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WORKSHOP ON

QUANTUM SYSTEMS OUT OF EQUILIBRIUM

(14 – 25 June 2004)

" Coulomb-blockade-induced bound quasiparticle states in double island qubits"

presented by:

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Coulomb-blockade-induced bound quasiparticle states in superconductor nanostructures

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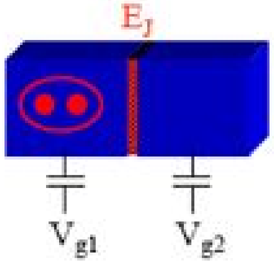
Discussions with: L. Glazman and K. Lehnert

cond-mat/0404450

ICTP, Trieste, June 2004

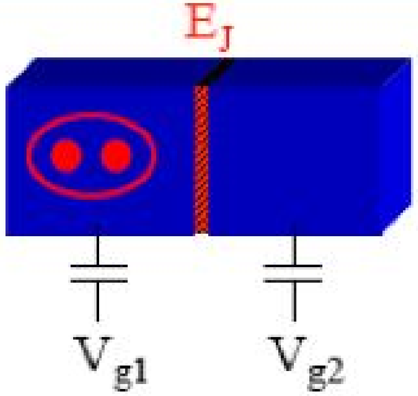
Outline



- Double island superconductor structure [1]: Model
  - Quantum charge fluctuations: shape of Coulomb blockade steps
- Even number of electrons
- 1) $\Delta > 2E_C$
  - Strong Coulomb enhancement of the Josephson energy
- 2) $\Delta < 2E_C$
  - Bound state of two quasiparticles
  - Gate voltage dependence of the island charge
- Odd number of electrons: bound state for a single quasiparticle
  - Gate voltage dependence of the island charge
  - Non-monotonic temperature dependence of the width of the Coulomb blockade step
- Summary

[1] E. Bibow, P. Lafarge and L. P. Levy, Phys. Rev. Lett. **88**, 017003 (2002)

Double island S-structure



Fixed total number of electrons.
Electrons can tunnel between the islands but not to the outside world.

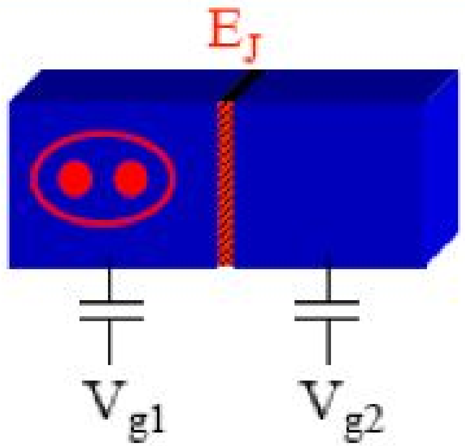
This structure can be used as a charge qubit [1,2,3]

- Here: Quantum fluctuations of the island charge
- Shape of Coulomb blockade steps at low temperature
  - Total number of electrons is fixed
  - No thermal quasiparticles in the system
  - Dependence of the island charge on the gate voltage difference, $V_{g1} - V_{g2}$

[1] Y. Nakamura, Yu. Pashkin and J. S. Tsai, Nature, **938,** 786 (1999)
[2] E. Bibow, P. Lafarge and L. P. Levy, Phys. Rev. Lett. **88**, 017003 (2002)
[3] A. Blais, R. Huang, A. Wallraff, S. M. Girvin, R. J. Schoelkopf, cond-mat/0402216

Double island S-structure (model)

- Notations (symmetric system):
  - δ - mean level spacing
  - E_C - charging energy
  - Δ - pairing gap (same in both islands)
  - $\hat{n}$ - island charge *difference* in units of *e*
  - $N \propto V_{g1} - V_{g2}$ - dimensionless gate voltage difference



$$H = H_0 + H_C + H_t$$

$$H_C = E_C(\hat{n} - N)^2$$

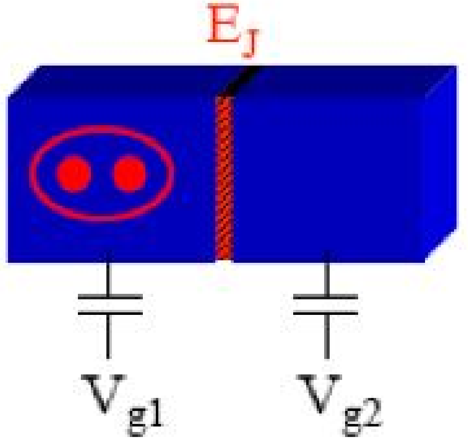
$$H_t = \sum_{pk\sigma} t_{kp} a^\dagger_{k\sigma} a_{p\sigma} + h.c.$$

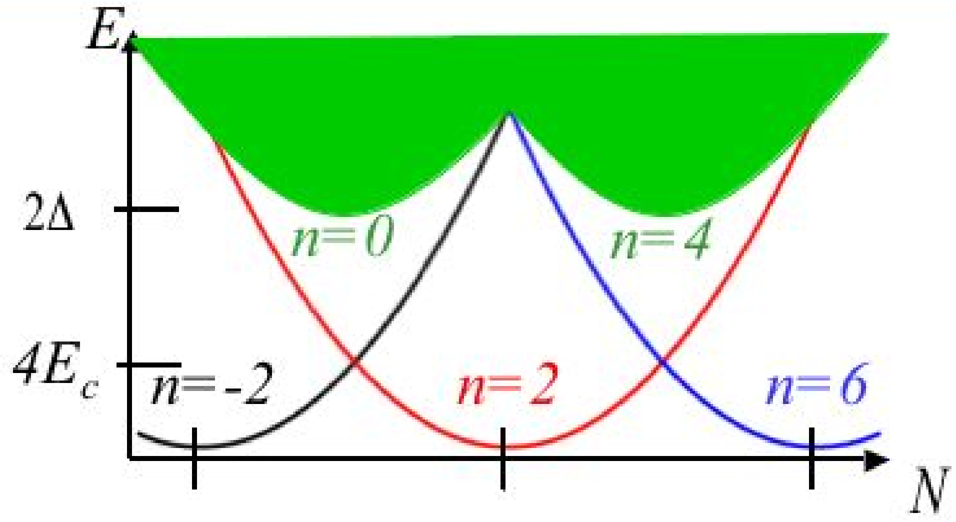
H_0 - BCS Hamiltonian

- Assumptions:
  - Total number of electrons is fixed
  - Constant interaction model
  - No thermal quasiparticles in the system, $T < \Delta / \ln(\Delta / \delta)$
  - Small mean level spacing, $\delta \quad T, \Delta, E_C$
  - Weak tunneling, $\langle t_{kp}^2 \rangle \quad \delta^2$

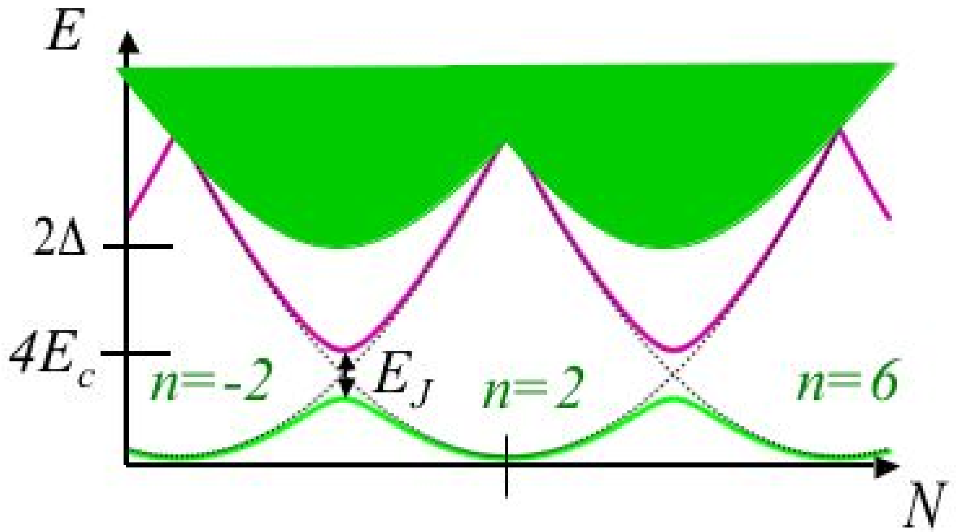
Even electron number

- Zero tunneling



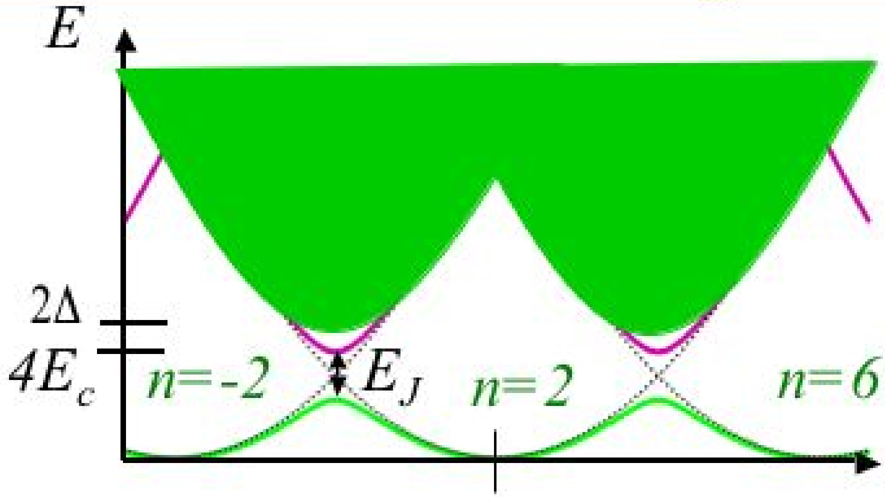


- Finite tunneling

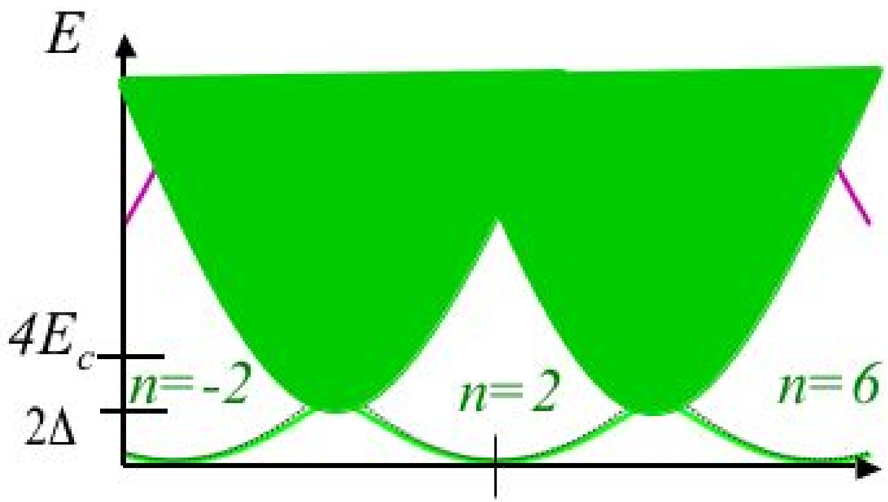


What happens as Δ approaches $2E_C$?

- $\Delta > 2E_C$: Two discrete states below the continuum, Josephson energy



- $\Delta < 2E_C$: What happens? Are there discrete states below the continuum?



Josephson energy (1): $\Delta > 2E_C$

- Perturbative result [1,2] diverges logarithmically

$$\tilde{E}_J = E_J \frac{4}{\pi^2}\int_0^\infty \frac{d\xi_k}{\varepsilon_k}\int_0^\infty \frac{d\xi_p}{\varepsilon_p}\frac{\Delta}{(\varepsilon_k+\varepsilon_p-4E_C)}$$

as $\Delta \to 2E_C$. Here $\varepsilon_p = \sqrt{\xi_p^2 + \Delta^2}$, $E_J = g\Delta/8$ [3] with g contuctance.

- The divergence is due to the coupling via virtual states with two quasiparticles with $\xi_k, \xi_p \quad \Delta$.
- For $\Delta \to 2E_C$ the states $|\pm 2\rangle$ with $n = \pm 2$ become strongly hybridized with the continuum of states $|k,\sigma;p,-\sigma\rangle = \alpha^\dagger_{k,\sigma}\alpha^\dagger_{p,-\sigma}|0\rangle_{BCS}$ with $n = 0$ and one quasiparticle in each grain.
- With logarithmic accuracy the wave function is

$$|\Psi\rangle = \Psi_2|2\rangle + \Psi_{-2}|-2\rangle + \sum_{kp\sigma} C_{kp\sigma}|k,\sigma;p,-\sigma\rangle$$

where $\xi_k, \xi_p \quad \Delta$

[1] K.A. Matveev, M. Gisselfalt, L.I. Glazman, M. Johnson, R.I. Shekhter, Phys. Rev. Lett. **70**, 2940 (1993)
[2] P. Joyez, P. Lafarge, A. Filipe, D. Esteve, M.H. Devoret, Phys. Rev. Lett. **72**, 2458 (1994)
[3] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963)

Josephson energy (2): $\Delta > 2E_C$

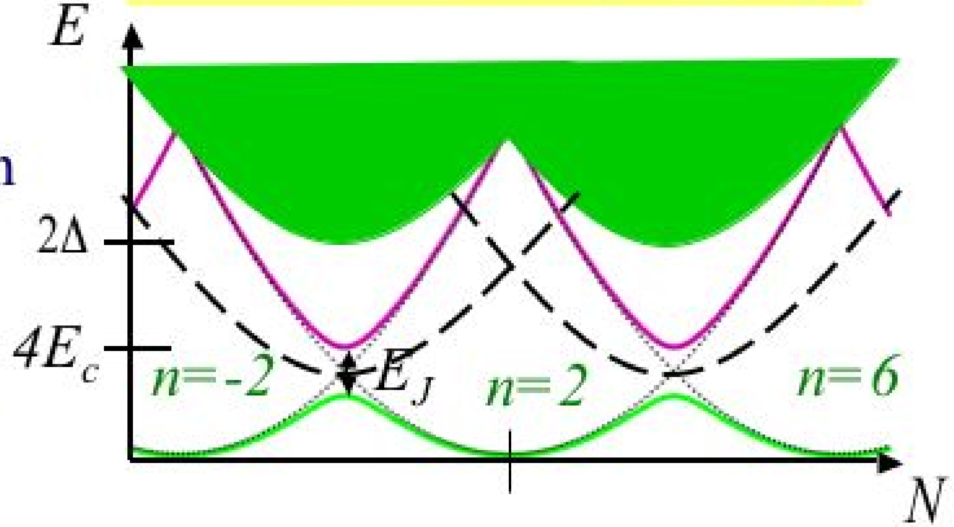
- The discrete spectrum is given by the implicit relation:

$$E = \frac{E_2 + E_{-2}}{2} + \frac{\pi (E - E_2)(E - E_{-2})}{8E_J \ln \frac{E_0 + 2\Delta - E}{E_0 + 3\Delta - E}}$$

- Here $E_{\pm 2}$ and E_0 are electrostatic energies for $n = \pm 2$ and $n = 0$
- Two discrete states: Energy splitting at the degeneracy point $N = 0$ is

$$\tilde{E}_J = E_J \frac{8}{\pi} \ln \frac{\Delta}{\max\{E_J, \Delta - 2E_C\}}$$

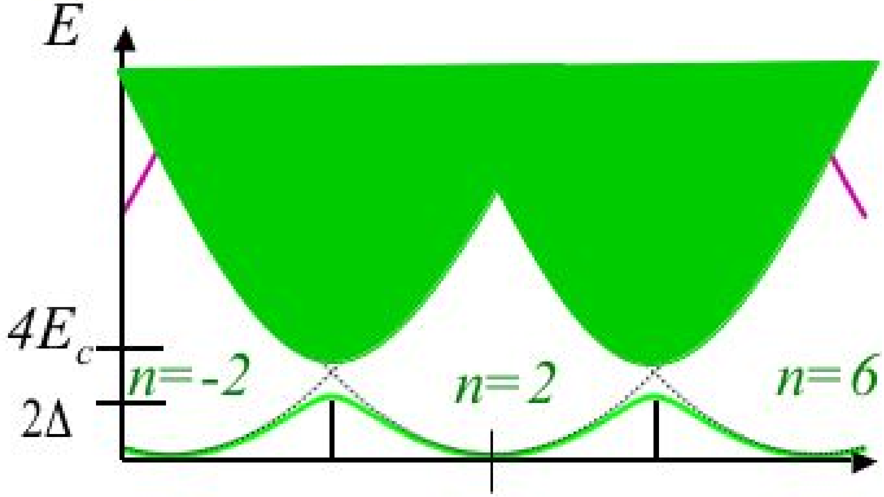
- Coulomb enhancement of the Josephson energy can be much stronger than that for a single grain in contact with a lead, where it is limited to 1.3 [1,2].
- In Ref. [3] an enhancement factor of 3 was reported.



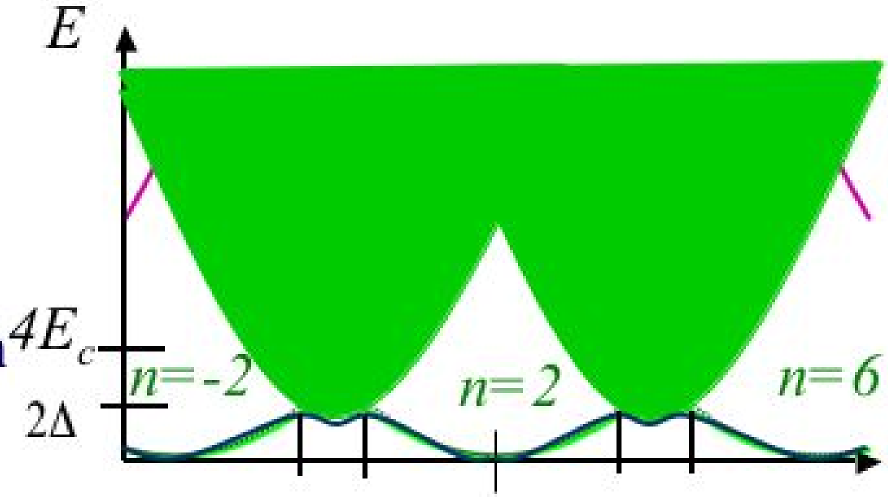
[1] K.A. Matveev, M. Gisselfalt, L.I. Glazman, M. Johnson, R.I. Shekhter, Phys. Rev. Lett. **70**, 2940 (1993)
[2] P. Joyez, P. Lafarge, A. Filipe, D. Esteve, M.H. Devoret, Phys. Rev. Lett. **72**, 2458 (1994)
[3] E. Bibow, P. Lafarge and L.P. Levy, Phys. Rev. Lett. **88**, 017003 (2002)

$\Delta < 2E_C$: Bound state of two quasiparticles

- One discrete state always remains



- 1) $0 < 2E_C - \Delta < \tilde{E}_J$



- 2) $2E_C - \Delta > \tilde{E}_J$
  charge degeneracy points split into two. Between them the two quasiparticles are bound to the contact at T=0.

$\Delta < 2E_C$: Binding energy + island charge

- Between adjacent degeneracy points the gap to the continuum is

$$E_0 + 2\Delta - E = \Delta \exp\left\{ -\frac{\pi (E_2 - E_0)(E_{-2} - E_0)}{4g\Delta E_C} \right\}$$

- Gate voltage dependence of the island charge difference near the degeneracy point 0-2 is given by the implicit relation

$$\frac{n}{2-n} + \ln\frac{2}{2-n} = \frac{\pi N^*}{2} + \ln\frac{2\pi}{g}$$

$$N^* = \frac{2E_C}{E_J}\left(N - 1 + \frac{\Delta}{2E_C} \right)$$

- N* is the rescaled distance from the degeneracy point
- No singularities
- Width of the transition region is $\delta N \quad E_J / E_C$

$\Delta < 2E_C$: Island charge

- Plot of the gate voltage dependence of the island charge difference

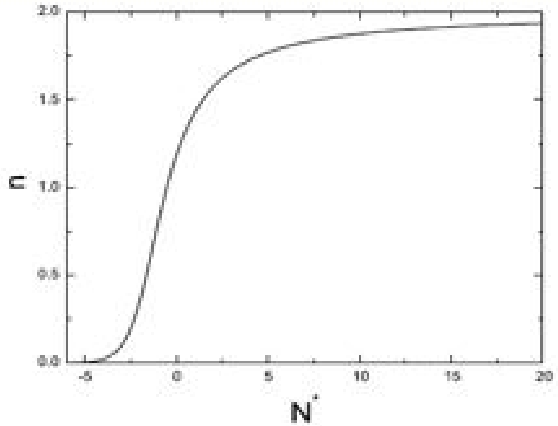


FIG. 3: The dependence of the island charge difference on the rescaled gate voltage $N^* = \frac{2E_C}{E_J}\left(N - 1 + \frac{\Delta}{2E_C}\right)$ for an even number of electrons in the device.

- Asymptotics far from degeneracy point:

$$n = \frac{4\pi}{g}\exp\left(-\frac{\pi\,|N^*|}{2}\right) \qquad N^* < 0$$

$$n = 2 - \frac{4}{\pi N^*} \qquad N^* > 0$$

Even charge summary

- Coulomb enhancement of the Josephson energy

$$\tilde{E}_J = E_J \frac{8}{\pi} \ln \frac{\Delta}{\max\{E_J, \Delta - 2E_C\}}$$

- $2E_C > \Delta$: bound state of two quasiparticles. Binding energy

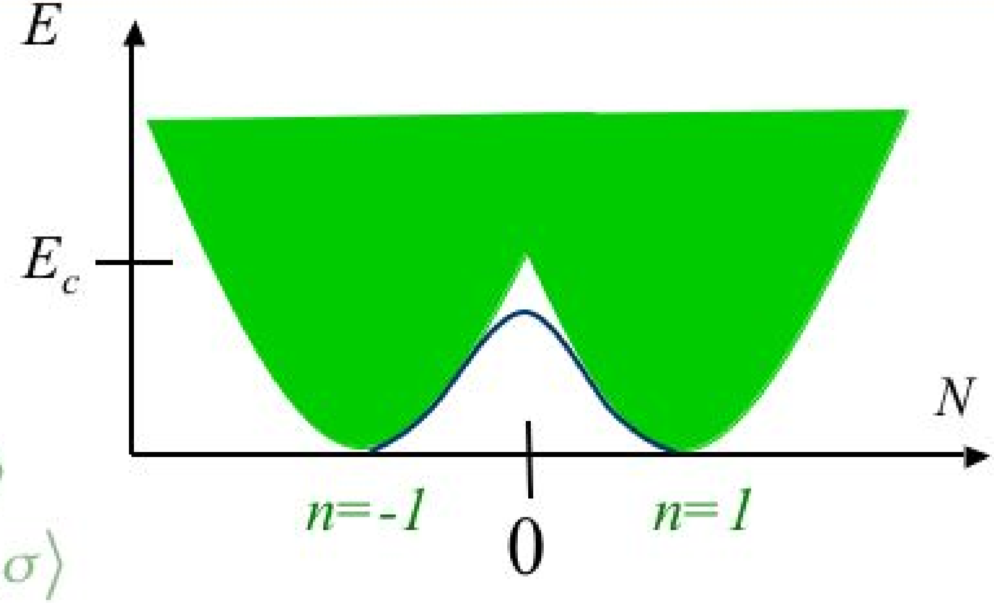
$$\Delta \exp\left\{-\frac{\pi(E_2 - E_0)(E_{-2} - E_0)}{32 E_J E_C}\right\}$$

- Analytic gate voltage dependence of the island charge
  - No singularities at the charge degeneracy point
  - Width of the transition region in dimensionless gate voltage is $\delta N \quad E_J / E_C$

$$\frac{n}{2-n} + \ln\frac{2}{2-n} = \frac{\pi N^*}{2} + \ln\frac{2\pi}{g}$$

Odd electron number

- Low temperature
- One intrinsic quasiparticle in the system
- No tunneling. Continuum of states:
  - n=1 + q.p in the left island $\left|k,\sigma;0\right\rangle$
  - n=-1 + q.p. in the right island $\left|0;p,\sigma\right\rangle$



- Near the degeneracy point between n=1 and n=-1 states

$$\left|\Psi\right\rangle = \sum_{k\sigma} C_{k\sigma}\left|k,\sigma;0\right\rangle + \sum_{p\sigma} C_{p\sigma}\left|0;p,\sigma\right\rangle$$

Single channel contact

- Continuum spectrum is unaffected by tunneling
- Ground state: intrinsic quasiparticle bound to the tunneling contact
- Binding energy
  - Rescaled gate voltage, $\tilde{N} = 2E_C N / E_J$
  - Binding energy, $\tilde{\varepsilon}_B = \varepsilon_B / E_J$
  - Temperature, $\tilde{T} = T / E_J$
- Island charge difference

$$E(N) = E_C(1+N^2) + \Delta - \sqrt{4E_C{}^2 N^2 + E_J{}^2}$$

$$\varepsilon_B = E_J\left(-|\tilde{N}| + \sqrt{\tilde{N}^2+1}\right)$$

$$n_0(\tilde{N}) = \frac{\tilde{N}}{\sqrt{\tilde{N}^2+1}} \qquad T = 0$$

$$\alpha = \frac{1}{2}\ln\frac{8\pi\Delta E_J \tilde{T}}{\delta^2}$$

$$n(\tilde{N}) = \frac{2\sinh\left(|\tilde{N}|/\tilde{T}\right)e^{-\frac{|\tilde{N}|}{\tilde{T}}} + n_0(\tilde{N})e^{-\frac{\tilde{\varepsilon}_B}{\tilde{T}}-\alpha}}{2\cosh\left(|\tilde{N}|/\tilde{T}\right)e^{-\frac{|\tilde{N}|}{\tilde{T}}} + e^{-\frac{\tilde{\varepsilon}_B}{\tilde{T}}-\alpha}}$$

Nonmonotonic temperature dependence of step width

- Width of the transition region at zero temperature:

$$\delta N_0 \quad E_J / E_C$$

- At the ionization $T_i \quad E_J / \ln \frac{\Delta E_J}{\delta^2}$ temperature of the bound state, the width is minimal,

$$\delta N_{\min} \quad T_i / E_C < \delta N_0$$

- At higher temperatures the width is

$$\delta N \quad T_i / E_C$$

- The temperature dependence of the step width is ***nonmonotonic***

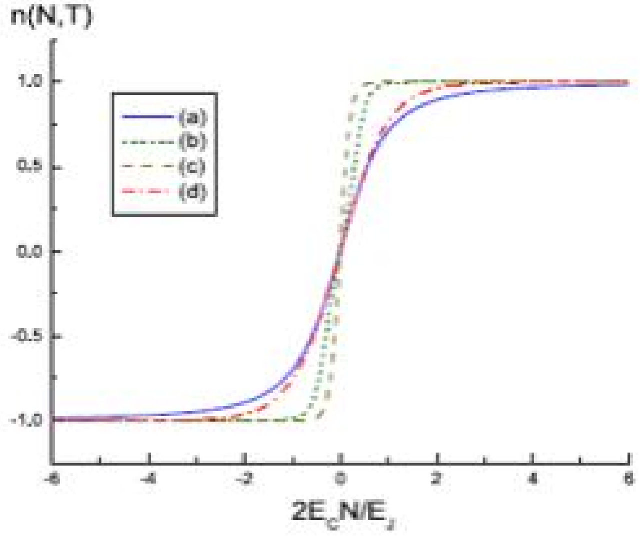


FIG. 2: The dependence of the island charge difference on the rescaled gate voltage $2E_C N/E_J$ for temperatures $T = 0$ (a), $T = 0.08E_J$ (b), $T = 0.2E_J$ (c), $T = E_J$ (d). The temperature dependence of the step width is clearly non-monotonic.

Multichannel contact

- N bound quasiparticle states for an N-channel contact
- Binding energy is approximately determined by the partial channel conductance $g_i\Delta$
- Corrections are small in $1/(k_F d)$ with d being the typical distance between the channels
- Bound states are unlikely to be observed in multichannel tunneling contacts but can be observed in break junctions with few conducting channels [1]

[1] R. Cron *et al.*, Phys. Rev. Lett. **86**, 4104 (2001)

SUMMARY

- Thermodynamic properties of double island structures differ significantly from those of a single island in contact with a bulk lead
- Even electron number
  - Strong Coulomb enhancement of the Josephson energy
  - Ground state is always separated by a finite gap from the continuum of excited states. For $2E_C - \Delta > \tilde{E}_J$ the two intrisic quasiparticles are bound to the contact
  - Gate voltage of the island charge is non-singular and is determined analytically
- Odd electron number
- Single channel contact
  - Intrinsic quasiparticle is bound to the contact in the ground state. Binding energy is of order of the Josephson energy.
  - Gate voltage and temperature dependence of the island charge is determined analytically
  - Due to the presence of the bound state the temperature dependence of the Coulomb blockade step width is nonmonotonic
- Experimental consequences
  - Predictions for the gate-voltage dependence of the island charge for the even electron number can be directly tested by sensitive electrometry techniques [1,2].
  - Bound state for the odd number of electrons may be observed in systems with break junctions

[1]R.J. Schoelkopf, *et al.*, Science, **280**, 1238 (1998)
[2] K.W. Lehnert, *et al.*, Phys. Rev. Lett. **91**, 106801 (1999)