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WORKSHOP ON

QUANTUM SYSTEMS OUT OF EQUILIBRIUM

(14 – 25 June 2004)

" Single-atom density of states and out-of-equilibrium effects in optical lattices"

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Single-atom density of states and out-of-equilibrium effects in optical lattices

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cond-mat/0312079

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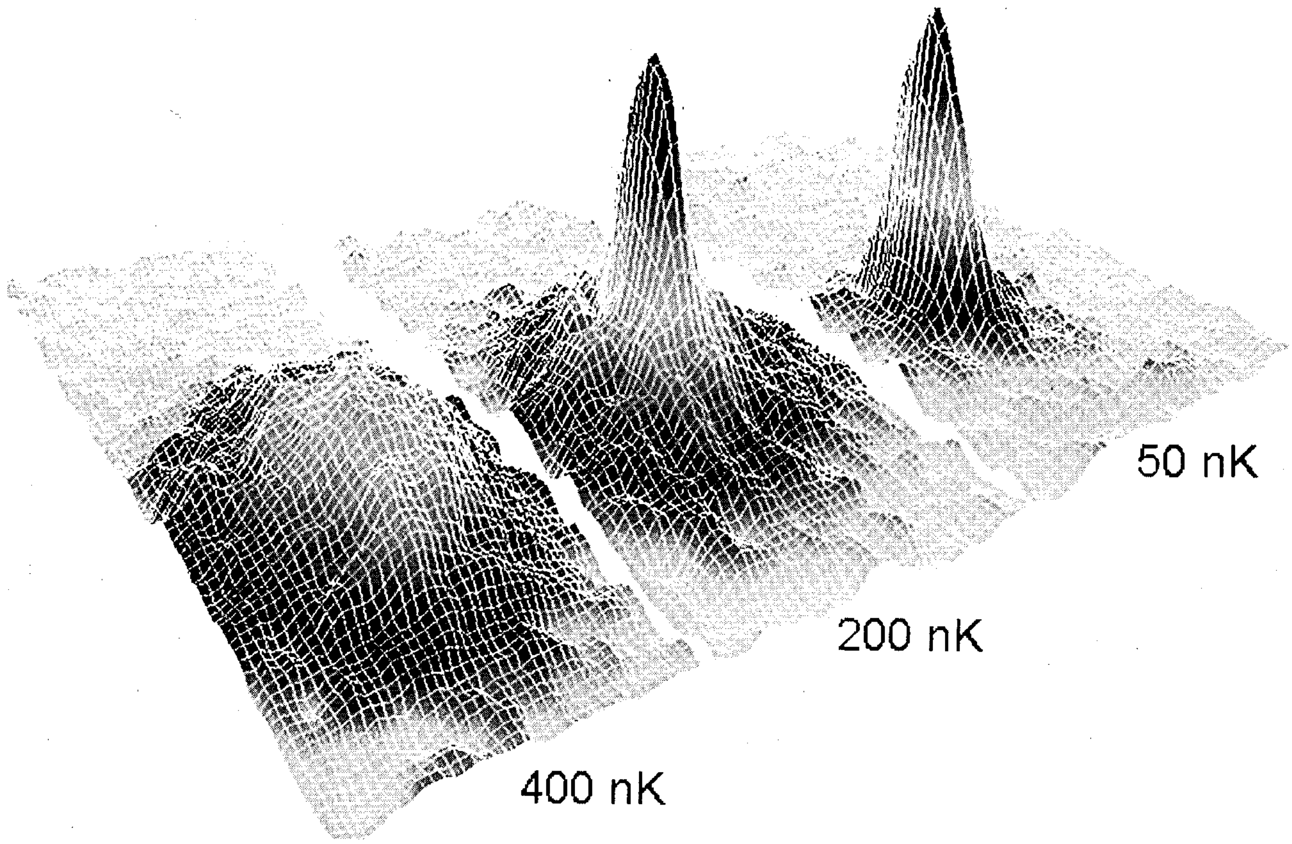
Outline

1. Introduction:
   a) Bose-Einstein condensation; atom traps.
   b) Optical lattices and "artificial crystals".
2. Single-atom density of states:
   a) Effective potential; tight-binding approximation.
   b) WKBJ-type approach.
   c) Results; comparison with numerics.
3. Experimental situation and outlook:
   a) Florence experiments (Inguscio et al.).
   b) Ordering and fingering.
   c) Conclusions.

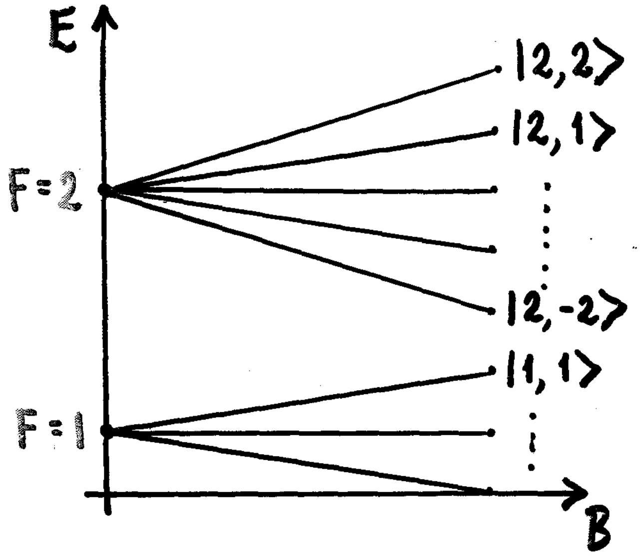
Bose-Einstein condensation



JILA group, 1995 (http://jilawww.colorado.edu/bec/)

How to build a (magnetic) atom-trap

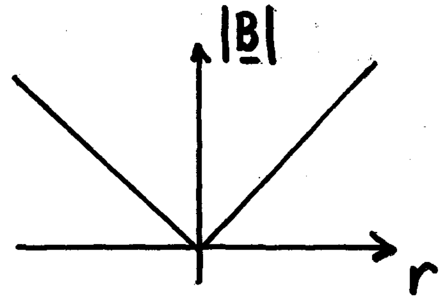
1. Hyperfine coupling: e.g. ^{87}Rb $\quad I = \frac{3}{2},\ S = \frac{1}{2} \Rightarrow F = 1, 2$



2. Spatially varying magnetic field,

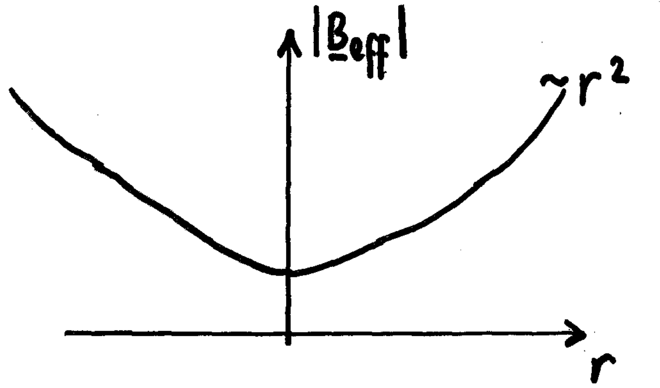
e.g. $\underline{B} = B_q\left(x\underline{i} + y\underline{j} - \eta z\underline{k}\right)$

(quadrupole)



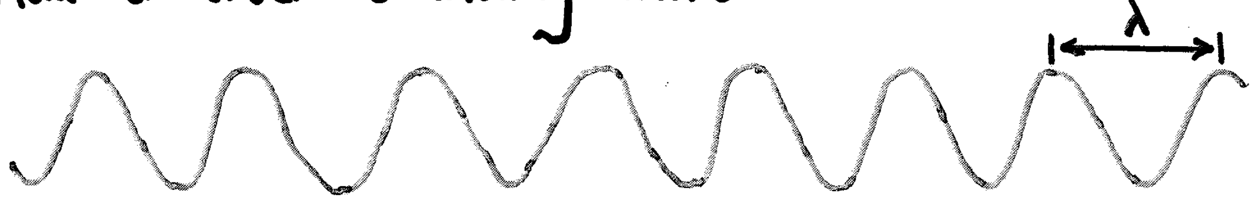
3. Time-varying trap centre: $\underline{B} \rightarrow \underline{B} + B_0\left(\cos(\omega_0 t)\underline{i} + \sin(\omega_0 t)\underline{j}\right)$

$$\left.\begin{aligned} \omega_{at.} &\sim 100\,\text{Hz} \\ \omega_0 &\sim 50\,\text{kHz} \\ \omega_L &\sim 10\,\text{MHz} \end{aligned}\right\} \Rightarrow$$



Optical lattices and "artificial crystals"

1. Add a laser standing wave:

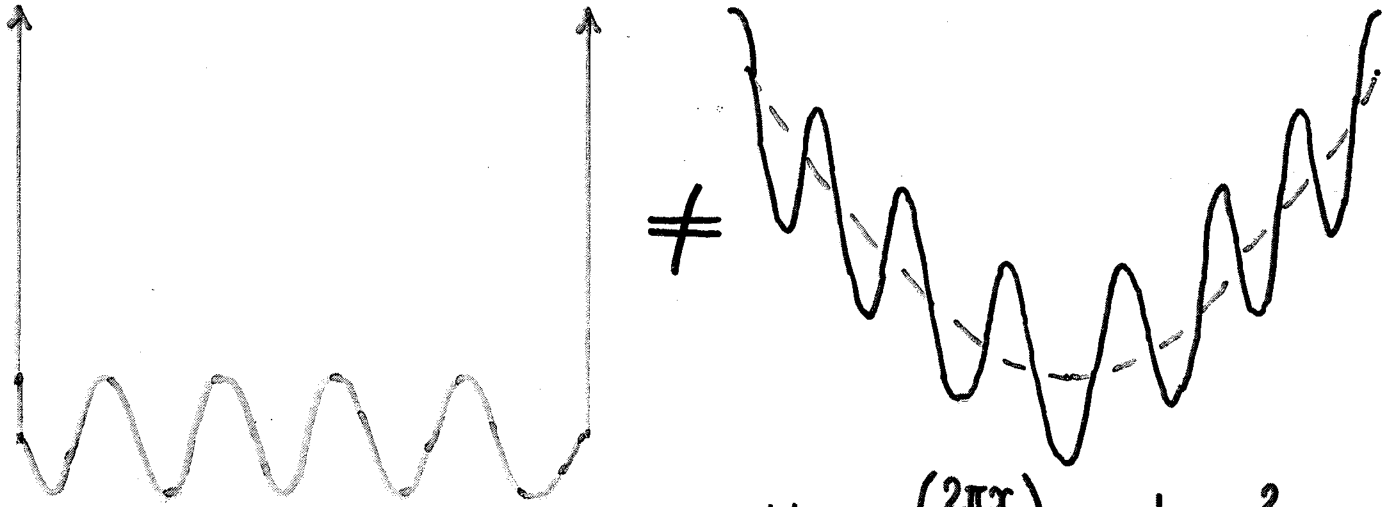


Atoms polarise ⇒ attracted to extrema of wave

$$\therefore\ V_{eff} = V_0 \cos\left(\frac{2\pi\, 2x}{\lambda}\right) \equiv \underbrace{V_0 \cos\left(\frac{2\pi x}{a}\right)}_{\text{optical lattice potential}}.$$

2. Much excitement: – perfect crystals;
   – Bose-Hubbard model;
   – SU(N) (spin as flavour); ....

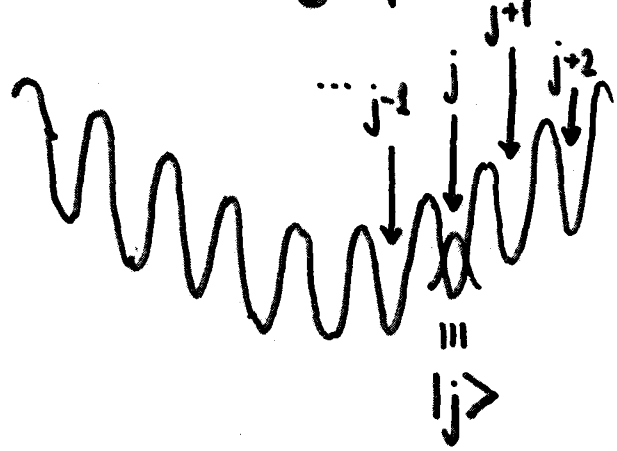
3. <u>BUT</u>



$$V_0 \cos\left(\frac{2\pi x}{a}\right) + \frac{1}{2}\kappa x^2$$

Tight-binding approximation; asymptotics

1. Strong-laser limit;
   use only ground state
   of each minimum:



$$\hat{H} = \underbrace{-t\sum_j \left(|j\rangle\langle j+1| + \text{H.c.}\right)}_{\text{hopping}} + \underbrace{\tfrac{1}{2}\kappa \sum_j (aj)^2 |j\rangle\langle j|}_{\text{harmonic trap}}$$

$\left(\varepsilon(k) = -2t\cos(ka)\right)$

2. Low energies, $\varepsilon \sim -2t \Rightarrow -2t\cos(ka) \approx -2t + ta^2k^2$
   $\Rightarrow$ continuum H.O. with

$$\rho(\varepsilon) \longrightarrow \text{const.} \quad \text{as} \quad \varepsilon \rightarrow -2t\,.$$

3. High energies, $\varepsilon \gg 2t \Rightarrow E \approx \frac{1}{2}\kappa x^2$
   $\Rightarrow$ localised wave functions with

$$\rho(\varepsilon) \sim \frac{1}{\sqrt{\varepsilon}} \quad \text{when} \quad \varepsilon \gg 2t\,.$$

Reminder: ordinary WKBJ

1. Asymptotic method $(\hbar \to 0)$:

$$\psi(x) \sim \exp\left(i \int^{x} k(x')\,dx'\right)$$

2. $k(x)$ describes classical orbits:

$$\frac{p^2}{2m} + V(x) = \underset{\substack{\uparrow \\ \text{constant}}}{E} \quad \Longrightarrow \quad k(x) = \frac{p}{\hbar} = \pm\frac{1}{\hbar}\sqrt{2m[E - V(x)]}$$

Note: turning points at $v = 0 \Rightarrow k = 0 \Rightarrow E = V(x)$

3. Quantisation: phase of ψ must be single-valued:

$$\oint k(x)\,dx = 2\pi\left(\underset{\substack{\uparrow \\ \text{quantum number of eigenstate}}}{n} + \overset{\text{Maslov index}}{\gamma}\right)$$

4. Density of states easy:

$$\rho(E) \equiv \frac{\partial n}{\partial E} = \boxed{\frac{1}{2\pi}\oint \frac{\partial k(x)}{\partial E}\,dx}$$

(Note: Maslov index not required.)

Our "WKBJ": a few changes

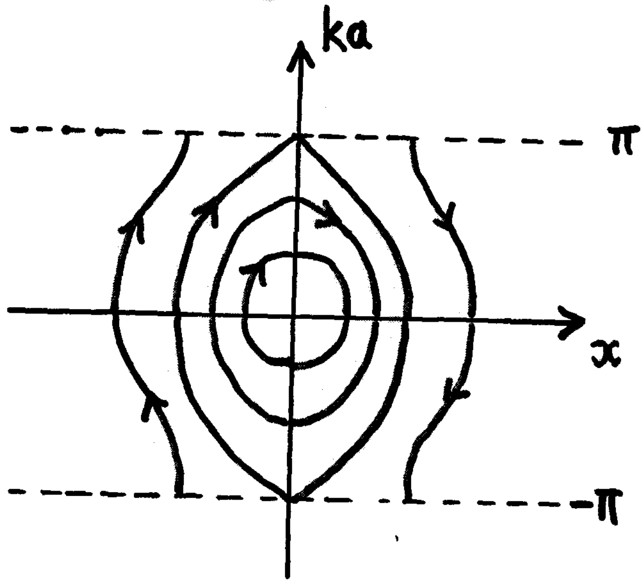
1. Orbits now given by:

$$-2t\cos(ka) + \tfrac{1}{2}\kappa x^2 = \varepsilon$$

$$\Downarrow$$

$$k(x) = \pm\frac{1}{a}\arccos\left(\frac{\kappa x^2 - 2\varepsilon}{4t}\right)$$

plays rôle of $\hbar$



2. Two types of turning point:

$$v = 0 \begin{cases} k = 0 & \Longrightarrow \quad \kappa x_c^2 = 2\varepsilon + 4t \quad \text{(classical)} \\ |ka| = \pi & \Longrightarrow \quad \kappa x_b^2 = 2\varepsilon - 4t \quad \text{(Bragg)} \end{cases}$$

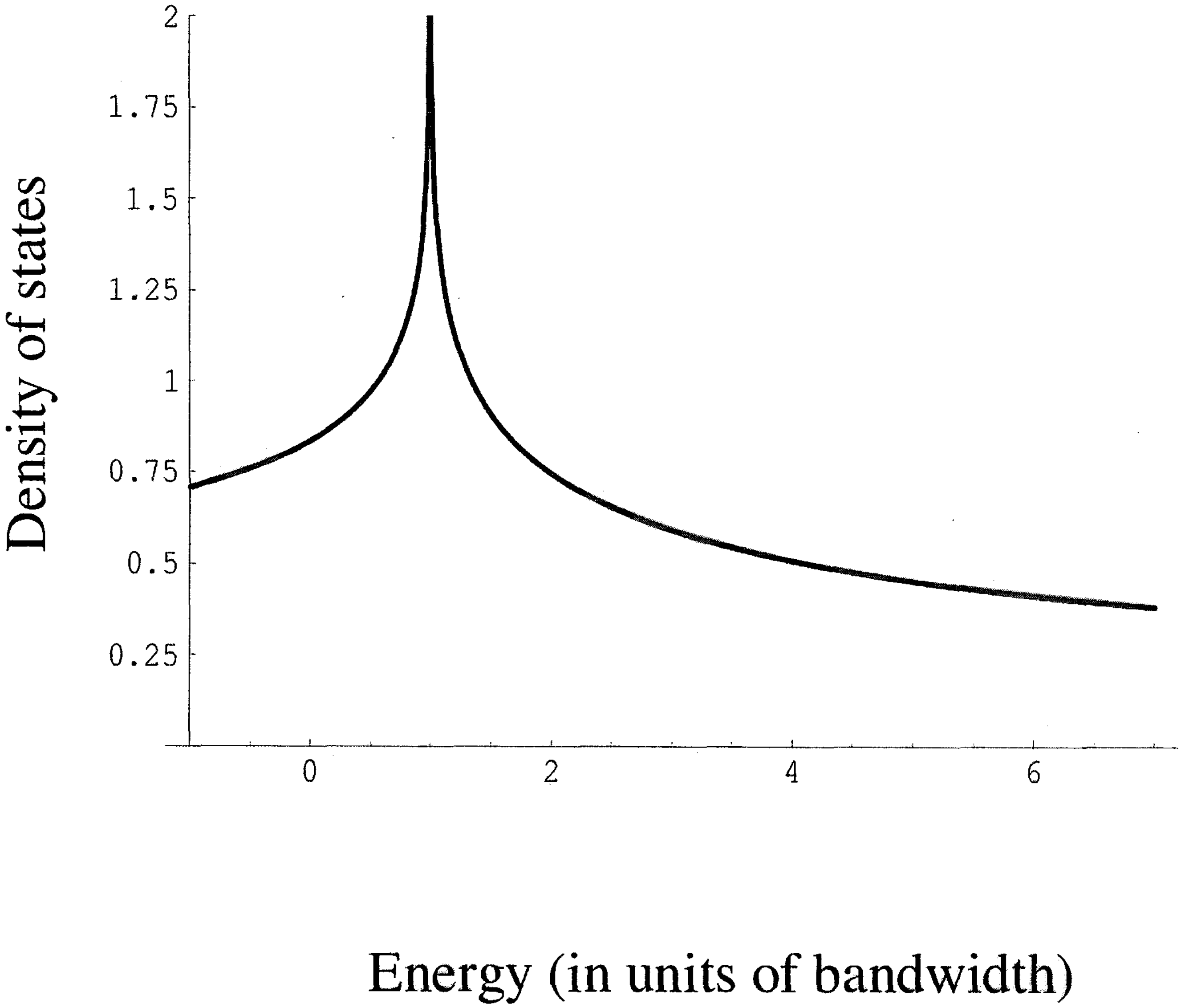
3. For $|x| < x_b$,

$$k(x) = \pi \pm \frac{i}{a}\operatorname{arccosh}\left(\frac{2\varepsilon - \kappa x^2}{4t}\right)$$

Wave function still oscillating in Bragg-forbidden region

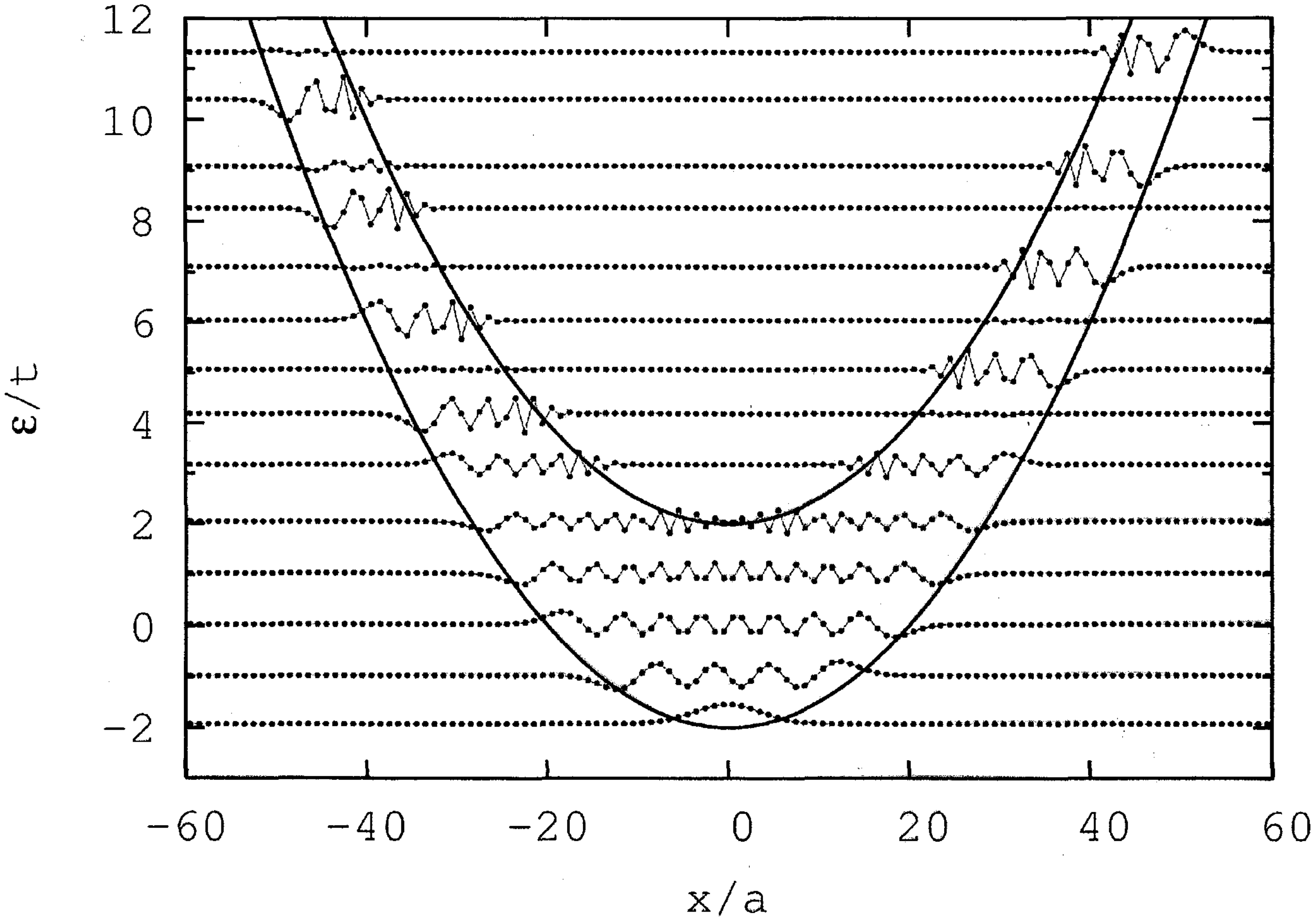
$\Rightarrow$ Must <u>include</u> this in quantisation condition.

One-dimensional density of states (analytical)



from C. H. and J. Quintanilla, cond-mat/0312079

Numerical wave functions



from C. H. and J. Quintanilla, cond-mat/0312079

The Florence experiment

1. Prepare ^{87}Rb $|2,2\rangle$; add strong optical lattice.

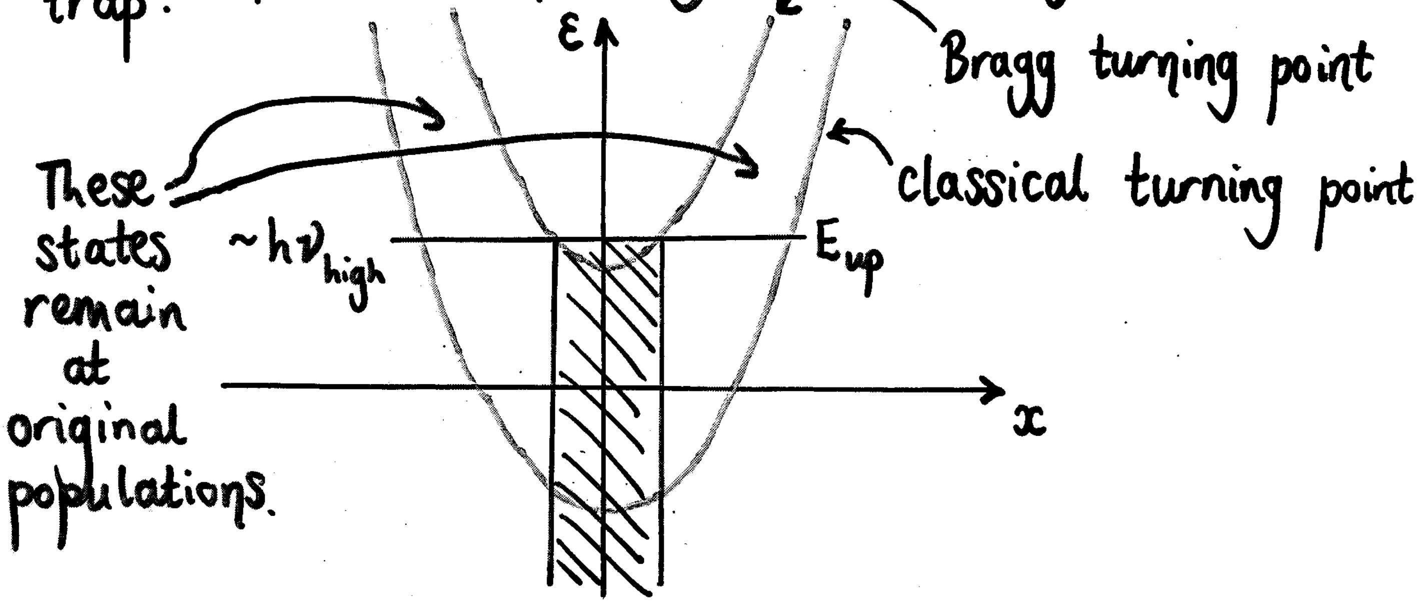
2. Apply high-power radio waves in frequency band

$$\nu_{low} \leq \nu \leq \nu_{high}.$$

These resonate with the $|2,2\rangle \longrightarrow |2,1\rangle$ transition when

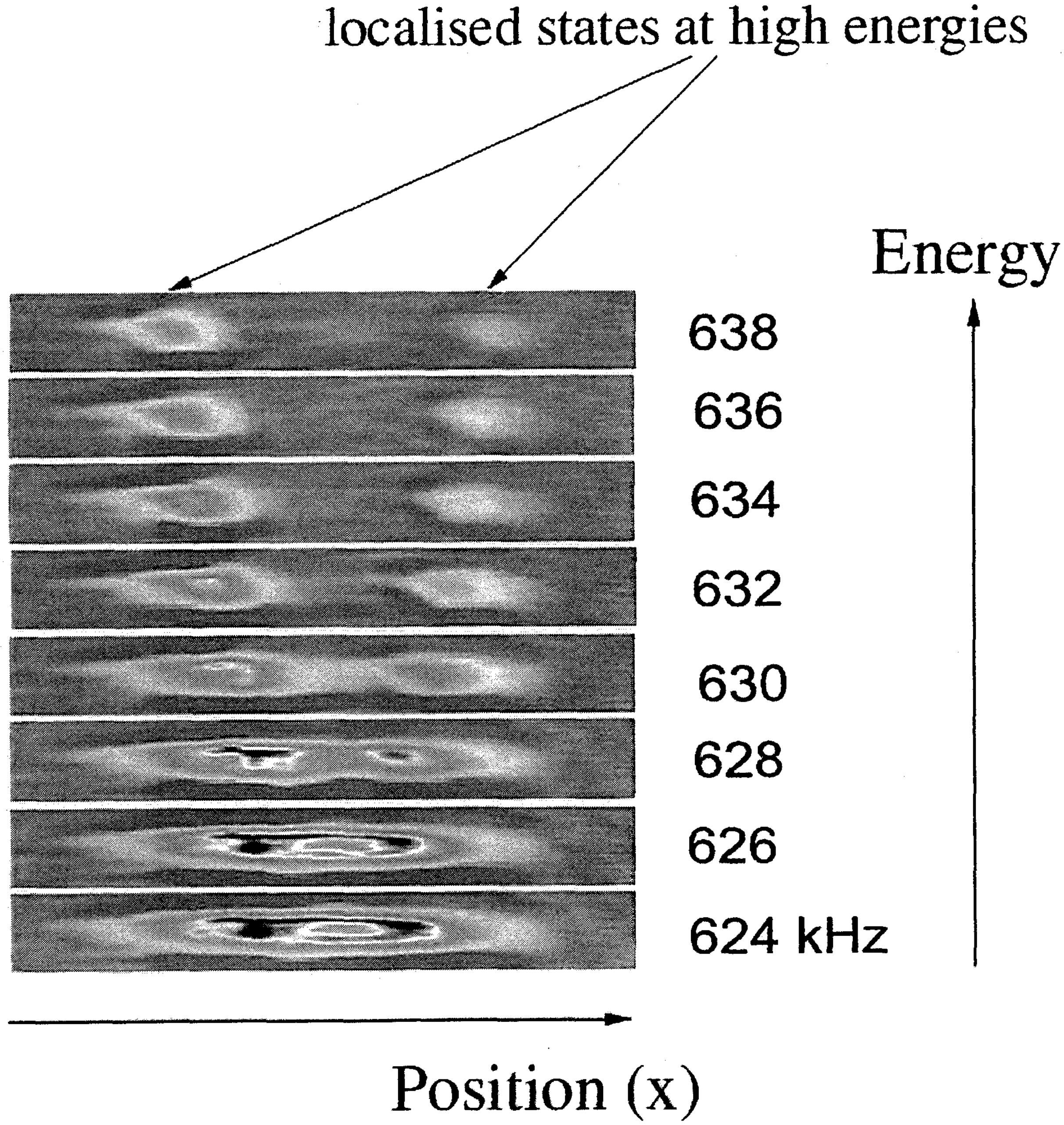
$$h\nu = \tfrac{1}{2}\mu_B B(\underline{r}).$$

3. This empties a spatially defined region of the trap:



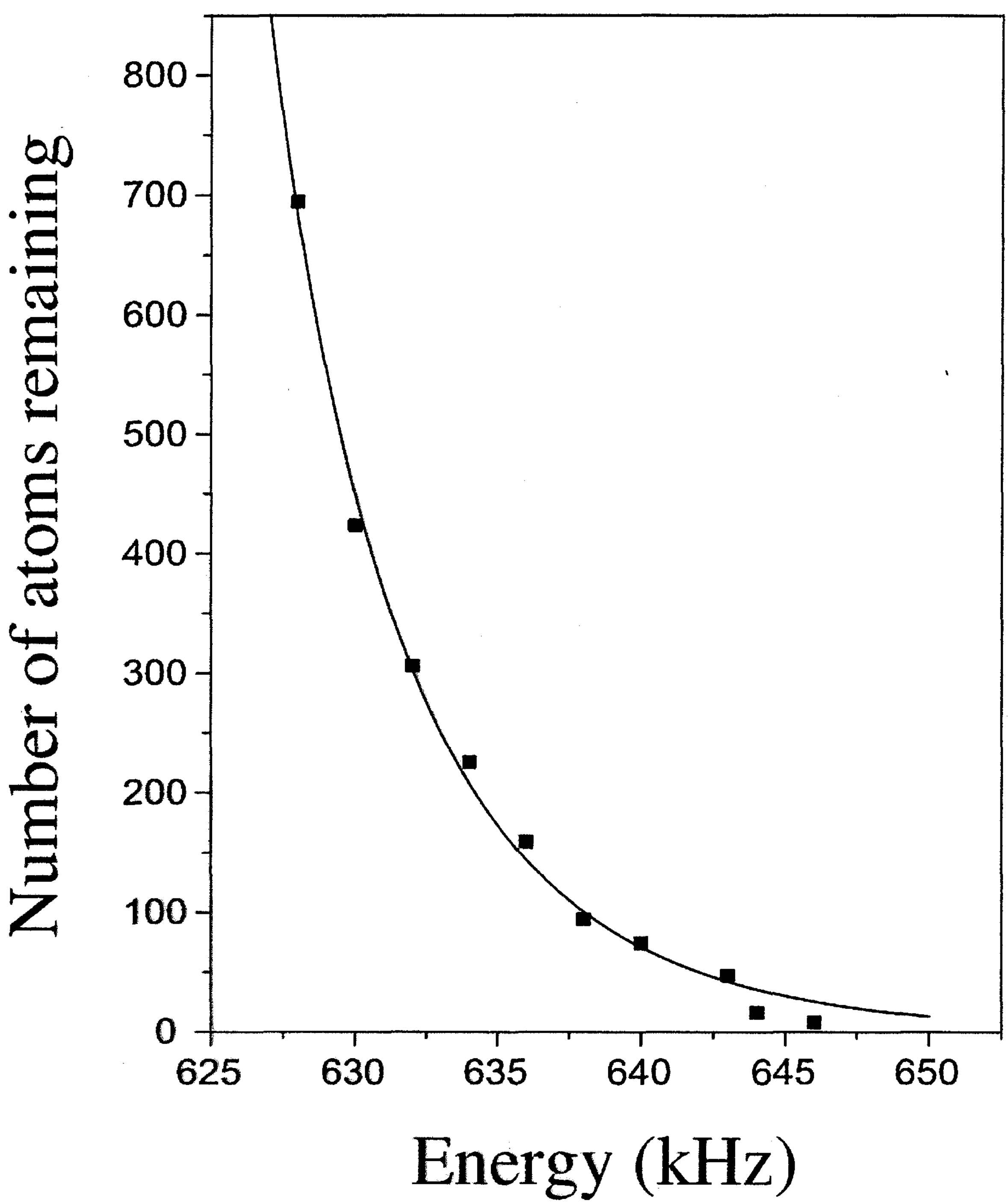
4. Expected remainder: $N \propto \int_{E_{up}}^{\infty} \rho(\varepsilon)\, n_B(\varepsilon)\, d\varepsilon$

Emptying the trap with radio waves



from H. Ott *et al.*, cond-mat/0404201

Measuring the density of states



from H. Ott *et al.*, cond-mat/0404201

Outlook

1. Ordering due to "van Hove" singularity.
2. Non-equilibrium growth behaviour.
3. Anti-bound states from non-adiabatic processes.

⋮

Conclusion

Q. Are optical lattices artificial realisations of crystalline physics?

A. No – they are much more interesting!