ?
  - Universal prefactor in Efros-Shklovskii conductivity ?

  (*Khondaker et al., Yakimov et al.*)

$$\sigma(T) = n\frac{h}{e^2}\exp\left[-\left(T_{ES}/T\right)^{1/2}\right]$$

- Quantum melting of the electron glass
  - Inclusion of hopping matrix elements in the locator approximation (*Pastor, Dobrosavljević*)
- Approach of the metal-insulator transition
  - Relation between Coulomb gap (insulating side) and zero bias anomaly (metal side); (*Altshuler, Aronov*)
- Small disorder limit
  - Discontinuous (1-step RSB) glass transition ?
  - From Coulomb glass to Wigner crystal ? *(Pankov et al.)*

From Coulomb gap to zero bias anomaly
Quantum melting of the electron glass



Boron-doped silicon matrix

- $n_c = 4.0 \cdot 10^{18}\, cm^{-3}$
- $\kappa = 100 \pm 10$
- $E_{Cb} = e^2 n^{1/3} / \kappa$
  $\approx 27\, K$

M. Lee et al., PRB ***60****, 1582 (1999)*

Coulomb gap : AC conductivity